NAME .....

STUDENT ID NUMBER .....

DISCUSSION SECTION TIME .....

TEACHING ASSISTANT .....

SIGNATURE .....

Math 21C Winter 2018 Final, March 22.

To receive full credit you must show all of your work. Please do not use any cell phones, notes or books. A calculator is allowed. Do not simplify your solutions

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1. Determine whether the following series converge. Specify the convergence tests you use.

   (a)
   $$\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^2 + \sqrt{n}}$$

   (b)
   $$\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n}$$

2. For each of the following find an upper bound for the error resulting from estimating the infinite sum with just the first 5 terms.

(a)

$$\sum_{n=1}^{\infty} \frac{2n}{(1+n^2)^2}$$

(b)

$$\sum_{n=1}^{\infty} (-1)^n \frac{3}{2^{2n}}$$

3. Determine the values of x for which the following series converges. Be sure to check the end points of the interval.

$$\sum_{n=1}^{\infty} \sqrt{n} \frac{x^n}{5^n}$$

4. Find the first three nonzero terms of the Taylor series about $x = 0$ for the following function.

$$f(x) = \cos(2x) - x\sin(x)$$

5. Find the ground speed (magnitude of the velocity vector) of a fly if the wind is blowing the fly $4\frac{\text{mi}}{\text{hr}}$ northeast while the fly is flying $3\frac{\text{mi}}{\text{hr}}$ west.

6. Find an equation for the plane through the points $(1, 1, 1)$, $(1, 2, 3)$ and $(-1, 0, 3)$.

7. Consider the function $f(x, y) = 3\sqrt{4 - x^2 - y^2}$.

   (a) Find and sketch the domain of f.

   (b) Find and sketch the range of f.

   (c) Describe the surface $z = f(x, y)$.

8. Consider again the surface $z = 3\sqrt{4 - x^2 - y^2}$. Find a parametric equation for the line normal to the surface at the point with $x = y = 1$.

9. Laplace's equation for heat in a plate is satisfied by $f(x, y)$ if $f_{xx} + f_{yy} = 0$. Determine whether each of the following satisfy Laplace's equation.

   (a) $f(x, y) = e^{-2y} \cos(3x)$

   (b) $f(x, y) = \ln(x^2 + y^2)$

10. Find all the local maxima, local minima and saddle points of the function

$$f(x, y) = x^3 - y^3 - 2xy + 6.$$

11. Find the maximum value of the function $f(x, y, z) = x - 2y + 3z$ on the sphere $x^2 + y^2 + z^2 = 14$.

12. (Optional extra credit problem.) Evaluate the sum

$$\sum_{n=0}^{\infty} \frac{1}{(4n)!}.$$