Math 108 Midterm Exam May 4, 2020- 3:10-4:00
To receive full credit you must show all of your work.

1. (10 pts: Logic)
   If P and Q are propositions consider the proposition
   $S = (P \wedge Q) \vee (\sim P \vee \ \sim Q)$.

   (a) Write the truth table for the proposition S.

   (b) Determine whether $P \implies S$ is true.

2. (8 pts: Contrapositive)
   Assume that n is a natural number.
   Prove by contraposition that if n is not prime then $n^2 \neq 49$.

3. (8 pts: Quantifiers)
   Which two of the following four are true? Explain your answer.
   In the universe of real numbers.

   (a) $(\forall x)(\exists y > x)(y^2 - 2y$ is positive.)

   (b) $\sim [(\forall x)(\exists y > x)(y^2 - 2y$ is positive.)]

   (c) $(\exists x)(\forall y < x)(y^2 - 2y$ is negative.)

   (d) $\sim [(\exists x)(\forall y < x)(y^2 - 2y$ is negative.)]

4. (8 pts: Venn)
   Prove that if A, B and C are sets then
   $C \subseteq A \cap B$ if and only if $C \subseteq A$ and $C \subseteq B$.

5. (8 pts: Induction)
   Prove that $(\forall n \in \mathbb{N})\,(\sum_{r=1}^{n}(2r) = n(n+1))$.

6. (8 pts: Power Set)
   Find sets A and B for which $\overline{\overline{(\mathcal{P}A) \cap (\mathcal{P}B)}} = 2$.
   (Here $\mathcal{P}A$ is the power set or set of subsets of A).