Math 21B Practice Midterm I Spring 2025

You may use one page of notes but not a calculator or textbook.
Please do not simplify your answers.

1. (8 points: Summation)
   Find $\sum_{k=2}^{6}(2k+3)$.

2. (18 points: Estimate)
   A car in an amusement park ride runs for twelve seconds along a straight track. The velocity of the car is recorded every three seconds and listed in this table:

time in seconds	0	3	6	9	12
velocity in feet per second	3	5	6	6	5

   Estimate the distance that the car travels during these twelve seconds in two ways.

   (a) (L_4): Use four equal intervals and the Left End rule.

   (b) Use the average of the three estimates:

      i. (L_4): Using four equal intervals and the Left End rule,

      ii. (R_4): using four equal intervals and the Right End rule and

      iii. (M_2): using two equal intervals and the Midpoint rule.

3. (18 points: FTC I)

   (a) Write a definite integral for the function $\text{erf}(x)$ which is the area under the curve $\frac{2}{\sqrt{\pi}}e^{-t^2}$ between $t=0$ and $t=x$.
   (This function is popular in statistics).

   (b) Find $\frac{d}{dx}[\text{erf}(x^2)]|_{x=2}$.

4. (16 points: Indefinite Integration) Find the following as functions with a constant of integration:

   (a) $\int (x^2 + \sqrt{x})dx$.

   (b) $\int \cos^2(2x)\sin(2x)dx$.

5. (40 points: Definite Integration) Compute the following numbers:

   (a) $\int_0^1 \left(x^3\sqrt{1-x^2}\right) dx$.

   (b) $\int_0^1 (1-x)\left(2x-x^2\right)^9 dx$.

   (c) $\int_0^1 \left(\sqrt{1-x^2}\right) dx$.

   (d) $\int_4^5 f(x)dx$ if:

      i. The average value of the function f over the interval $[0, 5]$ is 3.

      ii. $\int_0^4 [2f(x) + x]dx = 10$.

6. (10 points: Extra Credit... you may skip this problem)
   Show that if $f(x)$ is a degree two (quadratic) polynomial on an interval $[0, 12]$ then the approximation in problem 2b (the average of R_4, L_4 and M_2) is exactly the integral $\int_0^{12} f(x)dx$.
   (This would true in problem 2 if the ride had constant jerk).