Math 21B Midterm I Spring 2025: Wed April 16 3:10-4:00

You may use one page of notes but not a calculator or textbook.
Please do not simplify your answers.

Name:

ID:

Basic and Trigonometric Integrals

$\int x^n dx$	$=$	$\frac{1}{n+1}x^{n+1} + C$
$\int x^{-1} dx$	$=$	$\ln \lvert x \rvert + C$
$\int e^x dx$	$=$	$e^x + C$
$\int \sin(x) dx$	$=$	$-\cos(x) + C$
$\int \cos(x) dx$	$=$	$\sin(x) + C$
$\int \frac{dx}{\sqrt{1-x^2}} dx$	$=$	$\arcsin(x) + C = -\arccos(x) + C$
$\int \sec^2(x) dx$	$=$	$\tan(x) + C$
$\int \tan(x) dx$	$=$	$\ln \lvert \sec(x) \rvert + C$
$\int \csc^2(x) dx$	$=$	$-\cot(x) + C$
$\int \cot(x) dx$	$=$	$\ln \lvert \sin(x) \rvert + C$
$\int \frac{dx}{1+x^2} dx$	$=$	$\arctan(x) + C = -\text{arccot}(x) + C$
$\int \sec(x)\tan(x) dx$	$=$	$\sec(x) + C$
$\int \sec(x) dx$	$=$	$\ln \lvert \sec(x) + \tan(x) \rvert + C$
$\int \csc(x)\cot(x) dx$	$=$	$-\csc(x) + C$
$\int \csc(x) dx$	$=$	$-\ln \lvert \csc(x) + \cot(x) \rvert + C$
$\int \frac{dx}{\lvert x \rvert \sqrt{x^2-1}} dx$	$=$	$\text{arcsec}(x) + C = -\text{arccsc}(x) + C$

1. (12 points: Summation)
   Find the number:

$$\sum_{k=4}^{19}(k-3).$$

   You may use the fact that $\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$.

2. (11 points: Estimate)
Use four equal intervals and either the Left or Right End point rule to estimate the definite integral

$$\int_{10}^{30} f(x)dx$$

if the following values are known

x	0	5	10	15	20	25	30	35	40
$f(x)$	1.9	1.7	1.6	1.5	1.3	1.2	1.0	0.9	0.6

(Pay attention to the bounds of integration.)

3. (22 points: Area)
Find the area of the region bounded by the curves

$$y = x^2$$

and

$$y = \sqrt{x}.$$

This region lies between the x values: $x = 0$ and $x = 1$.

4. (33 points: FTCI vs FTCII)

   (a) Evaluate the derivative:

$$\frac{d}{dx}\int_1^x \left[\frac{\sqrt{t}+1}{\sqrt{t}}\right] dt$$

   at $x = 4$.

(b) Find the antiderivative with constant of integration:

$$\int \left[\frac{\sqrt{x}+1}{\sqrt{x}} \right] dx.$$

(c) Find the number:

$$\int_1^4 \left[\frac{\sqrt{x}+1}{\sqrt{x}}\right] dx.$$

5. (22 points: Substitution)

(a) Find the antiderivative with constant of integration:

$$\int \left[x \cos(x^2) \sin(x^2)\right] dx.$$

(b) Find the number:

$$\int_0^{\sqrt{\pi}} \left[x \cos(x^2) \sin(x^2)\right] \, dx.$$

6. (10 points: Extra Credit... you may skip this problem)
   Show that if $x \geq 1$ then:

$$\int_0^x e^{-t^2} dt \leq \frac{x+1}{2}.$$

   This is an improvement on the max-min inequality.