MAT 108 Homework 9 Solutions

Problems are from A Transition to Advanced Mathematics 8th edition by Smith, Eggen, and Andre.

Section 1.8 #6cd, 10, 13, 21abg

6. Find $d = \gcd(a, b)$ and integers x and y such that $d = ax + by$.

(c) $a = 9, b = 30$
Solution: $d = \gcd(a, b) = 3$ and we can choose $x = -3, y = 1$.

(d) $a = 77, b = 4$.
Solution: $\gcd(a, b) = 1$ and we can choose $x = 1, y = -19$.

10. Let q be a natural number greater than 1 with the property that q divides a or q divides b whenever q divides ab. Prove that q is prime.

Solution: Let $q > 1$ be a natural number such that $q|ab$ implies $q|a$ or $q|b$ for natural numbers a and b. Suppose that some number $m \in \mathbb{N}$ divides q. Then $q = mn$ for some $n \in \mathbb{N}$. Therefore, $q|m$ or $q|n$. Without loss of generality, assume $q|m$. Then $q|m$ and $m|q$, so $q = m$. Therefore $n = 1$ and the only factors of q are 1 and q itself. Thus, q is prime.

13. Let a and b be nonzero integers that are relatively prime, and let c be an integer. Prove that the equation $ax + by = c$ has an integer solution.

Solution: Let $a, b \in \mathbb{Z}\backslash\{0\}$ with $\gcd(a, b) = 1$. From class, we know that $\gcd(a, b) = 1$ implies there exist $x, y \in \mathbb{Z}$ such that $ax + by = 1$. Multiplying both sides by $c \in \mathbb{Z}$ yields $axc + byc = c$. If we define $x' = cx$ and $y' = cy$, then we have produced the equation $ax' + by' = c$, where x' and y' are integers.

21. 'Grade' the following proofs:

(a) (see textbook for proof)
Solution: F. The proof fails to show that 1 is the greatest of all possible common divisors.

(b) (see textbook for proof)
Solution: A. Good proof by contradiction.

(g) (see textbook for proof)
Solution: A. Proof is good so long as we assume the result from 17f.