Math 108 Practice Midterm
To receive full credit you must show all of your work.

1. If P, Q and R are propositions then take S to be the proposition $S = (P \vee (\sim Q)) \wedge R$.
   (a) Write out the truth table for S.
   (b) Find truth values for P, Q and R so that the truth value of S differs from that for R.

2. If A, B and C are sets then take D to be the set $D = (A \cup B^c) \cap C$.
   (a) Sketch a Venn (circle) diagram for D.
   (b) Find an example of sets A, B and C for which D differs from C.

3. For each statement below decide which of the universes $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$ it is true in.
   (a) $(\forall a)(\forall b)5a + 3b = 11$.
   (b) $(\forall a)(\exists b)5a + 3b = 11$.
   (c) $(\exists a)(\forall b)5a + 3b \neq 11$.
   (d) $(\exists! a)(\exists b)5a + 3b = 11$.
   (e) $(\exists a)(\exists b)5a + 3b = 11$.
   (f) $(\exists a)(\exists b)6a + 3b = 11$.

4. Prove that if a and b are intgers then ab is even iff either a is even or b is even.

5. Every even natural number is less than its square.
   (a) Rewrite this sentence using quantifiers and logic notation.
   (b) Prove that the sentence is true.

6. Either prove or find a counterexample to the following statement:
   If B is a set and $\mathbb{A} = \{A_\alpha | \alpha \in \Delta\}$ is an indexed family of sets then
   $$B - (\cap_{\alpha \in \Delta} A_\alpha) = \cap_{\alpha \in \Delta} (B - A_\alpha).$$