MAT 108 Homework 1 Solutions

Problems are from A Transition to Advanced Mathematics 8th edition by Smith, Eggen, and Andre.

Section 1.1 #1, 3dhj, 6, 7aeg

1. (a) Not a proposition. (has no truth value)

(b) Proposition. If P is the statement "π is a rational number", then we can write the proposition as $\sim\sim P$. The number π is irrational, so P is false and $\sim\sim P$ is false as well.

(c) Not a proposition (need to specify what x is)

(d) Not a proposition.

(e) Proposition. Let P be the same as in part b, Q be the statement "17 is prime", R be the statement "7¡13", and S be the statement "81 is a perfect square". The proposition can then be written as $(P \wedge Q) \vee (R \wedge S)$. Q, R and S are all true, so the proposition is true.

(f) Proposition. If P is the statement "2 is rational", Q be the statement "π is irrational" and R be the statement "2π is rational", we can write the proposition as $(P \wedge Q) \vee R$. Since P and Q are both true, the entire proposition is true.

(g) Proposition. Let P be the statement "5π is rational", Q be the statement "4.9 is rational", and R be the statement "There are exactly four primes less than 10". Then the proposition is $(P \wedge Q) \vee R$. Since R is true (2, 3, 5, and 7 are the four primes less than 10), the proposition is true.

(h) Proposition. If P is the statement "-3.7 is rational", Q is the statement "$3\pi < 10$" and R is the statement "$3\pi > 15$", then we can write the proposition as $P \wedge (Q \vee R)$. Since P is true and Q is true, the entire proposition is true.

(i) Proposition. Let P be the statement "39 is prime" and Q be the statement "64 is a power of 2". The proposition can then be written as $\sim (P \vee Q)$. Since P is true and Q is true, $P \vee Q$ is also true and the entire proposition is false.

(j) Not a proposition. (it's actually a paradox)

3.(d)(h)(j)

P	Q	$Q \vee \sim Q$	$P \wedge (Q \vee \sim Q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	T	F

P	Q	$\sim P \wedge \sim Q$
T	T	F
T	F	F
F	T	F
F	F	T

P	Q	R	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

6. Can create a truth table or argue using properties of propositions.

(a) Not equivalent.

(b) Not equivalent.

(c) Not equivalent.

(d) Not equivalent.

(e) Not equivalent.

(f) Equivalent.

7. (a) P is the statement "gold is a metal". The proposition is then $\sim\sim P$, which is true.

(e) P is the statement "51 divides 153"; Q is the statement "51 is prime", and R is the statement "51 is a divisor of 409". The proposition is then $P \wedge (\sim Q \vee \sim R)$, which is true.

(g) P is the statement "$-5 \in \mathbb{N}$"; Q is the statement "$13 \in \mathbb{N}$"; R is the statement "$4 \in \mathbb{Q}$". Then the proposition is $\sim (P \wedge Q) \wedge \sim R$. This is true.