USING TECHNOLOGY **Graphing Polar Curves Parametrically**

For complicated polar curves we may need to use a graphing calculator or computer to graph the curve. If the device does not plot polar graphs directly, we can convert $r = f(\theta)$ into parametric form using the equations

$$x = r\cos\theta = f(\theta)\cos\theta, \qquad y = r\sin\theta = f(\theta)\sin\theta.$$

Then we use the device to draw a parametrized curve in the Cartesian xy-plane. It may be required to use the parameter t rather than θ for the graphing device.

Graphs

:urves in Exercises 1–12. Then sketch

2. $r = 2 - 2\cos\theta$
4. $r = 1 + \sin\theta$
6. $r = 1 + 2\sin\theta$
8. $r = \cos(\theta/2)$
10. $r^2 = \sin\theta$
12. $r^2 = -\cos\theta$

ises 13–16. What symmetries do these

14. $r^2 = 4\sin 2\theta$
16. $r^2 = -\cos 2\theta$

ı Exercises 17–20 at the given points.
eir tangents at these points.

$\theta;\quad \theta = \pm\pi/2$

$\theta;\quad \theta = 0, \pi$

ı $2\theta;\quad \theta = \pm\pi/4, \pm 3\pi/4$

s $2\theta;\quad \theta = 0, \pm\pi/2, \pi$

es 21–24. Limaçon ("*lee*-ma-sahn") is
l understand the name when you graph
Equations for limaçons have the form
$\sin\theta$. There are four basic shapes.

oop

b. $r = \frac{1}{2} + \sin\theta$

b. $r = -1 + \sin\theta$

23. **Dimpled limaçons**
    a. $r = \frac{3}{2} + \cos\theta$ b. $r = \frac{3}{2} - \sin\theta$
24. **Oval limaçons**
    a. $r = 2 + \cos\theta$ b. $r = -2 + \sin\theta$

Graphing Polar Inequalities

25. Sketch the region defined by the inequalities $-1 \le r \le 2$ and $-\pi/2 \le \theta \le \pi/2$.
26. Sketch the region defined by the inequalities $0 \le r \le 2\sec\theta$ and $-\pi/4 \le \theta \le \pi/4$.

In Exercises 27 and 28, sketch the region defined by the inequality.

27. $0 \le r \le 2 - 2\cos\theta$ 28. $0 \le r^2 \le \cos\theta$

Intersections

29. Show that the point $(2, 3\pi/4)$ lies on the curve $r = 2\sin 2\theta$.
30. Show that $(1/2, 3\pi/2)$ lies on the curve $r = -\sin(\theta/3)$.

Find the points of intersection of the pairs of curves in Exercises 31–38.

31. $r = 1 + \cos\theta, \quad r = 1 - \cos\theta$
32. $r = 1 + \sin\theta, \quad r = 1 - \sin\theta$
33. $r = 2\sin\theta, \quad r = 2\sin 2\theta$
34. $r = \cos\theta, \quad r = 1 - \cos\theta$
35. $r = \sqrt{2}, \quad r^2 = 4\sin\theta$
36. $r^2 = \sqrt{2}\sin\theta, \quad r^2 = \sqrt{2}\cos\theta$
37. $r = 1, \quad r^2 = 2\sin 2\theta$
38. $r^2 = \sqrt{2}\cos 2\theta, \quad r^2 = \sqrt{2}\sin 2\theta$

T Find the points of intersection of the pairs of curves in Exercises 39–42.

39. $r^2 = \sin 2\theta, \quad r^2 = \cos 2\theta$
40. $r = 1 + \cos\frac{\theta}{2}, \quad r = 1 - \sin\frac{\theta}{2}$
41. $r = 1, \quad r = 2\sin 2\theta$ 42. $r = 1, \quad r^2 = 2\sin 2\theta$

T Grapher Explorations

43. Which of the following has the same graph as $r = 1 - \cos\theta$?
    a. $r = -1 - \cos\theta$ b. $r = 1 + \cos\theta$
    Confirm your answer with algebra.
44. Which of the following has the same graph as $r = \cos 2\theta$?
    a. $r = -\sin(2\theta + \pi/2)$ b. $r = -\cos(\theta/2)$
    Confirm your answer with algebra.
45. **A rose within a rose** Graph the equation $r = 1 - 2\sin 3\theta$.
46. **The nephroid of Freeth** Graph the nephroid of Freeth:
    $$r = 1 + 2\sin\frac{\theta}{2}.$$
47. **Roses** Graph the roses $r = \cos m\theta$ for $m = 1/3, 2, 3$, and 7.
48. **Spirals** Polar coordinates are just the thing for defining spirals. Graph the following spirals.
    a. $r = \theta$ b. $r = -\theta$
    c. *A logarithmic spiral:* $r = e^{\theta/10}$
    d. *A hyperbolic spiral:* $r = 8/\theta$
    e. *An equilateral hyperbola:* $r = \pm 10/\sqrt{\theta}$
    (Use different colors for the two branches.)

Theory and Examples

49. (*Continuation of Example 5.*) The simultaneous solution of the equations

$$r^2 = 4\cos\theta \qquad (1)$$
$$r = 1 - \cos\theta \qquad (2)$$

in the text did not reveal the points $(0, 0)$ and $(2, \pi)$ in which their graphs intersected.

a. We could have found the point $(2, \pi)$, however, by replacing the (r, θ) in Equation (1) by the equivalent $(-r, \theta + \pi)$ to obtain

$$r^2 = 4\cos\theta$$
$$(-r)^2 = 4\cos(\theta + \pi) \qquad (3)$$
$$r^2 = -4\cos\theta.$$

Solve Equations (2) and (3) simultaneously to show that $(2, \pi)$ is a common solution. (This will still not reveal that the graphs intersect at $(0, 0)$.)

b. The origin is still a special case. (It often is.) Here is one way to handle it: Set $r = 0$ in Equations (1) and (2) and solve each equation for a corresponding value of θ. Since $(0, \theta)$ is the origin for *any* θ, this will show that both curves pass through the origin even if they do so for different θ-values.

50. If a curve has any two of the symmetries listed at the beginning of the section, can anything be said about its having or not having the third symmetry? Give reasons for your answer.

*51. Find the maximum width of the petal of the four-leaved rose $r = \cos 2\theta$, which lies along the x-axis.

*52. Find the maximum height above the x-axis of the cardioid $r = 2(1 + \cos\theta)$.

10.7 Areas and Lengths in Polar Coordinates

This section shows how to calculate areas of plane regions, lengths of curves, and areas of surfaces of revolution in polar coordinates.

Area in the Plane

The region OTS in Figure 10.48 is bounded by the rays $\theta = \alpha$ and $\theta = \beta$ and the curve $r = f(\theta)$. We approximate the region with n nonoverlapping fan-shaped circular sectors based on a partition P of angle TOS. The typical sector has radius $r_k = f(\theta_k)$ and central angle of radian measure $\Delta\theta_k$. Its area is $\Delta\theta_k/2\pi$ times the area of a circle of radius r_k, or

$$A_k = \frac{1}{2}r_k^2\,\Delta\theta_k = \frac{1}{2}\left(f(\theta_k)\right)^2\Delta\theta_k.$$

The area of region OTS is approximately

$$\sum_{k=1}^{n} A_k = \sum_{k=1}^{n}\frac{1}{2}\left(f(\theta_k)\right)^2\Delta\theta_k.$$