**Math 21C Practice Midterm III Spring 2024**

You may not use a calculator.
You may use one page of notes.
You may not use the textbook.
Please do not simplify answers.

1. (15 pts: Partial Derivatives)
   Consider $z = 3x + \ln(x^2 + y)$. Compute the partial derivatives:

   (a) $z_x$

   (b) $z_y$

   (c) $z_{xx}$

2. (21 pts: Differential)
   Consider the surface given by $z = xy + y^3 - x^2$ and the point $P(2, 1)$.

   (a) What is the maximum possible value of $D_{\vec{u}} f$ at $P$ and for this maximum in what direction is $\vec{u}$?

   (b) Find a point $Q$ so that the directional derivative at $P(2, 1)$ in direction $\overrightarrow{PQ}$ is zero.

   (c) Find an equation for the tangent plane to the surface at the point $(2, 1)$.

3. (14 pts: Extant Limit)
Evaluate the following limit:

$$\lim_{(x,y)\to(1,-1)} \frac{1-\sqrt{x+y+1}}{x+y}.$$

4. (14 pts: Nonexistant Limit)
Verify that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^4}.$$

5. (15 pts: Chain Rule)
Consider $z = f(x,y)$, $x = 3r + s^2$ and $y = rs$.
Use the chain rule to find the second-order partial derivative $\frac{\partial^2 z}{\partial s^2}$.
Express your answer in terms of $r$, $s$ and partial derivatives of $z$ with respect to $x$ and $y$.

6. (21 pts: Critical Points)
Consider the function $f(x, y) = x^3 - 3xy + y^3$.

   (a) Find all its local maxima.

   (b) Find all its local minima.

   (c) Find all its saddle points.

7. (10 pts: Extra Credit... you may skip this problem)