Math 127BA Midterm Exam May 4, 2020- 9:00-9:50
To receive full credit you must show all of your work.

1. (8 pts: Derivative)
   Consider the following function:
   $$f(x) = \begin{cases} (e^x - 1)\sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}.$$
   (a) Show that $f(x)$ is continuous at 0.
   (b) Show that $f(x)$ is not differentiable at 0.

2. Consider the following sequence of differentiable functions on $\mathbb{R}$:
   $$f_n(x) = \begin{cases} \frac{-1}{n} & x < \frac{-\pi}{2n} \\ \frac{sin(nx)}{n} & |x| \leq \frac{\pi}{2n} \\ \frac{1}{n} & x > \frac{\pi}{2n} \end{cases}.$$
   (a) Find the pointwise limit of the sequence $(f_n(x))$.
   (b) Find the pointwise limit of the derivative sequence $(f'_n(x))$.
   (c) Determine which one (a or b) converges uniformly and which only pointwise.

3. (8 pts: Mean Value)
   Consider a power series $F(x) = \sum_{k=0}^{\infty} a_k x^k$.
   Assume that $a_0 = 0$ and
   $$\lim_{x \to 1^-} F(x) = \sum_{k=1}^{\infty} a_k = 1.$$
   Show that there is some c with $F'(c) = 1$.

4. (8 pts: Taylor Lagrange)
   Find a number ϵ so that if $|x| < \epsilon$ then $|\cos(x) - 1 + \frac{x^2}{2}| < \frac{x^2}{100}$.

5. (8 pts: M-Test)
   Find the radius of convergence for the power series
   $$\sum_{k=0}^{\infty} \frac{3^n x^{2n}}{(4n)! n^5}.$$

6. (8 pts: Smooth)
   Find a sequence a_k so that if $F(x) = \sum_{k=0}^{\infty} a_k x^k$ is the associated power series then
   $$\lim_{n \to \infty} F^{(n)}(0) = 3.$$
   Be sure to justify the fact that $F^{(n)}(0)$ exists.