Math 21C Practice Midterm II Answers Spring 2024

You may not use a calculator.
You may use one page of notes.
You may not use the textbook.
Please do not simplify answers.

1. (12 pts: Power Series)
   Determine the x values for which the following power series converges:
   $$\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n}(x-1)^n.$$

   See soln to 1 on practice exam 2.

2. (11 pts for each part: Taylor Polynomials)
   Find the first three nonzero terms for the following Taylor series associated to

   (a) $f(x) = \sqrt{x}$ about $x = 4$

   See soln to 2 on practice exam 2.

   (b) $f(x) = \cos(\sqrt{x})\sin(2x)$ about $x = 0$.

Hint: For this one you can use shortcuts and not compute any derivatives.

See soln to 3 on practice exam 2.

3. (11 pts: Taylor Remainder)
Estimate the error if the Maclauren polynomial $P_1(x) = x$ associated to $f(x) = \int_{t=0}^{x} e^{-\sin(t)} dt$ is used to make the estimate of $\frac{1}{3}$ for the integral $\int_{t=0}^{\frac{1}{3}} e^{-\sin(t)} dt$.

The quantity being estimated is $f(\frac{1}{3})$. The Taylor remainder theorem gives a bound of $M\frac{(\frac{1}{3})^2}{2!}$ for any M such that $M \geq |f^{(2)}(t)|$ with t between 0 and $\frac{1}{3}$. Here $f'(t) = e^{-\sin(t)}$ by the FTC so $f^{(2)}(t) = -\cos(t)e^{-\sin(t)}$ and hence I can choose $M = e > |f^{(2)}(t)|$ for every t value. This is true because $|\sin(t)|$ and $|\cos(t)|$ are both at most 1.
The estimate is thus $e\frac{(\frac{1}{3})^2}{2!}$.

4. (11 pts: Vectors)
Let $\mathbf{u} = \langle 1, -2 \rangle$ and $\mathbf{v} = \langle 3, 4 \rangle$.
Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.

See soln to 5 on practice exam 2.

5. (11 pts: Forces)
Consider a 100N weight suspended by two wires with slopes -1 and 2.
Find the magnitudes of the force vectors on the two wires.

The three force vectors must add to zero so $\langle 0,0\rangle = \langle 0,-100\rangle + A\langle -1,1\rangle + B\langle 1,2\rangle = \langle -A+B, A+2B-100\rangle$ so $A = B = \frac{100}{3}$ and the force vectors on the two wires are $\frac{100}{3}\langle -1,1\rangle$ and $\frac{100}{3}\langle 1,2\rangle$ which have magnitudes $\frac{100}{3}\sqrt{2}$ Newtons and $\frac{100}{3}\sqrt{5}$ Newtons respectively.

6. (11 pts: Lines)
Consider the following two intersecting lines: L_1 is given by $x = 1+t$, $y = 2t$ and $z = -1+3t$. L_2 is given by $x = 3+2s$, $y = 1+s$ and $z = -2-s$.

   (a) Find their point (x,y,z) of intersection.

   (b) Find the angle between the lines.

See soln to 7 on practice exam 2. 100pt

7. (11 pts: Planes)
Compute the distence from the origin $(0, 0, 0)$ to the plane $2x + y - 2z = 6$.

See soln to 8 on practice exam 2.

8. (11 pts: Functions)
Consider the function $f(x, y) = 4 - \sqrt{y - x^2}$.

(a) Determine and sketch the domain of f in the plane.

(b) Determine the range of f.

See soln to 5 on practice exam 3.

9. (10 pts: Extra Credit... you may skip this problem)