Math 127BB Midterm Exam May 4, 2020- 1:10-2:00
To receive full credit you must show all of your work.

1. (8 pts: Derivative)
   Consider the following function:

$$f(x) = \begin{cases} \sin(x)\sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

   (a) Show that $f(x)$ is continuous at 0.

   (b) Show that $f(x)$ is not differentiable at 0.

2. (10 pts: Convergence)
   Consider the following sequence of differentiable functions on $\mathbb{R}$:

$$f_n(x) = \begin{cases} -2(n^{-1}) & x < -n^{-1} \\ 3x - n^2x^3 & |x| \leq n^{-1} \\ 2(n^{-1}) & x > n^{-1} \end{cases}.$$

   (a) Find the pointwise limit of the sequence $(f_n(x))$.

   (b) Find the pointwise limit of the derivative sequence $(f_n'(x))$.

   (c) Determine which one (a or b) converges uniformly and which only pointwise.

3. (8 pts: Mean Value)
   Consider a power series $F(x) = \sum_{k=0}^{\infty} a_k x^k$.
   Assume that $a_0 = 0$, $\sum_{k=1}^{\infty} a_k = 1$ and $\sum_{k=1}^{\infty} 2^k a_k = 100$.

   Show that there is some c with $F'(c) = 1$.

4. (8 pts: Taylor Lagrange)
   Find a number ϵ so that if $|x| < \epsilon$ then $|\sin(2x) - 2x| < \frac{|x|}{100}$.

5. (8 pts: M-Test)
   Find the radius of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{n!x^{2n}}{3^n n^5}.$$

6. (8 pts: Smooth)
   Find a sequence a_k so that if $F(x) = \sum_{k=0}^{\infty} a_k x^k$ is the associated power series then for every $n > 0$ we have

$$2n > F^{(n)}(0) > n.$$

   Be sure to justify the fact that $F^{(n)}(0)$ exists.