Answers to Odd-Numbered Exercises

Chapter 1

Section 1.1, pp. 11–13

1. $D: (-\infty, \infty)$, $R: [1, \infty)$ **3.** $D: [-2, \infty)$, $R: [0, \infty)$

5. $D: (-\infty, 3) \cup (3, \infty)$, $R: (-\infty, 0) \cup (0, \infty)$

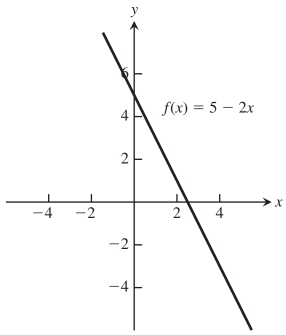
7. **(a)** Not a function of x because some values of x have two values of y

(b) A function of x because for every x there is only one possible y

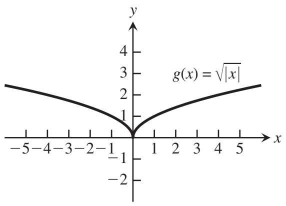
9. $A = \dfrac{\sqrt{3}}{4}x^2$, $p = 3x$ **11.** $x = \dfrac{d}{\sqrt{3}}$, $A = 2d^2$, $V = \dfrac{d^3}{3\sqrt{3}}$

13. $L = \dfrac{\sqrt{20x^2 - 20x + 25}}{4}$

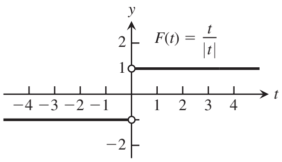
15. $(-\infty, \infty)$



17. $(-\infty, \infty)$

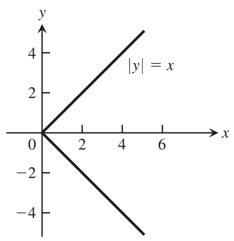


19. $(-\infty, 0) \cup (0, \infty)$

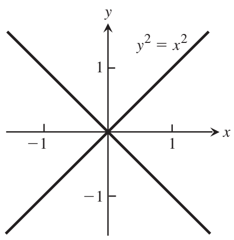


21. $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$

23. **(a)** For each positive value of x, there are two values of y.



(b) For each value of $x \neq 0$, there are two values of y.



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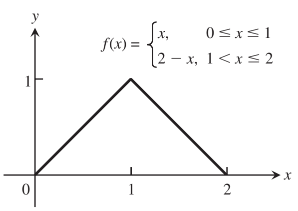

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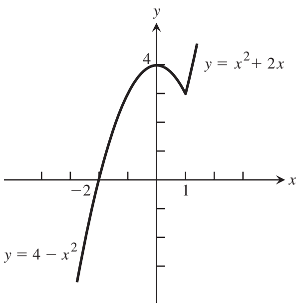


29. **(a)** $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x + 2, & 1 < x \le 2 \end{cases}$

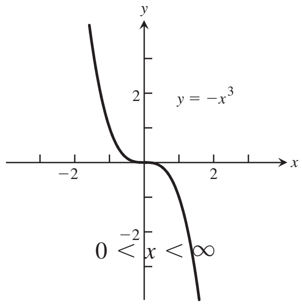
(b) $f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$

31. **(a)** $f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1, & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2}, & 1 < x < 3 \end{cases}$

(b) $f(x) = \begin{cases} \frac{1}{2}x, & -2 \le x \le 0 \\ -2x + 2, & 0 < x \le 1 \\ -1, & 1 < x \le 3 \end{cases}$

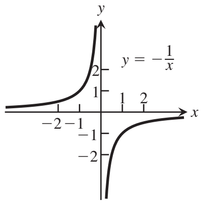
33. **(a)** $0 \le x < 1$ **(b)** $-1 < x \le 0$ **35.** Yes

37. Symmetric about the origin



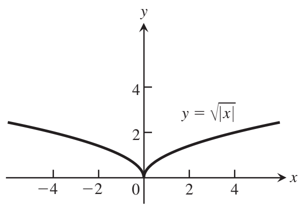
Dec. $-\infty < x < \infty$

39. Symmetric about the origin



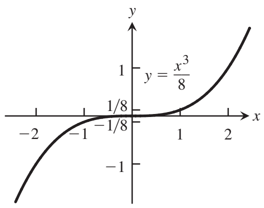
Inc. $-\infty < x < 0$ and $0 < x < \infty$

41. Symmetric about the y-axis



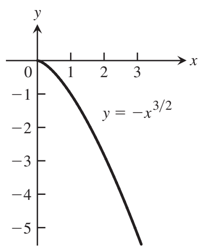
Dec. $-\infty < x \le 0$;
Inc. $0 \le x < \infty$

43. Symmetric about the origin



Inc. $-\infty < x < \infty$

45. No symmetry



Dec. $0 \le x < \infty$

47. Even **49.** Even **51.** Odd **53.** Even

55. Neither **57.** Neither **59.** $t = 180$ **61.** $s = 2.4$

63. $V = x(14 - 2x)(22 - 2x)$

65. **(a)** h **(b)** f **(c)** g **67.** **(a)** $(-2, 0) \cup (4, \infty)$

71. $C = 5(2 + \sqrt{2})h$

Section 1.2, pp. 18–21

1. $D_f: -\infty < x < \infty$, $D_g: x \geq 1$, $R_f: -\infty < y < \infty$, $R_g: y \geq 0$, $D_{f+g} = D_{f\cdot g} = D_g$, $R_{f+g}: y \geq 1$, $R_{f\cdot g}: y \geq 0$

3. $D_f: -\infty < x < \infty$, $D_g: -\infty < x < \infty$, $R_f: y = 2$, $R_g: y \geq 1$, $D_{f/g}: -\infty < x < \infty$, $R_{f/g}: 0 < y \leq 2$, $D_{g/f}: -\infty < x < \infty$, $R_{g/f}: y \geq 1/2$

5. **(a)** 2 **(b)** 22 **(c)** $x^2 + 2$ **(d)** $x^2 + 10x + 22$ **(e)** 5 **(f)** -2 **(g)** $x + 10$ **(h)** $x^4 - 6x^2 + 6$

7. $13 - 3x$ **9.** $\sqrt{\dfrac{5x+1}{4x+1}}$

11. **(a)** $f(g(x))$ **(b)** $j(g(x))$ **(c)** $g(g(x))$ **(d)** $j(j(x))$ **(e)** $g(h(f(x)))$ **(f)** $h(j(f(x)))$

13.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
(a)	$x - 7$	$\sqrt{x}$	$\sqrt{x-7}$
(b)	$x + 2$	$3x$	$3x + 6$
(c)	x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(d)	$\dfrac{x}{x-1}$	$\dfrac{x}{x-1}$	x
(e)	$\dfrac{1}{x-1}$	$1 + \dfrac{1}{x}$	x
(f)	$\dfrac{1}{x}$	$\dfrac{1}{x}$	x

15. **(a)** 1 **(b)** 2 **(c)** -2 **(d)** 0 **(e)** -1 **(f)** 0

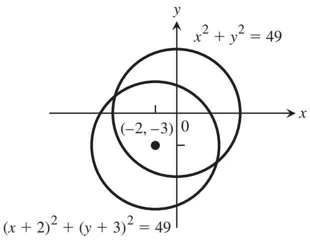
17. **(a)** $f(g(x)) = \sqrt{\dfrac{1}{x} + 1}$, $g(f(x)) = \dfrac{1}{\sqrt{x+1}}$
(b) $D_{f\circ g} = (-\infty, -1] \cup (0, \infty)$, $D_{g\circ f} = (-1, \infty)$
(c) $R_{f\circ g} = [0, 1) \cup (1, \infty)$, $R_{g\circ f} = (0, \infty)$

19. $g(x) = \dfrac{2x}{x-1}$

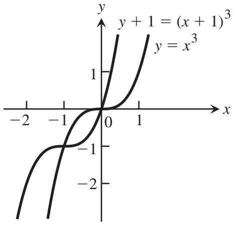
21. **(a)** $y = -(x+7)^2$ **(b)** $y = -(x-4)^2$

23. **(a)** Position 4 **(b)** Position 1 **(c)** Position 2 **(d)** Position 3

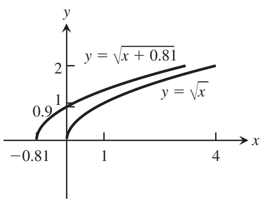
25. $(x+2)^2 + (y+3)^2 = 49$



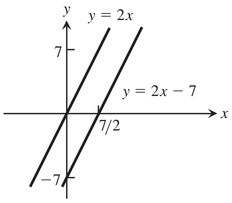
27. $y + 1 = (x+1)^3$



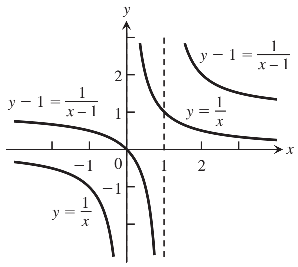
29. $y = \sqrt{x + 0.81}$



31. $y = 2x$



33. $y - 1 = \dfrac{1}{x-1}$



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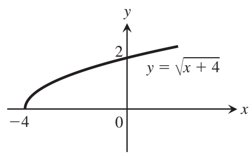

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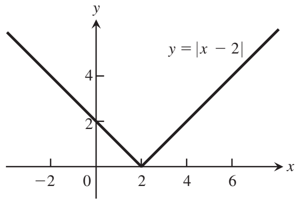

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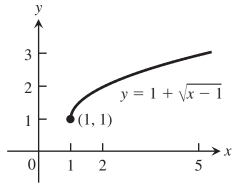

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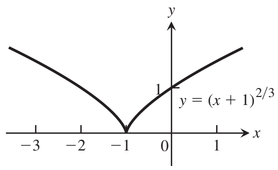

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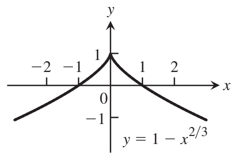

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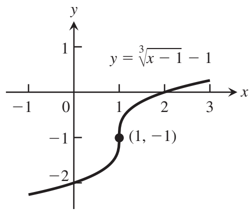

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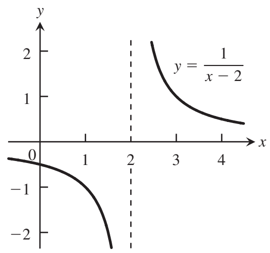

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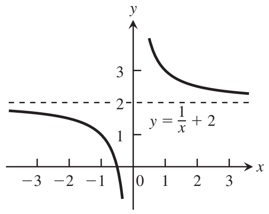

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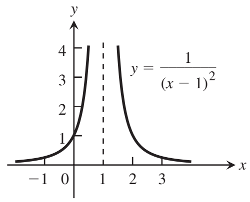

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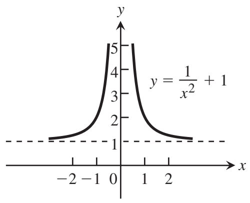
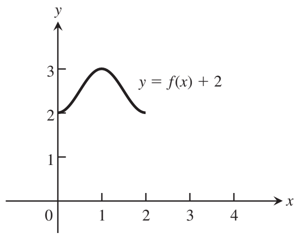
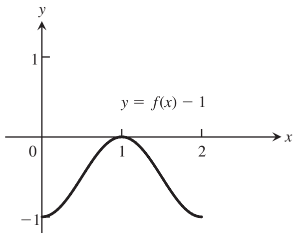


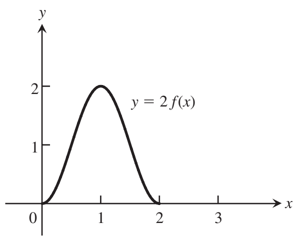
55. **(a)** $D: [0, 2]$, $R: [2, 3]$



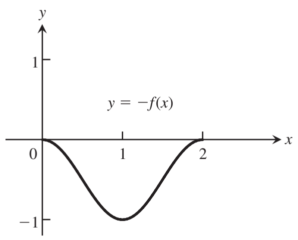
(b) $D: [0, 2]$, $R: [-1, 0]$



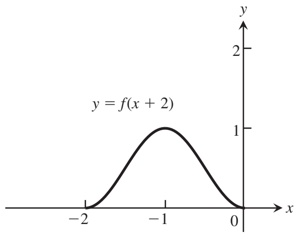
(c) $D: [0, 2], \quad R: [0, 2]$



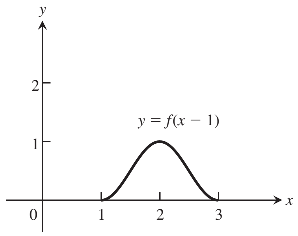
(d) $D: [0, 2], \quad R: [-1, 0]$



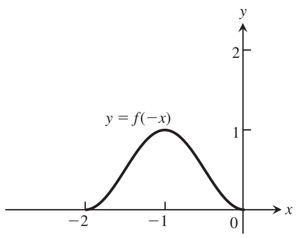
(e) $D: [-2, 0], \quad R: [0, 1]$



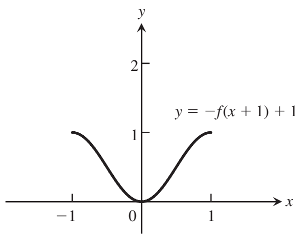
(f) $D: [1, 3], R: [0, 1]$



(g) $D: [-2, 0], \quad R: [0, 1]$



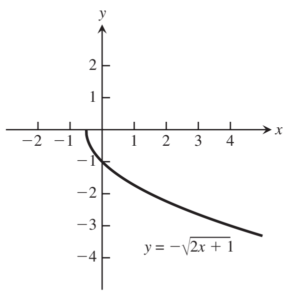
(h) $D: [-1, 1], \quad R: [0, 1]$



57. $y = 3x^2 - 3$ **59.** $y = \dfrac{1}{2} + \dfrac{1}{2x^2}$ **61.** $y = \sqrt{4x + 1}$

63. $y = \sqrt{4 - \dfrac{x^2}{4}}$ **65.** $y = 1 - 27x^3$

67.



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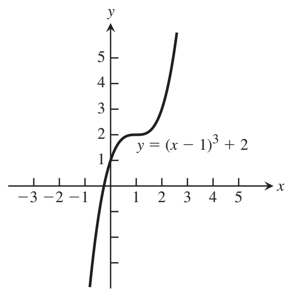

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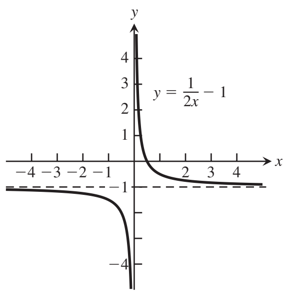

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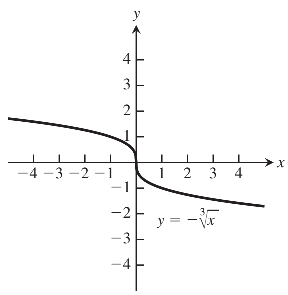

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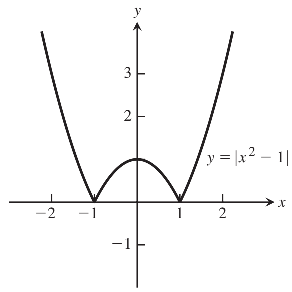


77. **(a)** Odd **(b)** Odd **(c)** Odd **(d)** Even **(e)** Even **(f)** Even **(g)** Even **(h)** Even **(i)** Odd

Section 1.3, pp. 27–29

1. **(a)** 8π m **(b)** $\dfrac{55\pi}{9}$ m **3.** 8.4 in.

5.

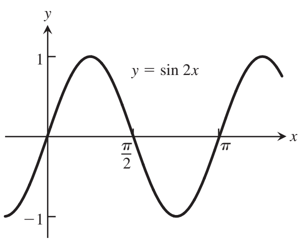
θ	$-\pi$	$-2\pi/3$	0	$\pi/2$	$3\pi/4$
$\sin\theta$	0	$-\dfrac{\sqrt{3}}{2}$	0	1	$\dfrac{1}{\sqrt{2}}$
$\cos\theta$	-1	$-\dfrac{1}{2}$	1	0	$-\dfrac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	UND	-1
$\cot\theta$	UND	$\dfrac{1}{\sqrt{3}}$	UND	0	-1
$\sec\theta$	-1	-2	1	UND	$-\sqrt{2}$
$\csc\theta$	UND	$-\dfrac{2}{\sqrt{3}}$	UND	1	$\sqrt{2}$

7. $\cos x = -4/5, \tan x = -3/4$

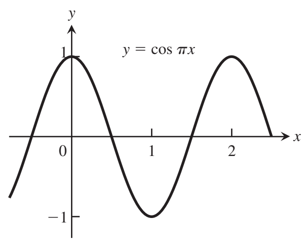
9. $\sin x = -\dfrac{\sqrt{8}}{3}, \tan x = -\sqrt{8}$

11. $\sin x = -\dfrac{1}{\sqrt{5}}, \cos x = -\dfrac{2}{\sqrt{5}}$

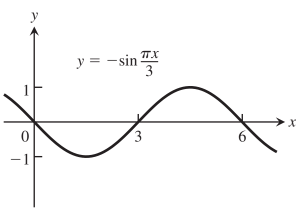
13. Period π



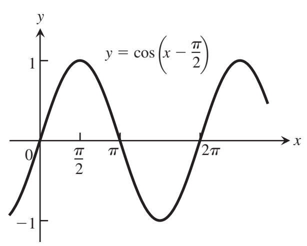
15. Period 2



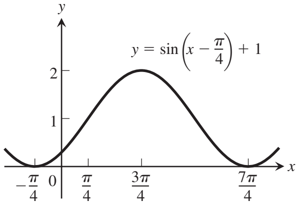
17. Period 6



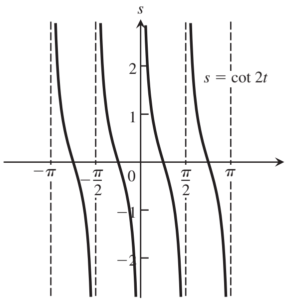
19. Period 2π



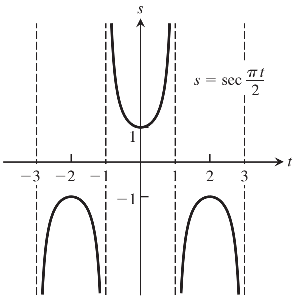
21. Period 2π



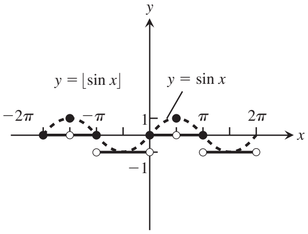
23. Period $\pi/2$, symmetric about the origin



25. Period 4, symmetric about the y-axis



29. $D: (-\infty, \infty)$, $R: y = -1, 0, 1$

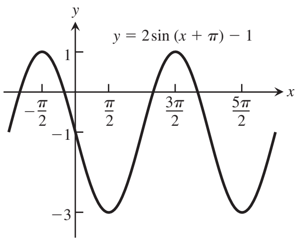


39. $-\cos x$ **41.** $-\cos x$ **43.** $\frac{\sqrt{6} + \sqrt{2}}{4}$ **45.** $\frac{\sqrt{2} + \sqrt{6}}{4}$

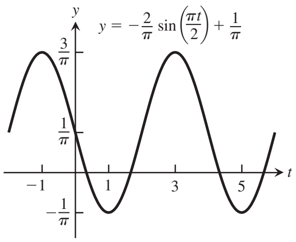
47. $\frac{2 + \sqrt{2}}{4}$ **49.** $\frac{2 - \sqrt{3}}{4}$ **51.** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

53. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **59.** $\sqrt{7} \approx 2.65$ **63.** $a = 1.464$

65. $A = 2, B = 2\pi,$
$C = -\pi, D = -1$



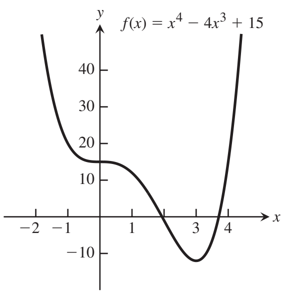
67. $A = -\frac{2}{\pi}, B = 4,$
$C = 0, D = \frac{1}{\pi}$



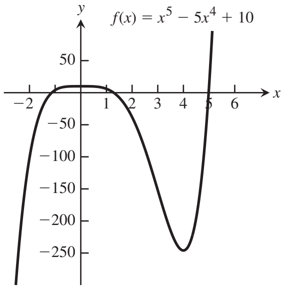
Section 1.4, pp. 34–36

1. d **3.** d

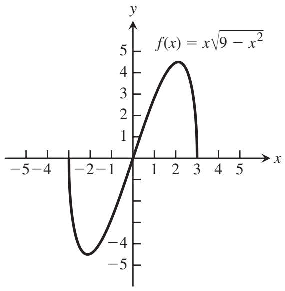
5. $[-3, 5]$ by $[-15, 40]$



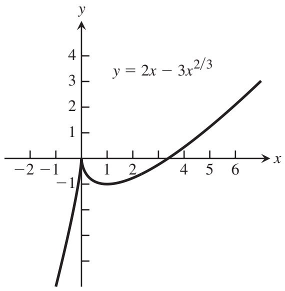
7. $[-3, 6]$ by $[-250, 50]$



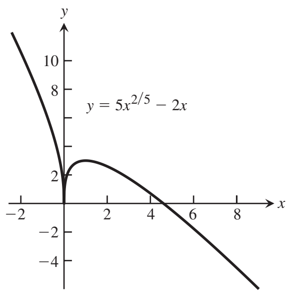
9. $[-5, 5]$ by $[-6, 6]$



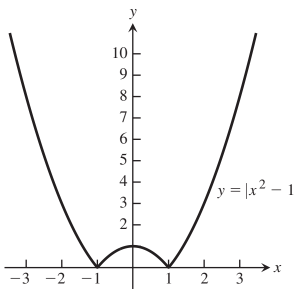
11. $[-2, 6]$ by $[-5, 4]$



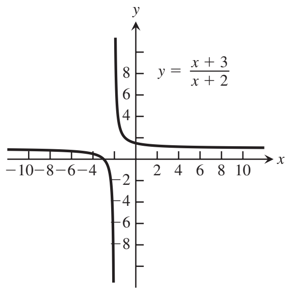
13. $[-2, 8]$ by $[-5, 10]$



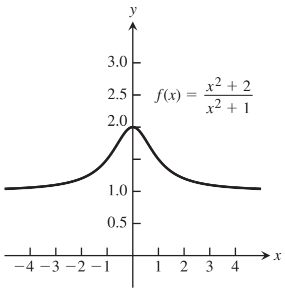
15. $[-3, 3]$ by $[0, 10]$



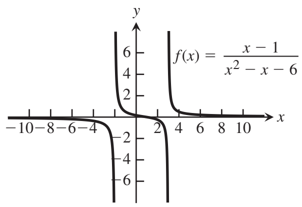
17. $[-10, 10]$ by $[-10, 10]$



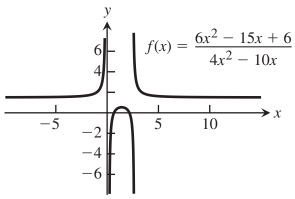
19. $[-4, 4]$ by $[0, 3]$



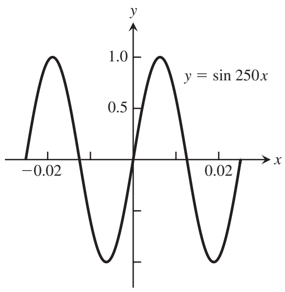
21. $[-10, 10]$ by $[-6, 6]$



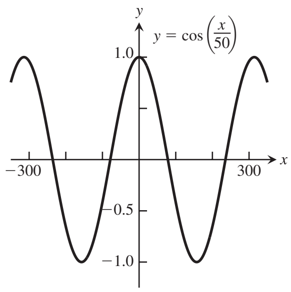
23. $[-6, 10]$ by $[-6, 6]$



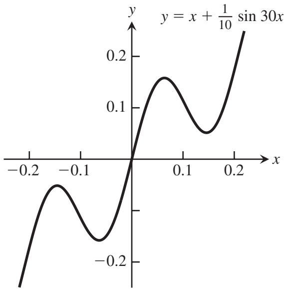
25. $\left[-\frac{\pi}{125}, \frac{\pi}{125}\right]$ by $[-1.25, 1.25]$



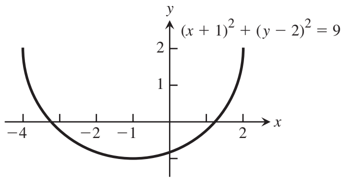
27. $[-100\pi, 100\pi]$ by $[-1.25, 1.25]$



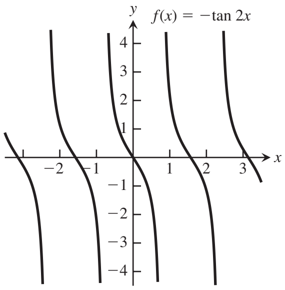
29. $\left[-\frac{\pi}{15}, \frac{\pi}{15}\right]$ by $[-0.25, 0.25]$



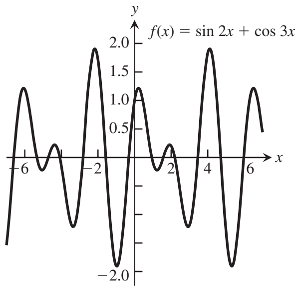
31.



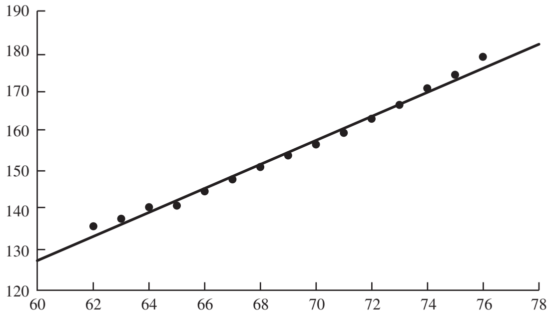
33.



35.

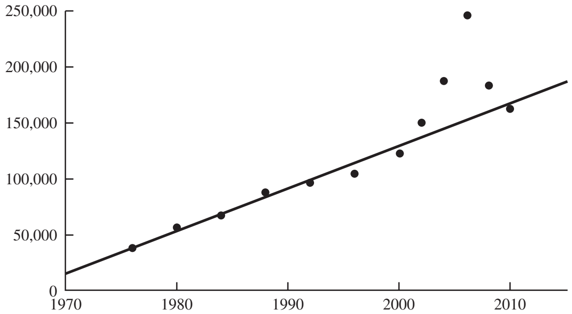


37. (a) & (b) $y = 3.0625x - 56.213$



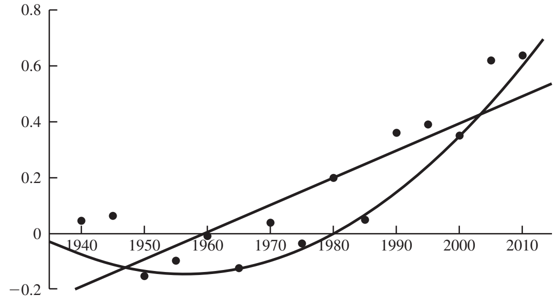
(c) Yes, $y(79) = 185.7$ lbs.

39. (a) & (b) $y = 3814x - 7.4988 \cdot 10^6$



(c) The price of a home within the "bubble" was inflated, in the sense that it exceeded the historical trend.

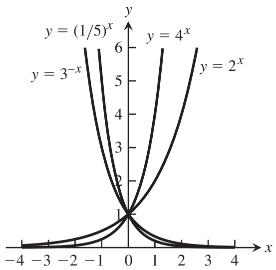
41. (a) & (b) $y = 9.7571 \cdot 10^{-3}x - 19.118$



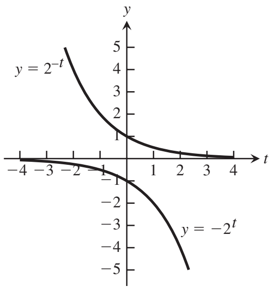
(c) $y = 2.6076 \cdot 10^{-4}x^2 - 1.0203x + 997.90$

Section 1.5, pp. 40–41

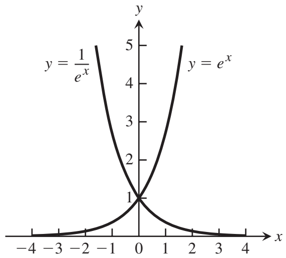
1.



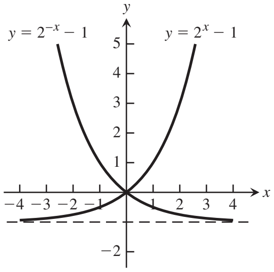
3.



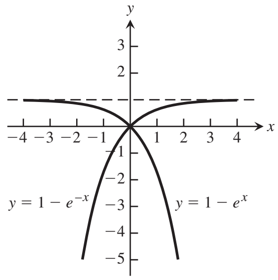
5.



7.



9.



11. $16^{1/4} = 2$ **13.** $4^{1/2} = 2$ **15.** 5 **17.** $14^{\sqrt{3}}$ **19.** 4

21. $D: -\infty < x < \infty$; $R: 0 < y < 1/2$

23. $D: -\infty < t < \infty$; $R: 1 < y < \infty$

25. $x \approx 2.3219$ **27.** $x \approx -0.6309$ **29.** After 19 years

31. (a) $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$ **(b)** About 38 days later

33. ≈ 11.433 years, or when interest is paid

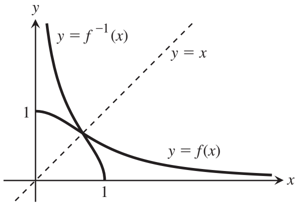
35. $2^{48} \approx 2.815 \times 10^{14}$

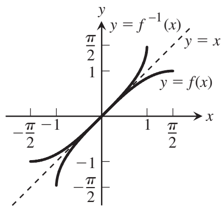
Section 1.6, pp. 51–53

1. One-to-one **3.** Not one-to-one **5.** One-to-one

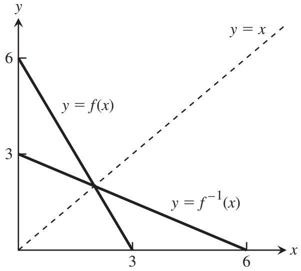
7. Not one-to-one **9.** One-to-one

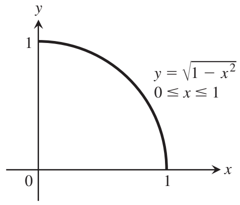
11. $D: (0, 1]$ $R: [0, \infty)$ **13.** $D: [-1, 1]$ $R: [-\pi/2, \pi/2]$





15. $D: [0, 6]$ $R: [0, 3]$ **17. (a)** Symmetric about the line $y = x$





19. $f^{-1}(x) = \sqrt{x - 1}$ **21.** $f^{-1}(x) = \sqrt[3]{x + 1}$

23. $f^{-1}(x) = \sqrt{x} - 1$

25. $f^{-1}(x) = \sqrt[5]{x}$; $D: -\infty < x < \infty$; $R: -\infty < y < \infty$

27. $f^{-1}(x) = \sqrt[3]{x - 1}$; $D: -\infty < x < \infty$; $R: -\infty < y < \infty$

29. $f^{-1}(x) = \dfrac{1}{\sqrt{x}}$; $D: x > 0$; $R: y > 0$

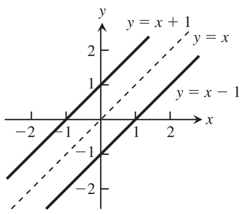
31. $f^{-1}(x) = \dfrac{2x + 3}{x - 1}$; $D: -\infty < x < \infty, x \neq 1$; $R: -\infty < y < \infty, y \neq 2$

33. $f^{-1}(x) = 1 - \sqrt{x + 1}$; $D: -1 \le x < \infty$; $R: -\infty < y \le 1$

35. $f^{-1}(x) = \dfrac{2x + b}{x - 1}$; $D: -\infty < x < \infty, x \neq 1$, $R: -\infty < y < \infty, y \neq 2$

37. (a) $f^{-1}(x) = \frac{1}{m}x$
(b) The graph of f^{-1} is the line through the origin with slope $1/m$.

39. (a) $f^{-1}(x) = x - 1$



(b) $f^{-1}(x) = x - b$. The graph of f^{-1} is a line parallel to the graph of f. The graphs of f and f^{-1} lie on opposite sides of the line $y = x$ and are equidistant from that line.
(c) Their graphs will be parallel to one another and lie on opposite sides of the line $y = x$ equidistant from that line.

41. (a) $\ln 3 - 2\ln 2$ **(b)** $2(\ln 2 - \ln 3)$ **(c)** $-\ln 2$ **(d)** $\frac{2}{3}\ln 3$ **(e)** $\ln 3 + \frac{1}{2}\ln 2$ **(f)** $\frac{1}{2}(3\ln 3 - \ln 2)$

43. (a) $\ln 5$ **(b)** $\ln(x - 3)$ **(c)** $\ln\left(\frac{2t^2}{b}\right)$

45. (a) 7.2 **(b)** $\frac{1}{x^2}$ **(c)** $\frac{x}{y}$

47. (a) 1 **(b)** 1 **(c)** $-x^2 - y^2$

49. e^{2t+4} **51.** $e^{5t} + b$ **53.** $y = 2xe^x + 1$

55. (a) $k = \ln 2$ **(b)** $k = (1/10)\ln 2$ **(c)** $k = 1000\ln a$

57. (a) $t = -10\ln 3$ **(b)** $t = -\frac{\ln 2}{k}$ **(c)** $t = \frac{\ln .4}{\ln .2}$

59. $4(\ln x)^2$

61. (a) 7 **(b)** $\sqrt{2}$ **(c)** 75 **(d)** 2 **(e)** 0.5 **(f)** -1

63. (a) $\sqrt{x}$ **(b)** x^2 **(c)** $\sin x$ **65. (a)** $\frac{\ln 3}{\ln 2}$ **(b)** 3 **(c)** 2

67. (a) $-\pi/6$ **(b)** $\pi/4$ **(c)** $-\pi/3$

69. (a) π **(b)** $\pi/2$

71. Yes, $g(x)$ is also one-to-one.

73. Yes, $f \circ g$ is also one-to-one.

75. (a) $f^{-1}(x) = \log_2\left(\frac{x}{100 - x}\right)$ **(b)** $f^{-1}(x) = \log_{1.1}\left(\frac{x}{50 - x}\right)$

77. (a) $y = \ln x - 3$ **(b)** $y = \ln(x - 1)$
(c) $y = 3 + \ln(x + 1)$ **(d)** $y = \ln(x - 2) - 4$
(e) $y = \ln(-x)$ **(f)** $y = e^x$

79. ≈ -0.7667

81. (a) Amount $= 8\left(\frac{1}{2}\right)^{t/12}$ **(b)** 36 hours

83. ≈ 44.081 years

Practice Exercises, pp. 54–56

1. $A = \pi r^2, C = 2\pi r, A = \frac{C^2}{4\pi}$ **3.** $x = \tan\theta, y = \tan^2\theta$

5. Origin **7.** Neither **9.** Even **11.** Even

13. Odd **15.** Neither

17. (a) Even **(b)** Odd **(c)** Odd **(d)** Even **(e)** Even

19. (a) Domain: all reals **(b)** Range: $[-2, \infty)$

21. (a) Domain: $[-4, 4]$ **(b)** Range: $[0, 4]$

23. (a) Domain: all reals **(b)** Range: $(-3, \infty)$

25. (a) Domain: all reals **(b)** Range: $[-3, 1]$

27. (a) Domain: $(3, \infty)$ **(b)** Range: all reals

29. (a) Increasing **(b)** Neither **(c)** Decreasing **(d)** Increasing

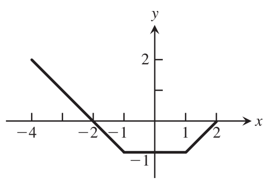
31. (a) Domain: $[-4, 4]$ **(b)** Range: $[0, 2]$

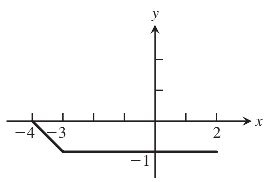
33. $f(x) = \begin{cases} 1 - x, & 0 \le x < 1 \\ 2 - x, & 1 \le x \le 2 \end{cases}$

35. (a) 1 **(b)** $\frac{1}{\sqrt{2.5}} = \sqrt{\frac{2}{5}}$ **(c)** $x, x \neq 0$
(d) $\dfrac{1}{\sqrt{1/\sqrt{x + 2} + 2}}$

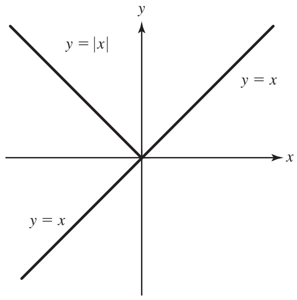
37. (a) $(f \circ g)(x) = -x, x \ge -2, (g \circ f)(x) = \sqrt{4 - x^2}$
(b) Domain $(f \circ g)$: $[-2, \infty)$, domain $(g \circ f)$: $[-2, 2]$
(c) Range $(f \circ g)$: $(-\infty, 2]$, range $(g \circ f)$: $[0, 2]$

39.





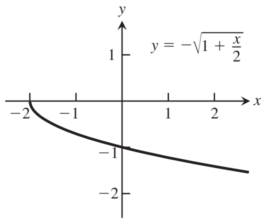
41. Replace the portion for $x < 0$ with the mirror image of the portion for $x > 0$ to make the new graph symmetric with respect to the y-axis.



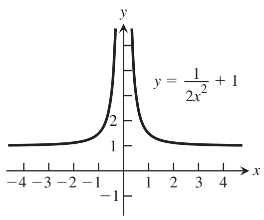
43. Reflects the portion for $y < 0$ across the x-axis
45. Reflects the portion for $y < 0$ across the x-axis
47. Adds the mirror image of the portion for $x > 0$ to make the new graph symmetric with respect to the y-axis

49. **(a)** $y = g(x - 3) + \frac{1}{2}$ **(b)** $y = g\left(x + \frac{2}{3}\right) - 2$
(c) $y = g(-x)$ **(d)** $y = -g(x)$ **(e)** $y = 5g(x)$
(f) $y = g(5x)$

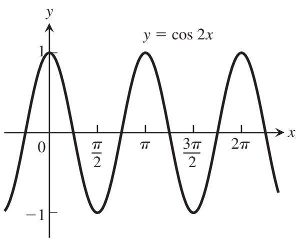
51.



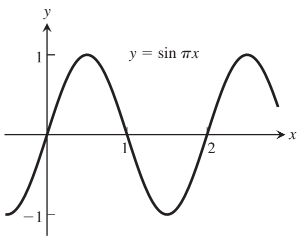
53.



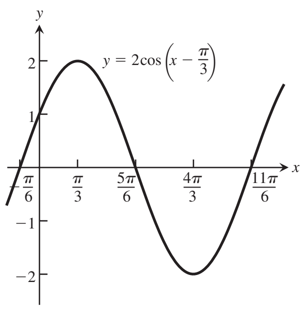
55. Period π



57. Period 2



59.



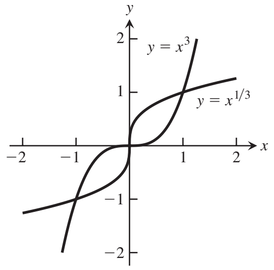
61. **(a)** $a = 1 \quad b = \sqrt{3}$ **(b)** $a = 2\sqrt{3}/3 \quad c = 4\sqrt{3}/3$

63. **(a)** $a = \dfrac{b}{\tan B}$ **(b)** $c = \dfrac{a}{\sin A}$

65. ≈ 16.98 m **67.** **(b)** 4π
69. **(a)** Domain: $-\infty < x < \infty$ **(b)** Domain: $x > 0$
71. **(a)** Domain: $-3 \le x \le 3$ **(b)** Domain: $0 \le x \le 4$
73. $(f \circ g)(x) = \ln(4 - x^2)$ and domain: $-2 < x < 2$;
$(g \circ f)(x) = 4 - (\ln x)^2$ and domain: $x > 0$;
$(f \circ f)(x) = \ln(\ln x)$ and domain: $x > 1$;
$(g \circ g)(x) = -x^4 + 8x^2 - 12$ and domain: $-\infty < x < \infty$.

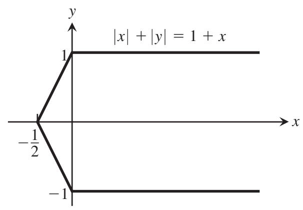
79. **(a)** D: $(-\infty, \infty)$ R: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ **(b)** D: $[-1, 1]$ R: $[-1, 1]$

81. **(a)** No **(b)** Yes
83. **(a)** $f(g(x)) = \left(\sqrt[3]{x}\right)^3 = x$, $g(f(x)) = \sqrt[3]{x^3} = x$
(b)



Additional and Advanced Exercises, pp. 57–58

1. Yes. For instance: $f(x) = 1/x$ and $g(x) = 1/x$, or $f(x) = 2x$ and $g(x) = x/2$, or $f(x) = e^x$ and $g(x) = \ln x$.
3. If $f(x)$ is odd, then $g(x) = f(x) - 2$ is not odd. Nor is $g(x)$ even, unless $f(x) = 0$ for all x. If f is even, then $g(x) = f(x) - 2$ is also even.
5.



19. **(a)** Domain: all reals. Range: If $a > 0$, then (d, ∞); if $a < 0$, then $(-\infty, d)$.
(b) Domain: (c, ∞), range: all reals
21. **(a)** $y = 100{,}000 - 10{,}000x$, $0 \le x \le 10$ **(b)** After 4.5 years
23. After $\dfrac{\ln(10/3)}{\ln 1.08} \approx 15.6439$ years. (If the bank only pays interest at the end of the year, it will take 16 years.)
25. $x = 2, x = 1$ **27.** $1/2$

Chapter 2

Section 2.1, pp. 64–66

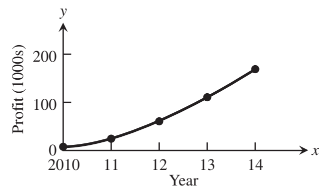
1. **(a)** 19 **(b)** 1
3. **(a)** $-\dfrac{4}{\pi}$ **(b)** $-\dfrac{3\sqrt{3}}{\pi}$ **5.** 1
7. **(a)** 4 **(b)** $y = 4x - 9$
9. **(a)** 2 **(b)** $y = 2x - 7$
11. **(a)** 12 **(b)** $y = 12x - 16$
13. **(a)** -9 **(b)** $y = -9x - 2$
15. Your estimates may not completely agree with these.
(a)

PQ_1	PQ_2	PQ_3	PQ_4
43	46	49	50

The appropriate units are m/sec.

(b) ≈ 50 m/sec or 180 km/h

17. **(a)**



(b) $\approx$ \$56,000/year

(c) $\approx$ \$42,000/year

19. **(a)** 0.414213, 0.449489, $\left(\sqrt{1+h}-1\right)/h$ **(b)** $g(x)=\sqrt{x}$

$1+h$	1.1	1.01	1.001	1.0001
$\sqrt{1+h}$	1.04880	1.004987	1.0004998	1.0000499
$\left(\sqrt{1+h}-1\right)/h$	0.4880	0.4987	0.4998	0.499

1.00001	1.000001
1.000005	1.0000005
0.5	0.5

(c) 0.5 **(d)** 0.5

21. **(a)** 15 mph, 3.3 mph, 10 mph **(b)** 10 mph, 0 mph, 4 mph

(c) 20 mph when $t = 3.5$ hr

Section 2.2, pp. 74–77

1. **(a)** Does not exist. As x approaches 1 from the right, $g(x)$ approaches 0. As x approaches 1 from the left, $g(x)$ approaches 1. There is no single number L that all the values $g(x)$ get arbitrarily close to as $x \to 1$.

(b) 1 **(c)** 0 **(d)** $1/2$

3. **(a)** True **(b)** True **(c)** False **(d)** False **(e)** False **(f)** True **(g)** True

5. As x approaches 0 from the left, $x/|x|$ approaches -1. As x approaches 0 from the right, $x/|x|$ approaches 1. There is no single number L that the function values all get arbitrarily close to as $x \to 0$.

7. Nothing can be said. **9.** No; no; no **11.** -4 **13.** -8 **15.** 3 **17.** $-25/2$ **19.** 16 **21.** $3/2$ **23.** $1/10$ **25.** -7 **27.** $3/2$ **29.** $-1/2$ **31.** -1 **33.** $4/3$ **35.** $1/6$ **37.** 4 **39.** $1/2$ **41.** $3/2$ **43.** -1 **45.** 1 **47.** $1/3$ **49.** $\sqrt{4-\pi}$

51. **(a)** Quotient Rule **(b)** Difference and Power Rules **(c)** Sum and Constant Multiple Rules

53. **(a)** -10 **(b)** -20 **(c)** -1 **(d)** $5/7$

55. **(a)** 4 **(b)** -21 **(c)** -12 **(d)** $-7/3$

57. 2 **59.** 3 **61.** $1/\left(2\sqrt{7}\right)$ **63.** $\sqrt{5}$

65. **(a)** The limit is 1.

67. **(a)** $f(x)=(x^2-9)/(x+3)$

x	-3.1	-3.01	-3.001	-3.0001	-3.00001	-3.000001
$f(x)$	-6.1	-6.01	-6.001	-6.0001	-6.00001	-6.000001

x	-2.9	-2.99	-2.999	-2.9999	-2.99999	-2.999999
$f(x)$	-5.9	-5.99	-5.999	-5.9999	-5.99999	-5.999999

(c) $\lim_{x\to -3} f(x) = -6$

69. **(a)** $G(x)=(x+6)/(x^2+4x-12)$

x	-5.9	-5.99	-5.999	-5.9999
$G(x)$	$-.126582$	$-.1251564$	$-.1250156$	$-.1250015$

-5.99999	-5.999999
$-.1250001$	$-.1250000$

x	-6.1	-6.01	-6.001	-6.0001
$G(x)$	$-.123456$	$-.124843$	$-.124984$	$-.124998$

-6.00001	-6.000001
$-.124999$	$-.124999$

(c) $\lim_{x\to -6} G(x) = -1/8 = -0.125$

71. **(a)** $f(x)=(x^2-1)/(|x|-1)$

x	-1.1	-1.01	-1.001	-1.0001	-1.00001	-1.000001
$f(x)$	2.1	2.01	2.001	2.0001	2.00001	2.000001

x	$-.9$	$-.99$	$-.999$	$-.9999$	$-.99999$	$-.999999$
$f(x)$	1.9	1.99	1.999	1.9999	1.99999	1.999999

(c) $\lim_{x\to -1} f(x) = 2$

73. **(a)** $g(\theta)=(\sin\theta)/\theta$

θ	.1	.01	.001	.0001	.00001	.000001
$g(\theta)$	.998334	.999983	.999999	.999999	.999999	.999999

θ	$-.1$	$-.01$	$-.001$	$-.0001$	$-.00001$	$-.000001$
$g(\theta)$	.998334	.999983	.999999	.999999	.999999	.999999

$\lim_{\theta\to 0} g(\theta) = 1$

75. **(a)** $f(x)=x^{1/(1-x)}$

x	.9	.99	.999	.9999	.99999	.999999
$f(x)$	.348678	.366032	.367695	.367861	.367877	.367879

x	1.1	1.01	1.001	1.0001	1.00001	1.000001
$f(x)$	.385543	.369711	.368063	.367897	.367881	.367878

$\lim_{x\to 1} f(x) \approx 0.36788$

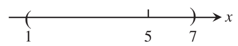
77. $c = 0, 1, -1$; the limit is 0 at $c = 0$, and 1 at $c = 1, -1$.

79. 7 **81.** **(a)** 5 **(b)** 5

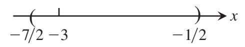
83. **(a)** 0 **(b)** 0

Section 2.3, pp. 83–86

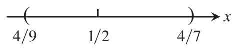
1. $\delta = 2$



3. $\delta = 1/2$



5. $\delta = 1/18$



7. $\delta = 0.1$ **9.** $\delta = 7/16$ **11.** $\delta = \sqrt{5}-2$

13. $\delta = 0.36$ **15.** $(3.99, 4.01)$, $\delta = 0.01$

17. $(-0.19, 0.21)$, $\delta = 0.19$ **19.** $(3, 15)$, $\delta = 5$

21. $(10/3, 5)$, $\delta = 2/3$

23. $\left(-\sqrt{4.5}, -\sqrt{3.5}\right)$, $\delta = \sqrt{4.5}-2 \approx 0.12$

25. $\left(\sqrt{15}, \sqrt{17}\right)$, $\delta = \sqrt{17}-4 \approx 0.12$

27. $\left(2-\frac{0.03}{m}, 2+\frac{0.03}{m}\right)$, $\delta = \frac{0.03}{m}$

29. $\left(\frac{1}{2}-\frac{c}{m}, \frac{c}{m}+\frac{1}{2}\right)$, $\delta = \frac{c}{m}$ **31.** $L = -3$, $\delta = 0.01$

33. $L = 4$, $\delta = 0.05$ **35.** $L = 4$, $\delta = 0.75$

55. $[3.384, 3.387]$. To be safe, the left endpoint was rounded up and the right endpoint rounded down.

59. The limit does not exist as x approaches 3.

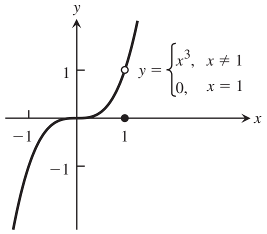
Section 2.4, pp. 91–93

1. **(a)** True **(b)** True **(c)** False **(d)** True
(e) True **(f)** True **(g)** False **(h)** False
(i) False **(j)** False **(k)** True **(l)** False

3. **(a)** 2, 1 **(b)** No, $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$
(c) 3, 3 **(d)** Yes, 3

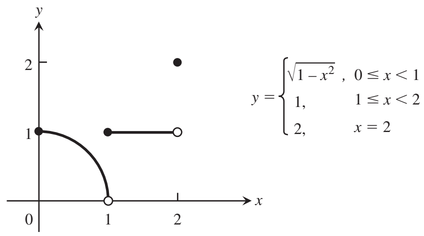
5. **(a)** No **(b)** Yes, 0 **(c)** No

7. **(a)**



(b) 1, 1 **(c)** Yes, 1

9. **(a)** $D: 0 \le x \le 2, R: 0 < y \le 1$ and $y = 2$
(b) $(0, 1) \cup (1, 2)$ **(c)** $x = 2$ **(d)** $x = 0$



11. $\sqrt{3}$ **13.** 1 **15.** $2/\sqrt{5}$ **17.** **(a)** 1 **(b)** −1
19. **(a)** 1 **(b)** 2/3 **21.** 1 **23.** 3/4 **25.** 2 **27.** 1/2
29. 2 **31.** 0 **33.** 1 **35.** 1/2 **37.** 0 **39.** 3/8
41. 3 **47.** $\delta = \epsilon^2$, $\lim_{x\to 5^+} \sqrt{x-5} = 0$
51. **(a)** 400 **(b)** 399 **(c)** The limit does not exist.

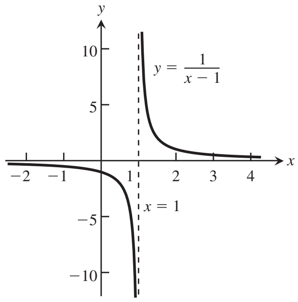
Section 2.5, pp. 102–104

1. No; discontinuous at $x = 2$; not defined at $x = 2$
3. Continuous **5.** **(a)** Yes **(b)** Yes **(c)** Yes **(d)** Yes
7. **(a)** No **(b)** No **9.** 0 **11.** 1, nonremovable; 0, removable
13. All x except $x = 2$ **15.** All x except $x = 3, x = 1$
17. All x **19.** All x except $x = 0$
21. All x except $n\pi/2$, n any integer
23. All x except $n\pi/2$, n an odd integer
25. All $x \ge -3/2$ **27.** All x **29.** All x
31. 0; continuous at $x = \pi$ **33.** 1; continuous at $y = 1$
35. $\sqrt{2}/2$; continuous at $t = 0$ **37.** 1; continuous at $x = 0$
39. $g(3) = 6$ **41.** $f(1) = 3/2$ **43.** $a = 4/3$ **45.** $a = -2, 3$
47. $a = 5/2, b = -1/2$ **71.** $x \approx 1.8794, -1.5321, -0.3473$
73. $x \approx 1.7549$ **75.** $x \approx 3.5156$ **77.** $x \approx 0.7391$

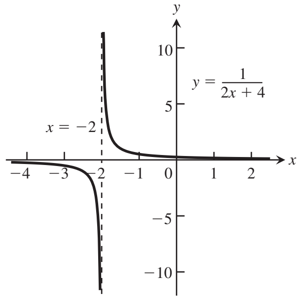
Section 2.6, pp. 115–117

1. **(a)** 0 **(b)** −2 **(c)** 2 **(d)** Does not exist **(e)** −1
(f) ∞ **(g)** Does not exist **(h)** 1 **(i)** 0
3. **(a)** −3 **(b)** −3 **5.** **(a)** 1/2 **(b)** 1/2 **7.** **(a)** −5/3
(b) −5/3 **9.** 0 **11.** −1 **13.** **(a)** 2/5 **(b)** 2/5
15. **(a)** 0 **(b)** 0 **17.** **(a)** 7 **(b)** 7 **19.** **(a)** 0 **(b)** 0
21. **(a)** ∞ **(b)** ∞ **23.** 2 **25.** ∞ **27.** 0
29. 1 **31.** ∞ **33.** 1 **35.** 1/2 **37.** ∞ **39.** −∞
41. −∞ **43.** ∞ **45.** **(a)** ∞ **(b)** −∞ **47.** ∞
49. ∞ **51.** −∞ **53.** **(a)** ∞ **(b)** −∞ **(c)** −∞ **(d)** ∞
55. **(a)** −∞ **(b)** ∞ **(c)** 0 **(d)** 3/2
57. **(a)** −∞ **(b)** 1/4 **(c)** 1/4 **(d)** 1/4 **(e)** It will be −∞.
59. **(a)** −∞ **(b)** ∞ **61.** **(a)** ∞ **(b)** ∞ **(c)** ∞ **(d)** ∞

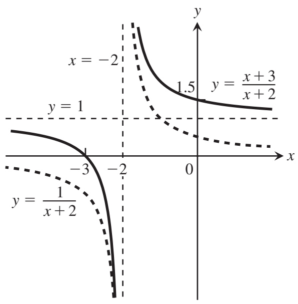
63.



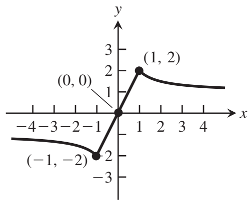
65.



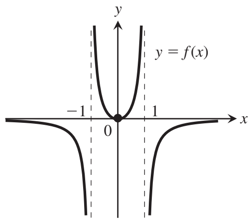
67.



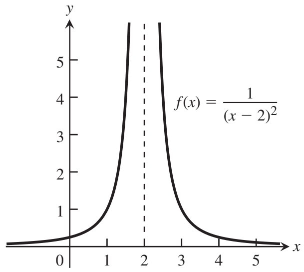
69. Here is one possibility.



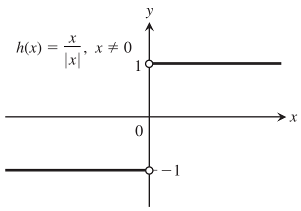
71. Here is one possibility.



73. Here is one possibility.



75. Here is one possibility.



79. At most one

81. 0 **83.** −3/4 **85.** 5/2

93. **(a)** For every positive real number B there exists a corresponding number $\delta > 0$ such that for all x

$$c - \delta < x < c \implies f(x) > B.$$

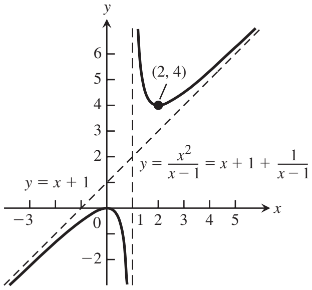
(b) For every negative real number $-B$ there exists a corresponding number $\delta > 0$ such that for all x

$$c < x < c + \delta \implies f(x) < -B.$$

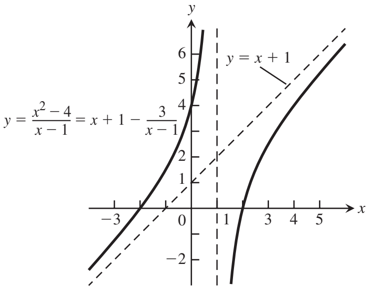
(c) For every negative real number $-B$ there exists a corresponding number $\delta > 0$ such that for all x

$$c - \delta < x < c \implies f(x) < -B.$$

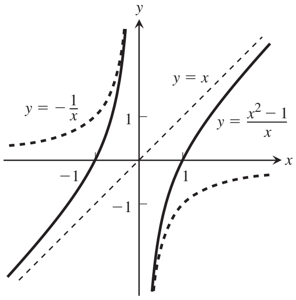
99.



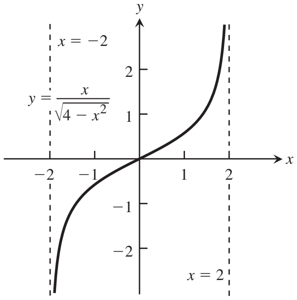
101.



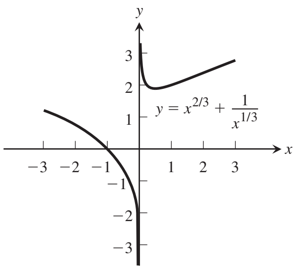
103.



105.



107.



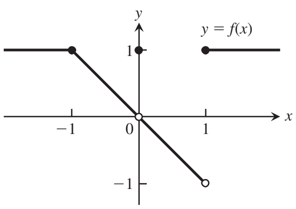
109. At ∞: ∞, at $-\infty$: 0

Practice Exercises, pp. 118–120

1. At $x = -1$: $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = 1$, so $\lim_{x\to -1} f(x) = 1 = f(-1)$; continuous at $x = -1$

At $x = 0$: $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 0$, so $\lim_{x\to 0} f(x) = 0$. However, $f(0) \neq 0$, so f is discontinuous at $x = 0$. The discontinuity can be removed by redefining $f(0)$ to be 0.

At $x = 1$: $\lim_{x\to 1^-} f(x) = -1$ and $\lim_{x\to 1^+} f(x) = 1$, so $\lim_{x\to 1} f(x)$ does not exist. The function is discontinuous at $x = 1$, and the discontinuity is not removable.



3. (a) -21 **(b)** 49 **(c)** 0 **(d)** 1 **(e)** 1 **(f)** 7 **(g)** -7 **(h)** $-\frac{1}{7}$ **5.** 4

7. (a) $(-\infty, +\infty)$ **(b)** $[0, \infty)$ **(c)** $(-\infty, 0)$ and $(0, \infty)$ **(d)** $(0, \infty)$

9. (a) Does not exist **(b)** 0 **11.** $\frac{1}{2}$ **13.** $2x$ **15.** $-\frac{1}{4}$

17. $2/3$ **19.** $2/\pi$ **21.** 1 **23.** 4 **25.** $-\infty$

27. 0 **29.** 2 **31.** 0

35. No in both cases, because $\lim_{x\to 1} f(x)$ does not exist, and $\lim_{x\to -1} f(x)$ does not exist.

37. Yes, f does have a continuous extension, to $a = 1$ with $f(1) = 4/3$.

39. No **41.** $2/5$ **43.** 0 **45.** $-\infty$ **47.** 0 **49.** 1

51. 1 **53.** $-\pi/2$ **55. (a)** $x = 3$ **(b)** $x = 1$ **(c)** $x = -4$

Additional and Advanced Exercises, pp. 120–122

3. 0; the left-hand limit was needed because the function is undefined for $v > c$. **5.** $65 < t < 75$; within 5°F

13. (a) B **(b)** A **(c)** A **(d)** A

21. (a) $\lim_{a\to 0} r_+(a) = 0.5$, $\lim_{a\to -1^+} r_+(a) = 1$

(b) $\lim_{a\to 0} r_-(a)$ does not exist, $\lim_{a\to -1^+} r_-(a) = 1$

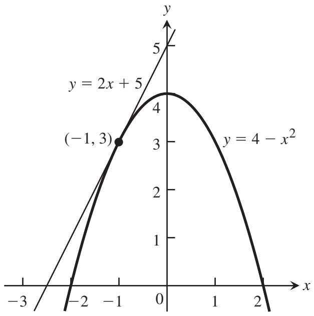
25. 0 **27.** 1 **29.** 4 **31.** $y = 2x$ **33.** $y = x, y = -x$

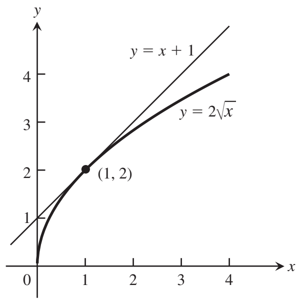
Chapter 3

Section 3.1, pp. 126–127

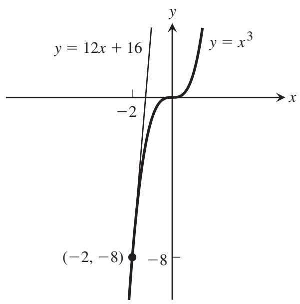
1. P_1: $m_1 = 1$, P_2: $m_2 = 5$ **3.** P_1: $m_1 = 5/2$, P_2: $m_2 = -1/2$

5. $y = 2x + 5$ **7.** $y = x + 1$





9. $y = 12x + 16$



11. $m = 4, y - 5 = 4(x - 2)$

13. $m = -2, y - 3 = -2(x - 3)$

15. $m = 12, y - 8 = 12(t - 2)$

17. $m = \frac{1}{4}, y - 2 = \frac{1}{4}(x - 4)$

19. $m = -1$ **21.** $m = -1/4$

23. (a) It is the rate of change of the number of cells when $t = 5$. The units are the number of cells per hour.

(b) $P'(3)$ because the slope of the curve is greater there.

(c) $51.72 \approx 52$ cells/h

25. $(-2, -5)$ **27.** $y = -(x + 1), y = -(x - 3)$

29. 19.6 m/sec **31.** 6π **35.** Yes **37.** Yes
39. **(a)** Nowhere **41.** **(a)** At $x = 0$ **43.** **(a)** Nowhere
45. **(a)** At $x = 1$ **47.** **(a)** At $x = 0$

Section 3.2, pp. 133–136

1. $-2x, 6, 0, -2$ **3.** $-\frac{2}{t^3}, 2, -\frac{1}{4}, -\frac{2}{3\sqrt{3}}$

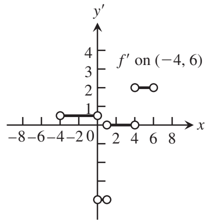
5. $\frac{3}{2\sqrt{3\theta}}, \frac{3}{2\sqrt{3}}, \frac{1}{2}, \frac{3}{2\sqrt{2}}$ **7.** $6x^2$ **9.** $\frac{1}{(2t+1)^2}$

11. $\frac{3}{2}q^{1/2}$ **13.** $1 - \frac{9}{x^2}, 0$ **15.** $3t^2 - 2t, 5$

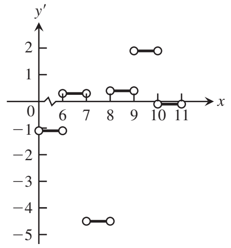
17. $\frac{-4}{(x-2)\sqrt{x-2}}$, $y - 4 = -\frac{1}{2}(x - 6)$ **19.** 6

21. $1/8$ **23.** $\frac{-1}{(x+2)^2}$ **25.** $\frac{-1}{(x-1)^2}$ **27.** (b) **29.** (d)

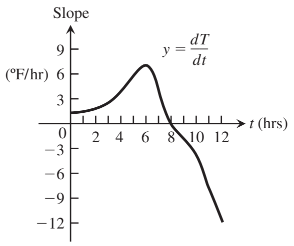
31. **(a)** $x = 0, 1, 4$
(b)



33.



35. **(a)** **i)** 1.5 °F/hr **ii)** 2.9 °F/hr
iii) 0 °F/hr **iv)** −3.7 °F/hr
(b) 7.3 °F/hr at 12 P.M., −11 °F/hr at 6 P.M.
(c)



37. Since $\lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h} = 1$
while $\lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h} = 0$,
$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$ does not exist and $f(x)$ is not differentiable at $x = 0$.

39. Since $\lim_{h\to 0^+} \frac{f(1+h) - f(1)}{h} = 2$ while
$\lim_{h\to 0^-} \frac{f(1+h) - f(1)}{h} = \frac{1}{2}$, $f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$
does not exist and $f(x)$ is not differentiable at $x = 1$.

41. Since $f(x)$ is not continuous at $x = 0$, $f(x)$ is not differentiable at $x = 0$.

43. **(a)** $-3 \le x \le 2$ **(b)** None **(c)** None
45. **(a)** $-3 \le x < 0, 0 < x \le 3$ **(b)** None **(c)** $x = 0$
47. **(a)** $-1 \le x < 0, 0 < x \le 2$ **(b)** $x = 0$ **(c)** None

Section 3.3, pp. 144–146

1. $\frac{dy}{dx} = -2x, \frac{d^2y}{dx^2} = -2$

3. $\frac{ds}{dt} = 15t^2 - 15t^4, \frac{d^2s}{dt^2} = 30t - 60t^3$

5. $\frac{dy}{dx} = 4x^2 - 1 + 2e^x, \frac{d^2y}{dx^2} = 8x + 2e^x$

7. $\frac{dw}{dz} = -\frac{6}{z^3} + \frac{1}{z^2}, \frac{d^2w}{dz^2} = \frac{18}{z^4} - \frac{2}{z^3}$

9. $\frac{dy}{dx} = 12x - 10 + 10x^{-3}, \frac{d^2y}{dx^2} = 12 - 30x^{-4}$

11. $\frac{dr}{ds} = \frac{-2}{3s^3} + \frac{5}{2s^2}, \frac{d^2r}{ds^2} = \frac{2}{s^4} - \frac{5}{s^3}$

13. $y' = -5x^4 + 12x^2 - 2x - 3$

15. $y' = 3x^2 + 10x + 2 - \frac{1}{x^2}$ **17.** $y' = \frac{-19}{(3x-2)^2}$

19. $g'(x) = \frac{x^2 + x + 4}{(x + 0.5)^2}$ **21.** $\frac{dv}{dt} = \frac{t^2 - 2t - 1}{(1+t^2)^2}$

23. $f'(s) = \frac{1}{\sqrt{s}(\sqrt{s}+1)^2}$ **25.** $v' = -\frac{1}{x^2} + 2x^{-3/2}$

27. $y' = \frac{-4x^3 - 3x^2 + 1}{(x^2-1)^2(x^2+x+1)^2}$ **29.** $y' = -2e^{-x} + 3e^{3x}$

31. $y' = 3x^2e^x + x^3e^x$ **33.** $y' = \frac{9}{4}x^{5/4} - 2e^{-2x}$

35. $\frac{ds}{dt} = 3t^{1/2}$ **37.** $y' = \frac{2}{7x^{5/7}} - ex^{e-1}$ **39.** $\frac{dr}{ds} = \frac{se^s - e^s}{s^2}$

41. $y' = 2x^3 - 3x - 1, y'' = 6x^2 - 3, y''' = 12x, y^{(4)} = 12,$
$y^{(n)} = 0$ for $n \ge 5$

43. $y' = 3x^2 + 8x + 1, y'' = 6x + 8, y''' = 6, y^{(n)} = 0$ for $n \ge 4$

45. $y' = 2x - 7x^{-2}, y'' = 2 + 14x^{-3}$

47. $\frac{dr}{d\theta} = 3\theta^{-4}, \frac{d^2r}{d\theta^2} = -12\theta^{-5}$ **49.** $\frac{dw}{dz} = -z^{-2} - 1, \frac{d^2w}{dz^2} = 2z^{-3}$

51. $\frac{dw}{dz} = 6ze^{2z}(1 + z), \frac{d^2w}{dz^2} = 6e^{2z}(1 + 4z + 2z^2)$

53. **(a)** 13 **(b)** −7 **(c)** 7/25 **(d)** 20

55. **(a)** $y = -\frac{x}{8} + \frac{5}{4}$ **(b)** $m = -4$ at $(0, 1)$
(c) $y = 8x - 15, y = 8x + 17$

57. $y = 4x, y = 2$ **59.** $a = 1, b = 1, c = 0$

61. $(2, 4)$ **63.** $(0, 0), (4, 2)$ **65.** **(a)** $y = 2x + 2$ **(c)** $(2, 6)$

67. 50 **69.** $a = -3$

71. $P'(x) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + 2a_2x + a_1$

73. The Product Rule is then the Constant Multiple Rule, so the latter is a special case of the Product Rule.

75. **(a)** $\frac{d}{dx}(uvw) = uvw' + uv'w + u'vw$
(b) $\frac{d}{dx}(u_1u_2u_3u_4) = u_1u_2u_3u_4' + u_1u_2u_3'u_4 + u_1u_2'u_3u_4 + u_1'u_2u_3u_4$
(c) $\frac{d}{dx}(u_1 \cdots u_n) = u_1u_2 \cdots u_{n-1}u_n' + u_1u_2 \cdots u_{n-2}u_{n-1}'u_n + \cdots + u_1'u_2 \cdots u_n$

77. $\frac{dP}{dV} = -\frac{nRT}{(V - nb)^2} + \frac{2an^2}{V^3}$

Section 3.4, pp. 153–156

1. **(a)** −2 m, −1 m/sec
(b) 3 m/sec, 1 m/sec; 2 m/sec², 2 m/sec²
(c) Changes direction at $t = 3/2$ sec

3. **(a)** −9 m, −3 m/sec
(b) 3 m/sec, 12 m/sec; 6 m/sec², −12 m/sec²
(c) No change in direction

5. **(a)** -20 m, -5 m/sec
(b) 45 m/sec, (1/5) m/sec; 140 m/sec^2, (4/25) m/sec^2
(c) No change in direction

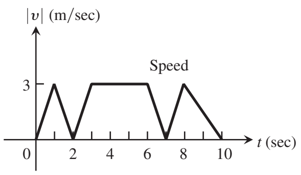
7. **(a)** $a(1) = -6$ m/sec^2, $a(3) = 6$ m/sec^2
(b) $v(2) = 3$ m/sec **(c)** 6 m

9. Mars: ≈ 7.5 sec, Jupiter: ≈ 1.2 sec

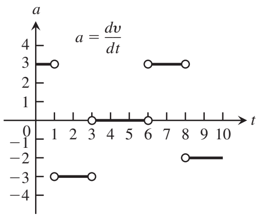
11. $g_s = 0.75$ m/sec^2

13. **(a)** $v = -32t$, $|v| = 32t$ ft/sec, $a = -32$ ft/sec^2
(b) $t \approx 3.3$ sec
(c) $v \approx -107.0$ ft/sec

15. **(a)** $t = 2, t = 7$ **(b)** $3 \le t \le 6$
(c)



(d)



17. **(a)** 190 ft/sec **(b)** 2 sec **(c)** 8 sec, 0 ft/sec
(d) 10.8 sec, 90 ft/sec **(e)** 2.8 sec
(f) Greatest acceleration happens 2 sec after launch
(g) Constant acceleration between 2 and 10.8 sec, -32 ft/sec^2

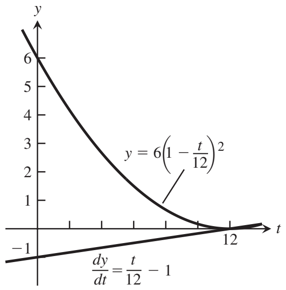
19. **(a)** $\frac{4}{7}$sec, 280 cm/sec **(b)** 560 cm/sec, 980 cm/sec^2
(c) 29.75 flashes/sec

21. C = position, A = velocity, B = acceleration

23. **(a)** \$110/machine **(b)** \$80 **(c)** \$79.90

25. **(a)** $b'(0) = 10^4$ bacteria/h **(b)** $b'(5) = 0$ bacteria/h
(c) $b'(10) = -10^4$ bacteria/h

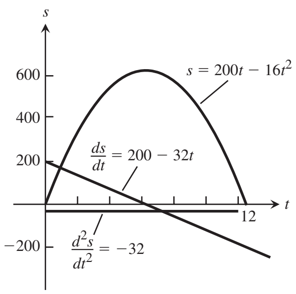
27. **(a)** $\frac{dy}{dt} = \frac{t}{12} - 1$
(b) The largest value of $\frac{dy}{dt}$ is 0 m/h when $t = 12$ and the smallest value of $\frac{dy}{dt}$ is -1 m/h when $t = 0$.
(c)



29. 4.88 ft, 8.66 ft, additional ft to stop car for 1 mph speed increase

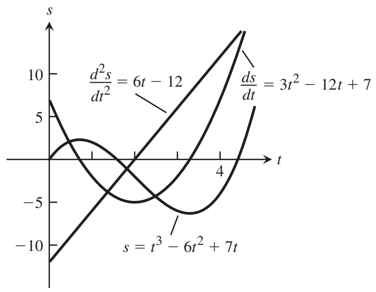
31. $t = 25$ sec, $D = \frac{6250}{9}$m

33.



(a) $v = 0$ when $t = 6.25$ sec
(b) $v > 0$ when $0 \le t < 6.25 \Rightarrow$ the object moves up; $v < 0$ when $6.25 < t \le 12.5 \Rightarrow$ the object moves down.
(c) The object changes direction at $t = 6.25$ sec.
(d) The object speeds up on $(6.25, 12.5]$ and slows down on $[0, 6.25)$.
(e) The object is moving fastest at the endpoints $t = 0$ and $t = 12.5$ when it is traveling 200 ft/sec. It's moving slowest at $t = 6.25$ when the speed is 0.
(f) When $t = 6.25$ the object is $s = 625$ m from the origin and farthest away.

35.



(a) $v = 0$ when $t = \frac{6 \pm \sqrt{15}}{3}$sec
(b) $v < 0$ when $\frac{6 - \sqrt{15}}{3} < t < \frac{6 + \sqrt{15}}{3} \Rightarrow$ the object moves left; $v > 0$ when $0 \le t < \frac{6 - \sqrt{15}}{3}$ or $\frac{6 + \sqrt{15}}{3} < t \le 4 \Rightarrow$ the object moves right.
(c) The object changes direction at $t = \frac{6 \pm \sqrt{15}}{3}$ sec.
(d) The object speeds up on $\left(\frac{6 - \sqrt{15}}{3}, 2\right) \cup \left(\frac{6 + \sqrt{15}}{3}, 4\right]$ and slows down on $\left[0, \frac{6 - \sqrt{15}}{3}\right) \cup \left(2, \frac{6 + \sqrt{15}}{3}\right)$.
(e) The object is moving fastest at $t = 0$ and $t = 4$ when it is moving 7 units/sec and slowest at $t = \frac{6 \pm \sqrt{15}}{3}$ sec.
(f) When $t = \frac{6 + \sqrt{15}}{3}$ the object is at position $s \approx -6.303$ units and farthest from the origin.

Section 3.5, pp. 160–162

1. $-10 - 3 \sin x$ **3.** $2x \cos x - x^2 \sin x$

5. $-\csc x \cot x - \frac{2}{\sqrt{x}} - \frac{7}{e^x}$ **7.** $\sin x \sec^2 x + \sin x$

9. $(e^{-x} \sec x)(1 - x + x \tan x)$ **11.** $\frac{-\csc^2 x}{(1 + \cot x)^2}$

13. $4 \tan x \sec x - \csc^2 x$ **15.** 0

17. $3x^2 \sin x \cos x + x^3 \cos^2 x - x^3 \sin^2 x$

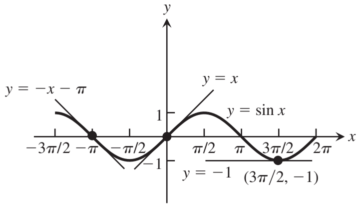
19. $\sec^2 t + e^{-t}$ **21.** $\frac{-2 \csc t \cot t}{(1 - \csc t)^2}$ **23.** $-\theta(\theta \cos \theta + 2 \sin \theta)$

25. $\sec \theta \csc \theta (\tan \theta - \cot \theta) = \sec^2 \theta - \csc^2 \theta$ **27.** $\sec^2 q$

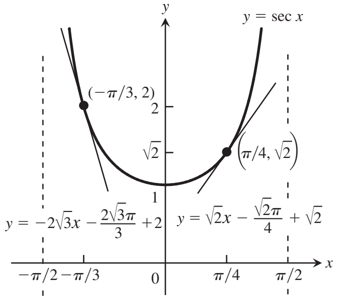
29. $\sec^2 q$ **31.** $\frac{q^3 \cos q - q^2 \sin q - q \cos q - \sin q}{(q^2 - 1)^2}$

33. **(a)** $2\csc^3 x - \csc x$ **(b)** $2\sec^3 x - \sec x$

35.

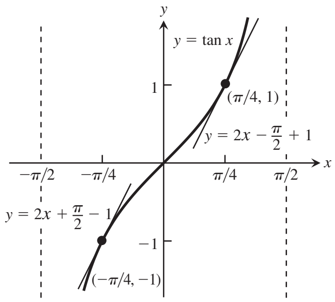


37.



39. Yes, at $x = \pi$ **41.** No

43. $\left(-\frac{\pi}{4}, -1\right); \left(\frac{\pi}{4}, 1\right)$



45. **(a)** $y = -x + \pi/2 + 2$ **(b)** $y = 4 - \sqrt{3}$

47. 0 **49.** $\sqrt{3}/2$ **51.** -1 **53.** 0

55. $-\sqrt{2}$ m/sec, $\sqrt{2}$ m/sec, $\sqrt{2}$ m/sec^2, $\sqrt{2}$ m/sec^3

57. $c = 9$ **59.** $\sin x$

61. **(a)** **i)** 10 cm **ii)** 5 cm **iii)** $-5\sqrt{2} \approx -7.1$ cm

(b) **i)** 0 cm/sec **ii)** $-5\sqrt{3} \approx -8.7$ cm/sec

iii) $-5\sqrt{2} \approx -7.1$ cm/sec

Section 3.6, pp. 168–171

1. $12x^3$ **3.** $3\cos(3x + 1)$ **5.** $\dfrac{\cos x}{2\sqrt{\sin x}}$

7. $2\pi x \sec^2(\pi x^2)$

9. With $u = (2x + 1)$, $y = u^5 : \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = 5u^4 \cdot 2 = 10(2x + 1)^4$

11. With $u = (1 - (x/7))$, $y = u^{-7} : \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = -7u^{-8} \cdot \left(-\dfrac{1}{7}\right) = \left(1 - \dfrac{x}{7}\right)^{-8}$

13. With $u = ((x^2/8) + x - (1/x))$, $y = u^4 : \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = 4u^3 \cdot \left(\dfrac{x}{4} + 1 + \dfrac{1}{x^2}\right) = 4\left(\dfrac{x^2}{8} + x - \dfrac{1}{x}\right)^3\left(\dfrac{x}{4} + 1 + \dfrac{1}{x^2}\right)$

15. With $u = \tan x$, $y = \sec u : \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\sec u \tan u)(\sec^2 x) = \sec(\tan x)\tan(\tan x)\sec^2 x$

17. With $u = \tan x$, $y = u^3 : \dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = 3u^2\sec^2 x = 3\tan^2 x(\sec^2 x)$

19. $y = e^u$, $u = -5x$, $\dfrac{dy}{dx} = -5e^{-5x}$

21. $y = e^u$, $u = 5 - 7x$, $\dfrac{dy}{dx} = -7e^{(5-7x)}$

23. $-\dfrac{1}{2\sqrt{3 - t}}$ **25.** $\dfrac{4}{\pi}(\cos 3t - \sin 5t)$ **27.** $\dfrac{\csc\theta}{\cot\theta + \csc\theta}$

29. $2x\sin^4 x + 4x^2\sin^3 x\cos x + \cos^{-2} x + 2x\cos^{-3} x\sin x$

31. $(3x - 2)^5 - \dfrac{1}{x^3\left(4 - \dfrac{1}{2x^2}\right)^2}$ **33.** $\dfrac{(4x + 3)^3(4x + 7)}{(x + 1)^4}$

35. $(1 - x)e^{-x} + 3x^2e^{x^3}$ **37.** $\left(\dfrac{5}{2}x^2 - 3x + 3\right)e^{5x/2}$

39. $\sqrt{x}\sec^2\left(2\sqrt{x}\right) + \tan\left(2\sqrt{x}\right)$ **41.** $\dfrac{x\sec x\tan x + \sec x}{2\sqrt{7 + x\sec x}}$

43. $\dfrac{2\sin\theta}{(1 + \cos\theta)^2}$ **45.** $-2\sin(\theta^2)\sin 2\theta + 2\theta\cos(2\theta)\cos(\theta^2)$

47. $\left(\dfrac{t + 2}{2(t + 1)^{3/2}}\right)\cos\left(\dfrac{t}{\sqrt{t + 1}}\right)$ **49.** $2\theta e^{-\theta^2}\sin\left(e^{-\theta^2}\right)$

51. $2\pi\sin(\pi t - 2)\cos(\pi t - 2)$ **53.** $\dfrac{8\sin(2t)}{(1 + \cos 2t)^5}$

55. $10t^{10}\tan^9 t\sec^2 t + 10t^9\tan^{10} t$

57. $\dfrac{dy}{dt} = -2\pi\sin(\pi t - 1)\cdot\cos(\pi t - 1)\cdot e^{\cos^2(\pi t - 1)}$

59. $\dfrac{-3t^6(t^2 + 4)}{(t^3 - 4t)^4}$ **61.** $-2\cos(\cos(2t - 5))(\sin(2t - 5))$

63. $\left(1 + \tan^4\left(\dfrac{t}{12}\right)\right)^2\left(\tan^3\left(\dfrac{t}{12}\right)\sec^2\left(\dfrac{t}{12}\right)\right)$

65. $-\dfrac{t\sin(t^2)}{\sqrt{1 + \cos(t^2)}}$ **67.** $6\tan(\sin^3 t)\sec^2(\sin^3 t)\sin^2 t\cos t$

69. $3(2t^2 - 5)^3(18t^2 - 5)$ **71.** $\dfrac{6}{x^3}\left(1 + \dfrac{1}{x}\right)\left(1 + \dfrac{2}{x}\right)$

73. $2\csc^2(3x - 1)\cot(3x - 1)$ **75.** $16(2x + 1)^2(5x + 1)$

77. $2(2x^2 + 1)e^{x^2}$ **79.** $5/2$ **81.** $-\pi/4$ **83.** 0 **85.** -5

87. **(a)** $2/3$ **(b)** $2\pi + 5$ **(c)** $15 - 8\pi$ **(d)** $37/6$ **(e)** -1

(f) $\sqrt{2}/24$ **(g)** $5/32$ **(h)** $-5/\left(3\sqrt{17}\right)$ **89.** 5

91. **(a)** 1 **(b)** 1 **93.** $y = 1 - 4x$

95. **(a)** $y = \pi x + 2 - \pi$ **(b)** $\pi/2$

97. It multiplies the velocity, acceleration, and jerk by 2, 4, and 8, respectively.

99. $v(6) = \dfrac{2}{5}$ m/sec, $a(6) = -\dfrac{4}{125}$ m/sec^2

Section 3.7, pp. 175–176

1. $\dfrac{-2xy - y^2}{x^2 + 2xy}$ **3.** $\dfrac{1 - 2y}{2x + 2y - 1}$

5. $\dfrac{-2x^3 + 3x^2y - xy^2 + x}{x^2y - x^3 + y}$ **7.** $\dfrac{1}{y(x + 1)^2}$ **9.** $\cos y\cot y$

11. $\frac{-\cos^2(xy) - y}{x}$ **13.** $\frac{-y^2}{y\sin\left(\frac{1}{y}\right) - \cos\left(\frac{1}{y}\right) + xy}$

15. $\frac{2e^{2x} - \cos(x + 3y)}{3\cos(x + 3y)}$ **17.** $-\frac{\sqrt{r}}{\sqrt{\theta}}$ **19.** $\frac{-r}{\theta}$

21. $y' = -\frac{x}{y},\ y'' = \frac{-y^2 - x^2}{y^3}$

23. $\frac{dy}{dx} = \frac{xe^{x^2} + 1}{y},\ \frac{d^2y}{dx^2} = \frac{(2x^2y^2 + y^2 - 2x)e^{x^2} - x^2e^{2x^2} - 1}{y^3}$

25. $y' = \frac{\sqrt{y}}{\sqrt{y} + 1},\ y'' = \frac{1}{2\left(\sqrt{y} + 1\right)^3}$

27. -2 **29.** $(-2, 1): m = -1, (-2, -1): m = 1$

31. **(a)** $y = \frac{7}{4}x - \frac{1}{2}$ **(b)** $y = -\frac{4}{7}x + \frac{29}{7}$

33. **(a)** $y = 3x + 6$ **(b)** $y = -\frac{1}{3}x + \frac{8}{3}$

35. **(a)** $y = \frac{6}{7}x + \frac{6}{7}$ **(b)** $y = -\frac{7}{6}x - \frac{7}{6}$

37. **(a)** $y = -\frac{\pi}{2}x + \pi$ **(b)** $y = \frac{2}{\pi}x - \frac{2}{\pi} + \frac{\pi}{2}$

39. **(a)** $y = 2\pi x - 2\pi$ **(b)** $y = -\frac{x}{2\pi} + \frac{1}{2\pi}$

41. Points: $\left(-\sqrt{7}, 0\right)$ and $\left(\sqrt{7}, 0\right)$, Slope: -2

43. $m = -1$ at $\left(\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2}\right)$, $m = \sqrt{3}$ at $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$

45. $(-3, 2): m = -\frac{27}{8}$; $(-3, -2): m = \frac{27}{8}$; $(3, 2): m = \frac{27}{8}$; $(3, -2): m = -\frac{27}{8}$

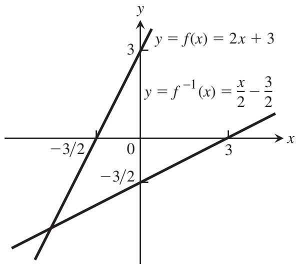
47. $(3, -1)$

53. $\frac{dy}{dx} = -\frac{y^3 + 2xy}{x^2 + 3xy^2}$, $\frac{dx}{dy} = -\frac{x^2 + 3xy^2}{y^3 + 2xy}$, $\frac{dx}{dy} = \frac{1}{dy/dx}$

Section 3.8, pp. 185–186

1. **(a)** $f^{-1}(x) = \frac{x}{2} - \frac{3}{2}$

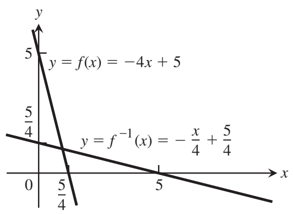
(b)



(c) 2, 1/2

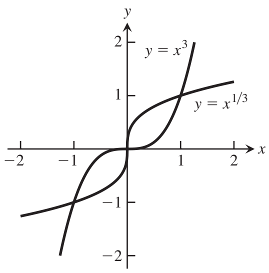
3. **(a)** $f^{-1}(x) = -\frac{x}{4} + \frac{5}{4}$

(b)



(c) $-4, -1/4$

5. **(b)**



(c) Slope of f at $(1, 1)$: 3; slope of g at $(1, 1)$: $1/3$; slope of f at $(-1, -1)$: 3; slope of g at $(-1, -1)$: $1/3$

(d) $y = 0$ is tangent to $y = x^3$ at $x = 0$; $x = 0$ is tangent to $y = \sqrt[3]{x}$ at $x = 0$.

7. $1/9$ **9.** 3 **11.** $\frac{1}{x} + 1$ **13.** $2/t$ **15.** $-1/x$

17. $\frac{1}{\theta + 1} - e^{\theta}$ **19.** $3/x$ **21.** $2(\ln t) + (\ln t)^2$

23. $x^3 \ln x$ **25.** $\frac{1 - \ln t}{t^2}$ **27.** $\frac{1}{x(1 + \ln x)^2}$ **29.** $\frac{1}{x \ln x}$

31. $2\cos(\ln \theta)$ **33.** $-\frac{3x + 2}{2x(x + 1)}$ **35.** $\frac{2}{t(1 - \ln t)^2}$

37. $\frac{\tan(\ln \theta)}{\theta}$ **39.** $\frac{10x}{x^2 + 1} + \frac{1}{2(1 - x)}$

41. $\left(\frac{1}{2}\right)\sqrt{x(x + 1)}\left(\frac{1}{x} + \frac{1}{x + 1}\right) = \frac{2x + 1}{2\sqrt{x(x + 1)}}$

43. $\left(\frac{1}{2}\right)\sqrt{\frac{t}{t + 1}}\left(\frac{1}{t} - \frac{1}{t + 1}\right) = \frac{1}{2\sqrt{t}(t + 1)^{3/2}}$

45. $\sqrt{\theta + 3}(\sin \theta)\left(\frac{1}{2(\theta + 3)} + \cot \theta\right)$

47. $t(t + 1)(t + 2)\left[\frac{1}{t} + \frac{1}{t + 1} + \frac{1}{t + 2}\right] = 3t^2 + 6t + 2$

49. $\frac{\theta + 5}{\theta \cos \theta}\left[\frac{1}{\theta + 5} - \frac{1}{\theta} + \tan \theta\right]$

51. $\frac{x\sqrt{x^2 + 1}}{(x + 1)^{2/3}}\left[\frac{1}{x} + \frac{x}{x^2 + 1} - \frac{2}{3(x + 1)}\right]$

53. $\frac{1}{3}\sqrt[3]{\frac{x(x - 2)}{x^2 + 1}}\left(\frac{1}{x} + \frac{1}{x - 2} - \frac{2x}{x^2 + 1}\right)$ **55.** $-2\tan \theta$

57. $\frac{1 - t}{t}$ **59.** $1/(1 + e^{\theta})$ **61.** $e^{\cos t}(1 - t\sin t)$

63. $\frac{ye^y \cos x}{1 - ye^y \sin x}$ **65.** $\frac{dy}{dx} = \frac{y^2 - xy\ln y}{x^2 - xy\ln x}$ **67.** $2^x \ln x$

69. $\left(\frac{\ln 5}{2\sqrt{s}}\right)5^{\sqrt{s}}$ **71.** $\pi x^{(\pi - 1)}$ **73.** $\frac{1}{\theta \ln 2}$ **75.** $\frac{3}{x \ln 4}$

77. $\frac{2(\ln r)}{r(\ln 2)(\ln 4)}$ **79.** $\frac{-2}{(x + 1)(x - 1)}$

81. $\sin(\log_7 \theta) + \frac{1}{\ln 7}\cos(\log_7 \theta)$ **83.** $\frac{1}{\ln 5}$

85. $\frac{1}{t}(\log_2 3)3^{\log_2 t}$ **87.** $\frac{1}{t}$ **89.** $(x + 1)^x\left(\frac{x}{x + 1} + \ln(x + 1)\right)$

91. $\left(\sqrt{t}\right)^t\left(\frac{\ln t}{2} + \frac{1}{2}\right)$ **93.** $(\sin x)^x(\ln \sin x + x\cot x)$

95. $(x^{\ln x})\left(\frac{\ln x^2}{x}\right)$

Section 3.9, pp. 192–193

1. **(a)** $\pi/4$ **(b)** $-\pi/3$ **(c)** $\pi/6$
3. **(a)** $-\pi/6$ **(b)** $\pi/4$ **(c)** $-\pi/3$
5. **(a)** $\pi/3$ **(b)** $3\pi/4$ **(c)** $\pi/6$
7. **(a)** $3\pi/4$ **(b)** $\pi/6$ **(c)** $2\pi/3$
9. $1/\sqrt{2}$ **11.** $-1/\sqrt{3}$ **13.** $\pi/2$ **15.** $\pi/2$ **17.** $\pi/2$
19. 0 **21.** $\dfrac{-2x}{\sqrt{1-x^4}}$ **23.** $\dfrac{\sqrt{2}}{\sqrt{1-2t^2}}$
25. $\dfrac{1}{|2s+1|\sqrt{s^2+s}}$ **27.** $\dfrac{-2x}{(x^2+1)\sqrt{x^4+2x^2}}$
29. $\dfrac{-1}{\sqrt{1-t^2}}$ **31.** $\dfrac{-1}{2\sqrt{t}(1+t)}$ **33.** $\dfrac{1}{(\tan^{-1}x)(1+x^2)}$
35. $\dfrac{-e^t}{|e^t|\sqrt{(e^t)^2-1}} = \dfrac{-1}{\sqrt{e^{2t}-1}}$ **37.** $\dfrac{-2s^2}{\sqrt{1-s^2}}$ **39.** 0
41. $\sin^{-1}x$
47. **(a)** Defined; there is an angle whose tangent is 2.
(b) Not defined; there is no angle whose cosine is 2.
49. **(a)** Not defined; no angle has secant 0.
(b) Not defined; no angle has sine $\sqrt{2}$.
59. **(a)** Domain: all real numbers except those having the form $\dfrac{\pi}{2}+k\pi$ where k is an integer; range: $-\pi/2 < y < \pi/2$
(b) Domain: $-\infty < x < \infty$; range: $-\infty < y < \infty$
61. **(a)** Domain: $-\infty < x < \infty$; range: $0 \le y \le \pi$
(b) Domain: $-1 \le x \le 1$; range: $-1 \le y \le 1$
63. The graphs are identical.

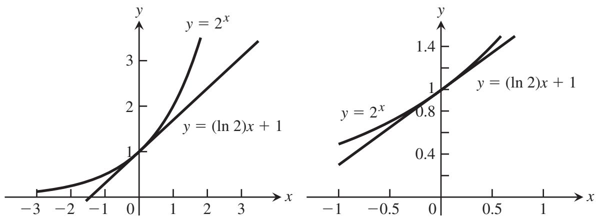
Section 3.10, pp. 198–202

1. $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$ **3.** 10 **5.** -6 **7.** $-3/2$
9. $31/13$ **11.** **(a)** $-180\ \text{m}^2/\text{min}$ **(b)** $-135\ \text{m}^3/\text{min}$
13. **(a)** $\dfrac{dV}{dt} = \pi r^2\dfrac{dh}{dt}$ **(b)** $\dfrac{dV}{dt} = 2\pi hr\dfrac{dr}{dt}$
(c) $\dfrac{dV}{dt} = \pi r^2\dfrac{dh}{dt} + 2\pi hr\dfrac{dr}{dt}$
15. **(a)** 1 volt/sec **(b)** $-\dfrac{1}{3}$ amp/sec
(c) $\dfrac{dR}{dt} = \dfrac{1}{I}\left(\dfrac{dV}{dt} - \dfrac{V}{I}\dfrac{dI}{dt}\right)$
(d) $3/2$ ohms/sec, R is increasing.
17. **(a)** $\dfrac{ds}{dt} = \dfrac{x}{\sqrt{x^2+y^2}}\dfrac{dx}{dt}$
(b) $\dfrac{ds}{dt} = \dfrac{x}{\sqrt{x^2+y^2}}\dfrac{dx}{dt} + \dfrac{y}{\sqrt{x^2+y^2}}\dfrac{dy}{dt}$ **(c)** $\dfrac{dx}{dt} = -\dfrac{y}{x}\dfrac{dy}{dt}$
19. **(a)** $\dfrac{dA}{dt} = \dfrac{1}{2}ab\cos\theta\dfrac{d\theta}{dt}$
(b) $\dfrac{dA}{dt} = \dfrac{1}{2}ab\cos\theta\dfrac{d\theta}{dt} + \dfrac{1}{2}b\sin\theta\dfrac{da}{dt}$
(c) $\dfrac{dA}{dt} = \dfrac{1}{2}ab\cos\theta\dfrac{d\theta}{dt} + \dfrac{1}{2}b\sin\theta\dfrac{da}{dt} + \dfrac{1}{2}a\sin\theta\dfrac{db}{dt}$
21. **(a)** $14\ \text{cm}^2/\text{sec}$, increasing **(b)** 0 cm/sec, constant
(c) $-14/13$ cm/sec, decreasing
23. **(a)** -12 ft/sec **(b)** $-59.5\ \text{ft}^2/\text{sec}$ **(c)** -1 rad/sec
25. 20 ft/sec
27. **(a)** $\dfrac{dh}{dt} = 11.19$ cm/min **(b)** $\dfrac{dr}{dt} = 14.92$ cm/min
29. **(a)** $\dfrac{-1}{24\pi}$ m/min **(b)** $r = \sqrt{26y - y^2}$ m
(c) $\dfrac{dr}{dt} = -\dfrac{5}{288\pi}$ m/min
31. 1 ft/min, $40\pi\ \text{ft}^2/\text{min}$ **33.** 11 ft/sec
35. Increasing at $466/1681\ L/\text{min}^2$
37. -5 m/sec **39.** -1500 ft/sec
41. $\dfrac{5}{72\pi}$ in./min, $\dfrac{10}{3}\ \text{in}^2/\text{min}$
43. **(a)** $-32/\sqrt{13} \approx -8.875$ ft/sec
(b) $d\theta_1/dt = 8/65$ rad/sec, $d\theta_2/dt = -8/65$ rad/sec
(c) $d\theta_1/dt = 1/6$ rad/sec, $d\theta_2/dt = -1/6$ rad/sec
45. -5.5 deg/min

Section 3.11, pp. 211–214

1. $L(x) = 10x - 13$ **3.** $L(x) = 2$ **5.** $L(x) = x - \pi$
7. $2x$ **9.** $-x - 5$ **11.** $\dfrac{1}{12}x + \dfrac{4}{3}$ **13.** $1 - x$
15. $f(0) = 1$. Also, $f'(x) = k(1+x)^{k-1}$, so $f'(0) = k$. This means the linearization at $x = 0$ is $L(x) = 1 + kx$.
17. **(a)** 1.01 **(b)** 1.003
19. $\left(3x^2 - \dfrac{3}{2\sqrt{x}}\right)dx$ **21.** $\dfrac{2-2x^2}{(1+x^2)^2}dx$
23. $\dfrac{1-y}{3\sqrt{y}+x}dx$ **25.** $\dfrac{5}{2\sqrt{x}}\cos\left(5\sqrt{x}\right)dx$
27. $(4x^2)\sec^2\left(\dfrac{x^3}{3}\right)dx$
29. $\dfrac{3}{\sqrt{x}}\left(\csc\left(1-2\sqrt{x}\right)\cot\left(1-2\sqrt{x}\right)\right)dx$
31. $\dfrac{1}{2\sqrt{x}}\cdot e^{\sqrt{x}}dx$ **33.** $\dfrac{2x}{1+x^2}dx$ **35.** $\dfrac{2xe^{x^2}}{1+e^{2x^2}}dx$
37. $\dfrac{-1}{\sqrt{e^{-2x}-1}}dx$
39. **(a)** 0.41 **(b)** 0.4 **(c)** 0.01
41. **(a)** 0.231 **(b)** 0.2 **(c)** 0.031
43. **(a)** $-1/3$ **(b)** $-2/5$ **(c)** $1/15$
45. $dV = 4\pi r_0^2\,dr$ **47.** $dS = 12x_0\,dx$ **49.** $dV = 2\pi r_0 h\,dr$
51. **(a)** $0.08\pi\ \text{m}^2$ **(b)** 2% **53.** $dV \approx 565.5\ \text{in}^3$
55. **(a)** 2% **(b)** 4% **57.** $\dfrac{1}{3}\%$ **59.** 3%
61. The ratio equals 37.87, so a change in the acceleration of gravity on the moon has about 38 times the effect that a change of the same magnitude has on Earth.
63. Increase $V \approx 40\%$
65. **(a)** **i)** $b_0 = f(a)$ **ii)** $b_1 = f'(a)$ **iii)** $b_2 = \dfrac{f''(a)}{2}$
(b) $Q(x) = 1 + x + x^2$ **(d)** $Q(x) = 1 - (x-1) + (x-1)^2$
(e) $Q(x) = 1 + \dfrac{x}{2} - \dfrac{x^2}{8}$
(f) The linearization of any differentiable function $u(x)$ at $x = a$ is $L(x) = u(a) + u'(a)(x-a) = b_0 + b_1(x-a)$, where b_0 and b_1 are the coefficients of the constant and linear terms of the quadratic approximation. Thus, the linearization for $f(x)$ at $x = 0$ is $1 + x$; the linearization for $g(x)$ at $x = 1$ is $1 - (x-1)$ or $2 - x$; and the linearization for $h(x)$ at $x = 0$ is $1 + \dfrac{x}{2}$.

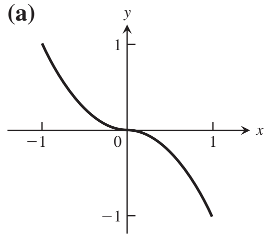
67. (a) $L(x) = x \ln 2 + 1 \approx 0.69x + 1$
(b)



Practice Exercises, pp. 215–219

1. $5x^4 - 0.25x + 0.25$ **3.** $3x(x - 2)$
5. $2(x + 1)(2x^2 + 4x + 1)$
7. $3(\theta^2 + \sec\theta + 1)^2(2\theta + \sec\theta\tan\theta)$
9. $\dfrac{1}{2\sqrt{t}\left(1 + \sqrt{t}\right)^2}$ **11.** $2\sec^2 x \tan x$
13. $8\cos^3(1 - 2t)\sin(1 - 2t)$ **15.** $5(\sec t)(\sec t + \tan t)^5$
17. $\dfrac{\theta\cos\theta + \sin\theta}{\sqrt{2\theta\sin\theta}}$ **19.** $\dfrac{\cos\sqrt{2\theta}}{\sqrt{2\theta}}$
21. $x\csc\left(\dfrac{2}{x}\right) + \csc\left(\dfrac{2}{x}\right)\cot\left(\dfrac{2}{x}\right)$
23. $\dfrac{1}{2}x^{1/2}\sec(2x)^2\left[16\tan(2x)^2 - x^{-2}\right]$
25. $-10x\csc^2(x^2)$ **27.** $8x^3\sin(2x^2)\cos(2x^2) + 2x\sin^2(2x^2)$
29. $\dfrac{-(t + 1)}{8t^3}$ **31.** $\dfrac{1 - x}{(x + 1)^3}$ **33.** $\dfrac{-1}{2x^2\left(1 + \dfrac{1}{x}\right)^{1/2}}$
35. $\dfrac{-2\sin\theta}{(\cos\theta - 1)^2}$ **37.** $3\sqrt{2x + 1}$ **39.** $-9\left[\dfrac{5x + \cos 2x}{(5x^2 + \sin 2x)^{5/2}}\right]$
41. $-2e^{-x/5}$ **43.** xe^{4x} **45.** $\dfrac{2\sin\theta\cos\theta}{\sin^2\theta} = 2\cot\theta$
47. $\dfrac{2}{(\ln 2)x}$ **49.** $-8^{-t}(\ln 8)$ **51.** $18x^{2.6}$
53. $(x + 2)^{x+2}(\ln(x + 2) + 1)$ **55.** $-\dfrac{1}{\sqrt{1 - u^2}}$
57. $\dfrac{-1}{\sqrt{1 - x^2}\cos^{-1}x}$ **59.** $\tan^{-1}(t) + \dfrac{t}{1 + t^2} - \dfrac{1}{2t}$
61. $\dfrac{1 - z}{\sqrt{z^2 - 1}} + \sec^{-1}z$ **63.** -1 **65.** $-\dfrac{y + 2}{x + 3}$
67. $\dfrac{-3x^2 - 4y + 2}{4x - 4y^{1/3}}$ **69.** $-\dfrac{y}{x}$ **71.** $\dfrac{1}{2y(x + 1)^2}$
73. $-1/2$ **75.** y/x **77.** $-\dfrac{2e^{-\tan^{-1}x}}{1 + x^2}$ **79.** $\dfrac{dp}{dq} = \dfrac{6q - 4p}{3p^2 + 4q}$
81. $\dfrac{dr}{ds} = (2r - 1)(\tan 2s)$
83. (a) $\dfrac{d^2y}{dx^2} = \dfrac{-2xy^3 - 2x^4}{y^5}$ **(b)** $\dfrac{d^2y}{dx^2} = \dfrac{-2xy^2 - 1}{x^4y^3}$
85. (a) 7 **(b)** -2 **(c)** $5/12$ **(d)** $1/4$ **(e)** 12 **(f)** $9/2$
(g) $3/4$
87. 0 **89.** $\dfrac{3\sqrt{2}e^{\sqrt{3}/2}}{4}\cos\left(e^{\sqrt{3}/2}\right)$ **91.** $-\dfrac{1}{2}$ **93.** $\dfrac{-2}{(2t + 1)^2}$

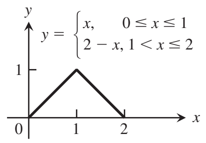
95. (a)



$f(x) = \begin{cases} x^2, & -1 \le x < 0 \\ -x^2, & 0 \le x < 1 \end{cases}$

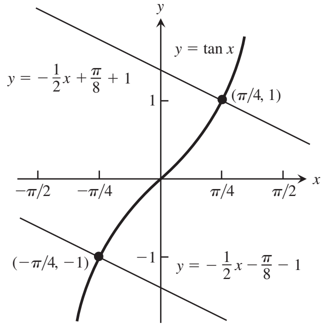
(b) Yes **(c)** Yes

97. (a)

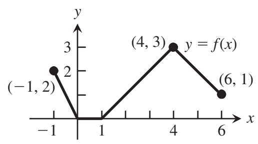


(b) Yes **(c)** No

99. $\left(\dfrac{5}{2}, \dfrac{9}{4}\right)$ and $\left(\dfrac{3}{2}, -\dfrac{1}{4}\right)$ **101.** $(-1, 27)$ and $(2, 0)$
103. (a) $(-2, 16)$, $(3, 11)$ **(b)** $(0, 20)$, $(1, 7)$
105.



107. $\dfrac{1}{4}$ **109.** 4
111. Tangent: $y = -\dfrac{1}{4}x + \dfrac{9}{4}$, normal: $y = 4x - 2$
113. Tangent: $y = 2x - 4$, normal: $y = -\dfrac{1}{2}x + \dfrac{7}{2}$
115. Tangent: $y = -\dfrac{5}{4}x + 6$, normal: $y = \dfrac{4}{5}x - \dfrac{11}{5}$
117. $(1, 1)$: $m = -\dfrac{1}{2}$; $(1, -1)$: m not defined
119. B = graph of f, A = graph of f'
121.



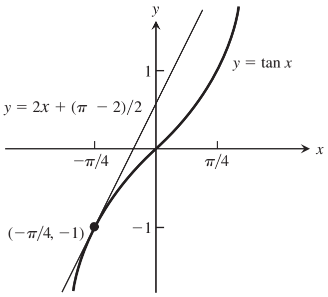
123. (a) 0, 0 **(b)** 1700 rabbits, $\approx$1400 rabbits
125. -1 **127.** $1/2$ **129.** 4 **131.** 1
133. To make g continuous at the origin, define $g(0) = 1$.
135. $\dfrac{2(x^2 + 1)}{\sqrt{\cos 2x}}\left[\dfrac{2x}{x^2 + 1} + \tan 2x\right]$
137. $5\left[\dfrac{(t + 1)(t - 1)}{(t - 2)(t + 3)}\right]^5\left[\dfrac{1}{t + 1} + \dfrac{1}{t - 1} - \dfrac{1}{t - 2} - \dfrac{1}{t + 3}\right]$
139. $\dfrac{1}{\sqrt{\theta}}(\sin\theta)^{\sqrt{\theta}}\left(\dfrac{\ln\sin\theta}{2} + \theta\cot\theta\right)$
141. (a) $\dfrac{dS}{dt} = (4\pi r + 2\pi h)\dfrac{dr}{dt}$ **(b)** $\dfrac{dS}{dt} = 2\pi r\dfrac{dh}{dt}$
(c) $\dfrac{dS}{dt} = (4\pi r + 2\pi h)\dfrac{dr}{dt} + 2\pi r\dfrac{dh}{dt}$
(d) $\dfrac{dr}{dt} = -\dfrac{r}{2r + h}\dfrac{dh}{dt}$

143. $-40\ \mathrm{m}^2/\mathrm{sec}$ **145.** 0.02 ohm/sec **147.** 2 m/sec

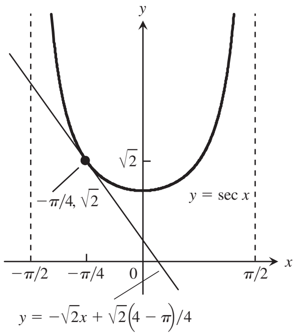
149. (a) $r = \frac{2}{5}h$ **(b)** $-\frac{125}{144\pi}$ ft/min

151. (a) $\frac{3}{5}$ km/sec or 600 m/sec **(b)** $\frac{18}{\pi}$ rpm

153. (a) $L(x) = 2x + \frac{\pi - 2}{2}$



(b) $L(x) = -\sqrt{2}x + \frac{\sqrt{2}(4 - \pi)}{4}$



155. $L(x) = 1.5x + 0.5$ **157.** $dS = \frac{\pi r h_0}{\sqrt{r^2 + h_0^2}}\, dh$

159. (a) 4% **(b)** 8% **(c)** 12%

Additional and Advanced Exercises, pp. 219–222

1. (a) $\sin 2\theta = 2\sin\theta\cos\theta$; $2\cos 2\theta = 2\sin\theta(-\sin\theta) + \cos\theta(2\cos\theta)$; $2\cos 2\theta = -2\sin^2\theta + 2\cos^2\theta$; $\cos 2\theta = \cos^2\theta - \sin^2\theta$

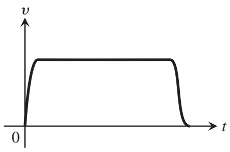
(b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$; $-2\sin 2\theta = 2\cos\theta(-\sin\theta) - 2\sin\theta(\cos\theta)$; $\sin 2\theta = \cos\theta\sin\theta + \sin\theta\cos\theta$; $\sin 2\theta = 2\sin\theta\cos\theta$

3. (a) $a = 1, b = 0, c = -\frac{1}{2}$ **(b)** $b = \cos a, c = \sin a$

5. $h = -4, k = \frac{9}{2}, a = \frac{5\sqrt{5}}{2}$

7. (a) $0.09y$ **(b)** Increasing at 1% per year

9. Answers will vary. Here is one possibility.



11. (a) 2 sec, 64 ft/sec **(b)** 12.31 sec, 393.85 ft

15. (a) $m = -\frac{b}{\pi}$ **(b)** $m = -1, b = \pi$

17. (a) $a = \frac{3}{4}, b = \frac{9}{4}$ **19.** f odd $\Rightarrow f'$ is even

23. h' is defined but not continuous at $x = 0$; k' is defined *and* continuous at $x = 0$.

27. (a) 0.8156 ft **(b)** 0.00613 sec
(c) It will lose about 8.83 min/day.

Chapter 4

Section 4.1, pp. 228–231

1. Absolute minimum at $x = c_2$; absolute maximum at $x = b$

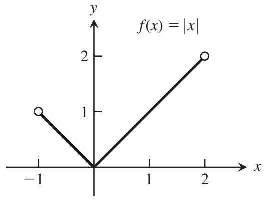
3. Absolute maximum at $x = c$; no absolute minimum

5. Absolute minimum at $x = a$; absolute maximum at $x = c$

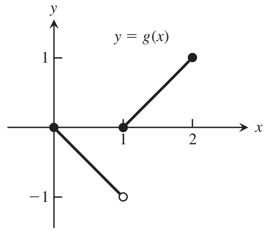
7. No absolute minimum; no absolute maximum

9. Absolute maximum at (0, 5) **11.** (c) **13.** (d)

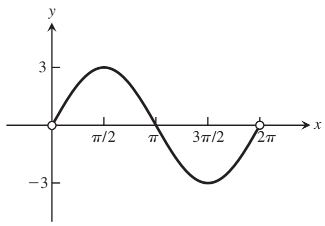
15. Absolute minimum at $x = 0$; no absolute maximum



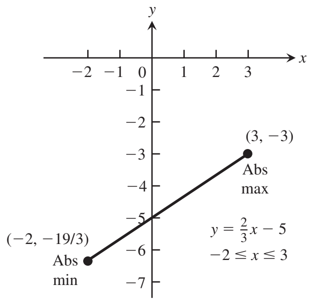
17. Absolute maximum at $x = 2$; no absolute minimum



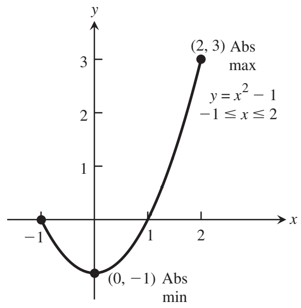
19. Absolute maximum at $x = \pi/2$; absolute minimum at $x = 3\pi/2$



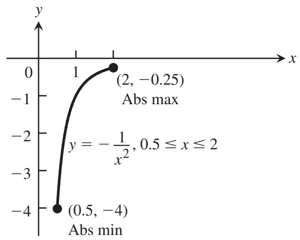
21. Absolute maximum: -3; absolute minimum: $-19/3$



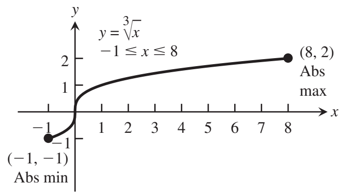
23. Absolute maximum: 3; absolute minimum: -1



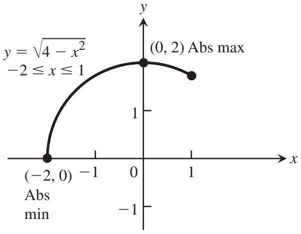
25. Absolute maximum: -0.25; absolute minimum: -4



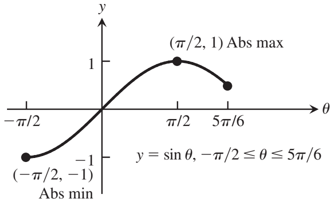
27. Absolute maximum: 2; absolute minimum: -1



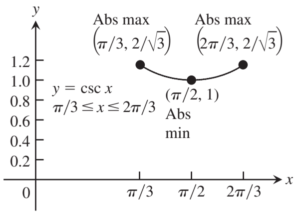
29. Absolute maximum: 2; absolute minimum: 0



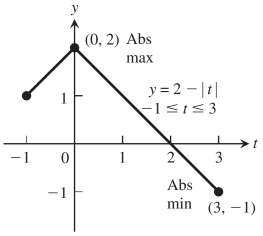
31. Absolute maximum: 1; absolute minimum: −1



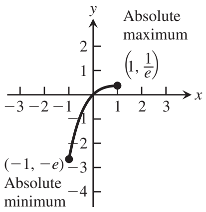
33. Absolute maximum: $2/\sqrt{3}$; absolute minimum: 1



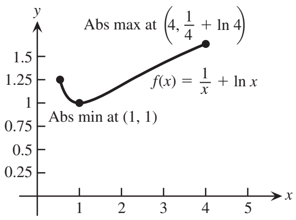
35. Absolute maximum: 2; absolute minimum: −1



37. Absolute maximum is $1/e$ at $x = 1$; absolute minimum is $-e$ at $x = -1$.



39. Absolute maximum value is $(1/4) + \ln 4$ at $x = 4$; absolute minimum value is 1 at $x = 1$; local maximum at $(1/2, 2 - \ln 2)$.



41. Increasing on (0, 8), decreasing on (−1, 0); absolute maximum: 16 at $x = 8$; absolute minimum: 0 at $x = 0$

43. Increasing on (−32, 1); absolute maximum: 1 at $\theta = 1$; absolute minimum: −8 at $\theta = -32$

45. $x = 3$

47. $x = 1, x = 4$

49. $x = 1$

51. $x = 0$ and $x = 4$

53. Minimum value is 1 at $x = 2$.

55. Local maximum at (−2, 17); local minimum at $\left(\frac{4}{3}, -\frac{41}{27}\right)$

57. Minimum value is 0 at $x = -1$ and $x = 1$.

59. There is a local minimum at (0, 1).

61. Maximum value is $\frac{1}{2}$ at $x = 1$; minimum value is $-\frac{1}{2}$ at $x = -1$.

63. The minimum value is 2 at $x = 0$.

65. The minimum value is $-\frac{1}{e}$ at $x = \frac{1}{e}$.

67. The maximum value is $\frac{\pi}{2}$ at $x = 0$; an absolute minimum value is 0 at $x = 1$ and $x = -1$.

69.

Critical point or endpoint	Derivative	Extremum	Value
$x = -\frac{4}{5}$	0	Local max	$\frac{12}{25}10^{1/3} \approx 1.034$
$x = 0$	Undefined	Local min	0

71.

Critical point or endpoint	Derivative	Extremum	Value
$x = -2$	Undefined	Local max	0
$x = -\sqrt{2}$	0	Minimum	−2
$x = \sqrt{2}$	0	Maximum	2
$x = 2$	Undefined	Local min	0

73.

Critical point or endpoint	Derivative	Extremum	Value
$x = 1$	Undefined	Minimum	2

75.

Critical point or endpoint	Derivative	Extremum	Value
$x = -1$	0	Maximum	5
$x = 1$	Undefined	Local min	1
$x = 3$	0	Maximum	5

77. **(a)** No
(b) The derivative is defined and nonzero for $x \neq 2$. Also, $f(2) = 0$ and $f(x) > 0$ for all $x \neq 2$.
(c) No, because $(-\infty, \infty)$ is not a closed interval.
(d) The answers are the same as parts (a) and (b), with 2 replaced by a.

79. Yes

81. g assumes a local maximum at $-c$.

83. **(a)** Maximum value is 144 at $x = 2$.
(b) The largest volume of the box is 144 cubic units, and it occurs when $x = 2$.

85. $\frac{v_0^2}{2g} + s_0$

87. Maximum value is 11 at $x = 5$; minimum value is 5 on the interval [−3, 2]; local maximum at (−5, 9).

89. Maximum value is 5 on the interval [3, ∞); minimum value is −5 on the interval (−∞, −2].

Section 4.2, pp. 237–239

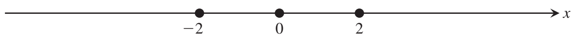
1. 1/2 **3.** 1 **5.** $\pm\sqrt{1 - \frac{4}{\pi^2}} \approx \pm 0.771$

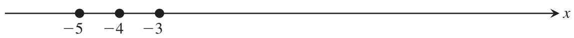
7. $\frac{1}{3}\left(1 + \sqrt{7}\right) \approx 1.22, \frac{1}{3}\left(1 - \sqrt{7}\right) \approx -0.549$

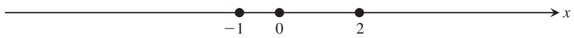
9. Does not; f is not differentiable at the interior domain point $x = 0$.

11. Does **13.** Does not; f is not differentiable at $x = -1$.

17. **(a)**

i)


ii)


iii)


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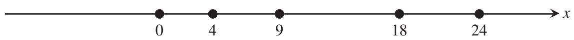


29. Yes **31. (a)** 4 **(b)** 3 **(c)** 3

33. (a) $\frac{x^2}{2} + C$ **(b)** $\frac{x^3}{3} + C$ **(c)** $\frac{x^4}{4} + C$

35. (a) $\frac{1}{x} + C$ **(b)** $x + \frac{1}{x} + C$ **(c)** $5x - \frac{1}{x} + C$

37. (a) $-\frac{1}{2}\cos 2t + C$ **(b)** $2\sin\frac{t}{2} + C$

(c) $-\frac{1}{2}\cos 2t + 2\sin\frac{t}{2} + C$

39. $f(x) = x^2 - x$ **41.** $f(x) = 1 + \frac{e^{2x}}{2}$

43. $s = 4.9t^2 + 5t + 10$ **45.** $s = \frac{1 - \cos(\pi t)}{\pi}$

47. $s = e^t + 19t + 4$ **49.** $s = \sin(2t) - 3$

51. If $T(t)$ is the temperature of the thermometer at time t, then $T(0) = -19$ °C and $T(14) = 100$ °C. From the Mean Value Theorem, there exists a $0 < t_0 < 14$ such that $\frac{T(14) - T(0)}{14 - 0} = 8.5$ °C/sec $= T'(t_0)$, the rate at which the temperature was changing at $t = t_0$ as measured by the rising mercury on the thermometer.

53. Because its average speed was approximately 7.667 knots, and by the Mean Value Theorem, it must have been going that speed at least once during the trip.

57. The conclusion of the Mean Value Theorem yields

$$\frac{\frac{1}{b} - \frac{1}{a}}{b - a} = -\frac{1}{c^2} \Rightarrow c^2\left(\frac{a - b}{ab}\right) = a - b \Rightarrow c = \sqrt{ab}.$$

61. $f(x)$ must be zero at least once between a and b by the Intermediate Value Theorem. Now suppose that $f(x)$ is zero twice between a and b. Then, by the Mean Value Theorem, $f'(x)$ would have to be zero at least once between the two zeros of $f(x)$, but this can't be true since we are given that $f'(x) \neq 0$ on this interval. Therefore, $f(x)$ is zero once and only once between a and b.

71. $1.09999 \leq f(0.1) \leq 1.1$

Section 4.3, pp. 242–244

1. (a) 0, 1
(b) Increasing on $(-\infty, 0)$ and $(1, \infty)$; decreasing on $(0, 1)$
(c) Local maximum at $x = 0$; local minimum at $x = 1$

3. (a) $-2, 1$
(b) Increasing on $(-2, 1)$ and $(1, \infty)$; decreasing on $(-\infty, -2)$
(c) No local maximum; local minimum at $x = -2$

5. (a) Critical point at $x = 1$
(b) Decreasing on $(-\infty, 1)$, increasing on $(1, \infty)$
(c) Local (and absolute) minimum at $x = 1$

7. (a) 0, 1
(b) Increasing on $(-\infty, -2)$ and $(1, \infty)$; decreasing on $(-2, 0)$ and $(0, 1)$
(c) Local minimum at $x = 1$

9. (a) $-2, 2$
(b) Increasing on $(-\infty, -2)$ and $(2, \infty)$; decreasing on $(-2, 0)$ and $(0, 2)$
(c) Local maximum at $x = -2$; local minimum at $x = 2$

11. (a) $-2, 0$
(b) Increasing on $(-\infty, -2)$ and $(0, \infty)$; decreasing on $(-2, 0)$
(c) Local maximum at $x = -2$; local minimum at $x = 0$

13. (a) $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$
(b) Increasing on $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$; decreasing on $\left(0, \frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$, and $\left(\frac{4\pi}{3}, 2\pi\right)$
(c) Local maximum at $x = 0$ and $x = \frac{4\pi}{3}$; local minimum at $x = \frac{2\pi}{3}$ and $x = 2\pi$

15. (a) Increasing on $(-2, 0)$ and $(2, 4)$; decreasing on $(-4, -2)$ and $(0, 2)$
(b) Absolute maximum at $(-4, 2)$; local maximum at $(0, 1)$ and $(4, -1)$; absolute minimum at $(2, -3)$; local minimum at $(-2, 0)$

17. (a) Increasing on $(-4, -1)$, $(1/2, 2)$, and $(2, 4)$; decreasing on $(-1, 1/2)$
(b) Absolute maximum at $(4, 3)$; local maximum at $(-1, 2)$ and $(2, 1)$; no absolute minimum; local minimum at $(-4, -1)$ and $(1/2, -1)$

19. (a) Increasing on $(-\infty, -1.5)$; decreasing on $(-1.5, \infty)$
(b) Local maximum: 5.25 at $t = -1.5$; absolute maximum: 5.25 at $t = -1.5$

21. (a) Decreasing on $(-\infty, 0)$; increasing on $(0, 4/3)$; decreasing on $(4/3, \infty)$
(b) Local minimum at $x = 0$ $(0, 0)$; local maximum at $x = 4/3$ $(4/3, 32/27)$; no absolute extrema

23. (a) Decreasing on $(-\infty, 0)$; increasing on $(0, 1/2)$; decreasing on $(1/2, \infty)$
(b) Local minimum at $\theta = 0$ $(0, 0)$; local maximum at $\theta = 1/2$ $(1/2, 1/4)$; no absolute extrema

25. (a) Increasing on $(-\infty, \infty)$; never decreasing
(b) No local extrema; no absolute extrema

27. (a) Increasing on $(-2, 0)$ and $(2, \infty)$; decreasing on $(-\infty, -2)$ and $(0, 2)$
(b) Local maximum: 16 at $x = 0$; local minimum: 0 at $x = \pm 2$; no absolute maximum; absolute minimum: 0 at $x = \pm 2$

29. (a) Increasing on $(-\infty, -1)$; decreasing on $(-1, 0)$; increasing on $(0, 1)$; decreasing on $(1, \infty)$
(b) Local maximum: 0.5 at $x = \pm 1$; local minimum: 0 at $x = 0$; absolute maximum: $1/2$ at $x = \pm 1$; no absolute minimum

31. (a) Increasing on $(10, \infty)$; decreasing on $(1, 10)$
(b) Local maximum: 1 at $x = 1$; local minimum: -8 at $x = 10$; absolute minimum: -8 at $x = 10$

33. (a) Decreasing on $\left(-2\sqrt{2}, -2\right)$; increasing on $(-2, 2)$; decreasing on $\left(2, 2\sqrt{2}\right)$
(b) Local minima: $g(-2) = -4$, $g\left(2\sqrt{2}\right) = 0$; local maxima: $g\left(-2\sqrt{2}\right) = 0$, $g(2) = 4$; absolute maximum: 4 at $x = 2$; absolute minimum: -4 at $x = -2$

35. (a) Increasing on $(-\infty, 1)$; decreasing when $1 < x < 2$, decreasing when $2 < x < 3$; discontinuous at $x = 2$; increasing on $(3, \infty)$
(b) Local minimum at $x = 3$ $(3, 6)$; local maximum at $x = 1$ $(1, 2)$; no absolute extrema

37. (a) Increasing on $(-2, 0)$ and $(0, \infty)$; decreasing on $(-\infty, -2)$
(b) Local minimum: $-6\sqrt[3]{2}$ at $x = -2$; no absolute maximum; absolute minimum: $-6\sqrt[3]{2}$ at $x = -2$

39. (a) Increasing on $\left(-\infty, -2/\sqrt{7}\right)$ and $\left(2/\sqrt{7}, \infty\right)$; decreasing on $\left(-2/\sqrt{7}, 0\right)$ and $\left(0, 2/\sqrt{7}\right)$
(b) Local maximum: $24\sqrt[3]{2}/7^{7/6} \approx 3.12$ at $x = -2/\sqrt{7}$; local minimum: $-24\sqrt[3]{2}/7^{7/6} \approx -3.12$ at $x = 2/\sqrt{7}$; no absolute extrema

41. **(a)** Increasing on $((1/3)\ln(1/2), \infty)$, decreasing on $(-\infty, (1/3)\ln(1/2))$
(b) Local minimum is $\frac{3}{2^{2/3}}$ at $x = (1/3)\ln(1/2)$; no local maximum; absolute minimum is $\frac{3}{2^{2/3}}$ at $x = (1/3)\ln(1/2)$; no absolute maximum

43. **(a)** Increasing on (e^{-1}, ∞), decreasing on $(0, e^{-1})$
(b) A local minimum is $-e^{-1}$ at $x = e^{-1}$, no local maximum; absolute minimum is $-e^{-1}$ at $x = e^{-1}$, no absolute maximum

45. **(a)** Local maximum: 1 at $x = 1$; local minimum: 0 at $x = 2$
(b) Absolute maximum: 1 at $x = 1$; no absolute minimum

47. **(a)** Local maximum: 1 at $x = 1$; local minimum: 0 at $x = 2$
(b) No absolute maximum; absolute minimum: 0 at $x = 2$

49. **(a)** Local maxima: -9 at $t = -3$ and 16 at $t = 2$; local minimum: -16 at $t = -2$
(b) Absolute maximum: 16 at $t = 2$; no absolute minimum

51. **(a)** Local minimum: 0 at $x = 0$
(b) No absolute maximum; absolute minimum: 0 at $x = 0$

53. **(a)** Local maximum: 5 at $x = 0$; local minimum: 0 at $x = -5$ and $x = 5$
(b) Absolute maximum: 5 at $x = 0$; absolute minimum: 0 at $x = -5$ and $x = 5$

55. **(a)** Local maximum: 2 at $x = 0$;
local minimum: $\frac{\sqrt{3}}{4\sqrt{3} - 6}$ at $x = 2 - \sqrt{3}$
(b) No absolute maximum; an absolute minimum at $x = 2 - \sqrt{3}$

57. **(a)** Local maximum: 1 at $x = \pi/4$;
local maximum: 0 at $x = \pi$;
local minimum: 0 at $x = 0$;
local minimum: -1 at $x = 3\pi/4$

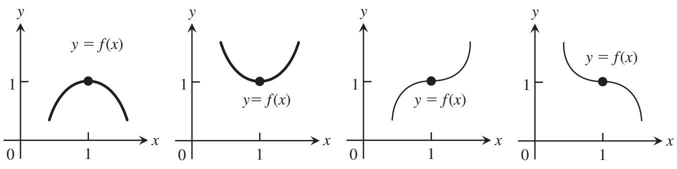
59. Local maximum: 2 at $x = \pi/6$;
local maximum: $\sqrt{3}$ at $x = 2\pi$;
local minimum: -2 at $x = 7\pi/6$;
local minimum: $\sqrt{3}$ at $x = 0$

61. **(a)** Local minimum: $(\pi/3) - \sqrt{3}$ at $x = 2\pi/3$;
local maximum: 0 at $x = 0$;
local maximum: π at $x = 2\pi$

63. **(a)** Local minimum: 0 at $x = \pi/4$

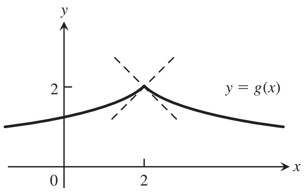
65. Local maximum: 3 at $\theta = 0$;
local minimum: -3 at $\theta = 2\pi$

67.

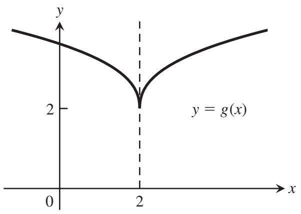


(a) **(b)** **(c)** **(d)**

69. **(a)**



(b)



73. $a = -2, b = 4$

75. **(a)** Absolute minimum occurs at $x = \pi/3$ with $f(\pi/3) = -\ln 2$, and the absolute maximum occurs at $x = 0$ with $f(0) = 0$.
(b) Absolute minimum occurs at $x = 1/2$ and $x = 2$ with $f(1/2) = f(2) = \cos(\ln 2)$, and the absolute maximum occurs at $x = 1$ with $f(1) = 1$.

77. Minimum of $2 - 2\ln 2 \approx 0.613706$ at $x = \ln 2$; maximum of 1 at $x = 0$

79. Absolute maximum value of $1/2e$ assumed at $x = 1/\sqrt{e}$

83. Increasing; $\frac{df^{-1}}{dx} = \frac{1}{9}x^{-2/3}$

85. Decreasing; $\frac{df^{-1}}{dx} = -\frac{1}{3}x^{-2/3}$

Section 4.4, pp. 252–255

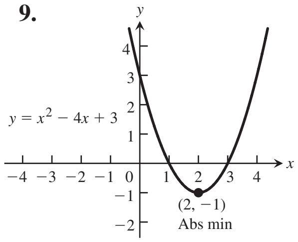
1. Local maximum: $3/2$ at $x = -1$; local minimum: -3 at $x = 2$; point of inflection at $(1/2, -3/4)$; rising on $(-\infty, -1)$ and $(2, \infty)$; falling on $(-1, 2)$; concave up on $(1/2, \infty)$; concave down on $(-\infty, 1/2)$

3. Local maximum: $3/4$ at $x = 0$; local minimum: 0 at $x = \pm 1$; points of inflection at $\left(-\sqrt{3}, \frac{3\sqrt[3]{4}}{4}\right)$ and $\left(\sqrt{3}, \frac{3\sqrt[3]{4}}{4}\right)$; rising on $(-1, 0)$ and $(1, \infty)$; falling on $(-\infty, -1)$ and $(0, 1)$; concave up on $\left(-\infty, -\sqrt{3}\right)$ and $\left(\sqrt{3}, \infty\right)$; concave down on $\left(-\sqrt{3}, \sqrt{3}\right)$

5. Local maxima: $\frac{-2\pi}{3} + \frac{\sqrt{3}}{2}$ at $x = -2\pi/3$, $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ at $x = \pi/3$; local minima: $-\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ at $x = -\pi/3$, $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ at $x = 2\pi/3$; points of inflection at $(-\pi/2, -\pi/2)$, $(0, 0)$, and $(\pi/2, \pi/2)$; rising on $(-\pi/3, \pi/3)$; falling on $(-2\pi/3, -\pi/3)$ and $(\pi/3, 2\pi/3)$; concave up on $(-\pi/2, 0)$ and $(\pi/2, 2\pi/3)$; concave down on $(-2\pi/3, -\pi/2)$ and $(0, \pi/2)$

7. Local maxima: 1 at $x = -\pi/2$ and $x = \pi/2$, 0 at $x = -2\pi$ and $x = 2\pi$; local minima: -1 at $x = -3\pi/2$ and $x = 3\pi/2$, 0 at $x = 0$; points of inflection at $(-\pi, 0)$ and $(\pi, 0)$; rising on $(-3\pi/2, -\pi/2)$, $(0, \pi/2)$, and $(3\pi/2, 2\pi)$; falling on $(-2\pi, -3\pi/2)$, $(-\pi/2, 0)$, and $(\pi/2, 3\pi/2)$; concave up on $(-2\pi, -\pi)$ and $(\pi, 2\pi)$; concave down on $(-\pi, 0)$ and $(0, \pi)$

9.



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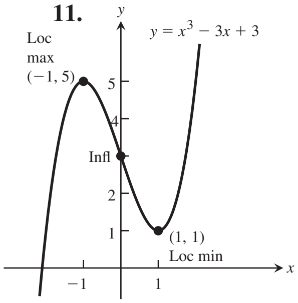

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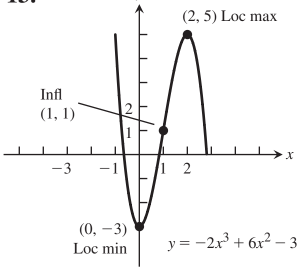

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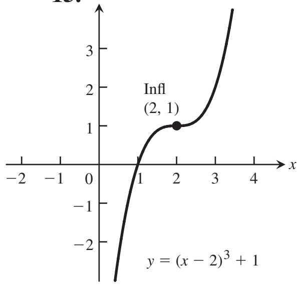

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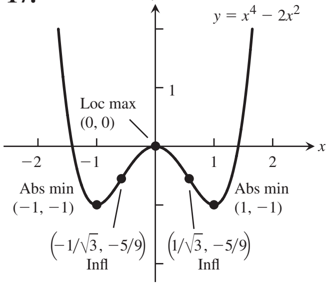

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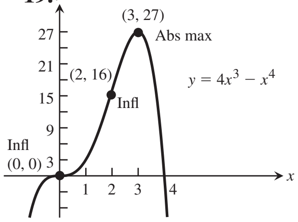

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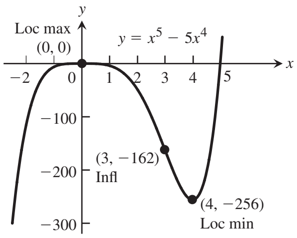

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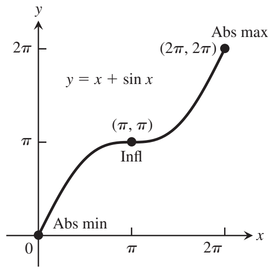

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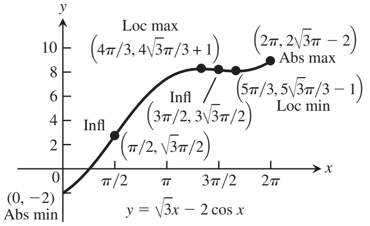

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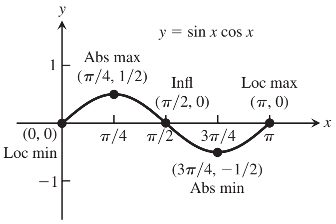

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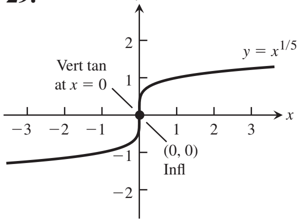

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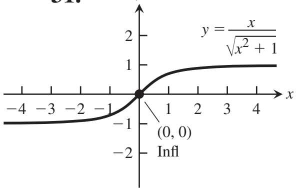

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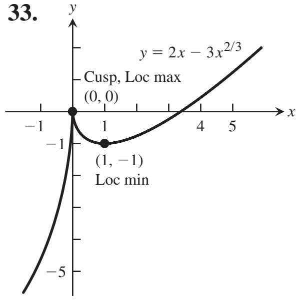

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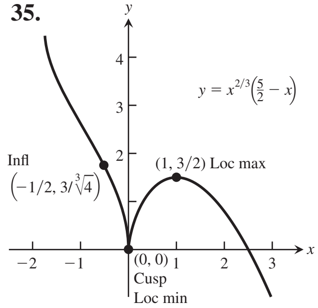

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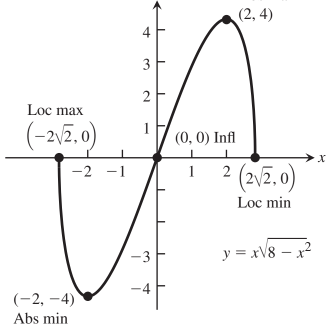

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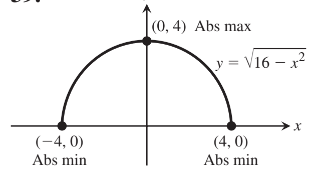

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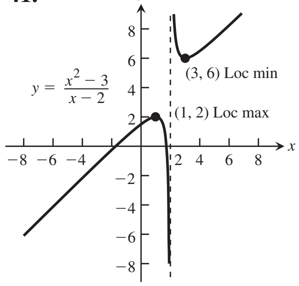

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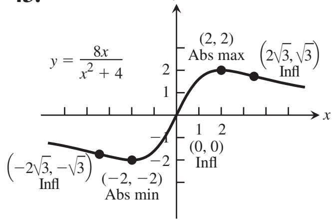

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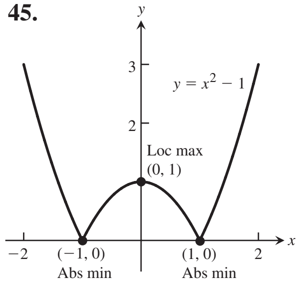

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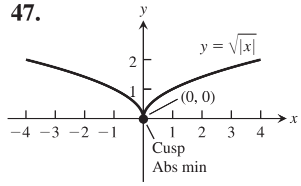

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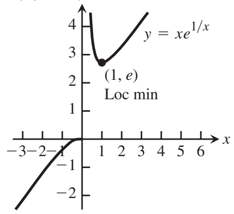

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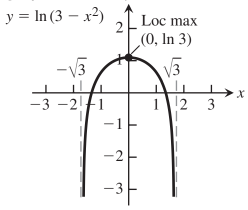

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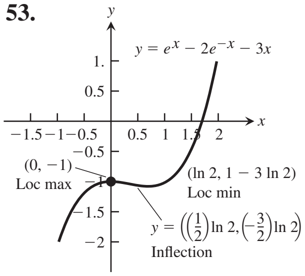

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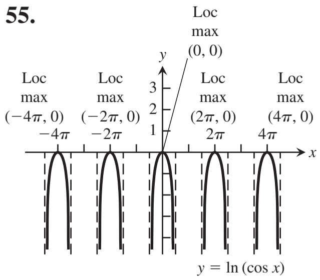

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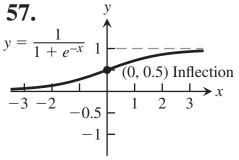
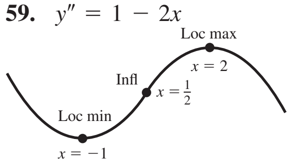
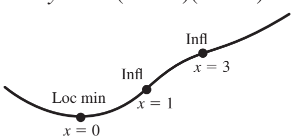


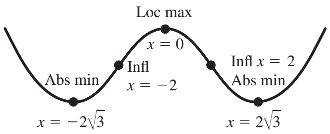
59. $y'' = 1 - 2x$



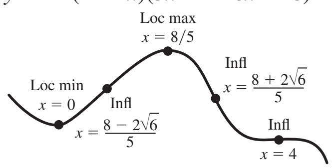
61. $y'' = 3(x - 3)(x - 1)$



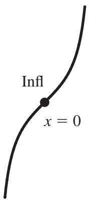
63. $y'' = 3(x - 2)(x + 2)$



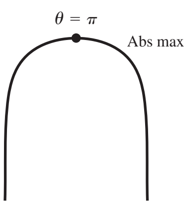
65. $y'' = 4(4 - x)(5x^2 - 16x + 8)$



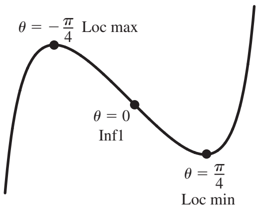
67. $y'' = 2 \sec^2 x \tan x$



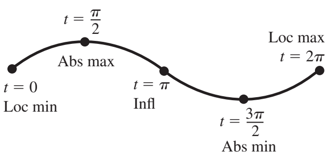
69. $y'' = -\frac{1}{2} \csc^2 \frac{\theta}{2}$, $0 < \theta < 2\pi$



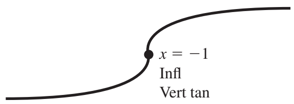
71. $y'' = 2 \tan \theta \sec^2 \theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$



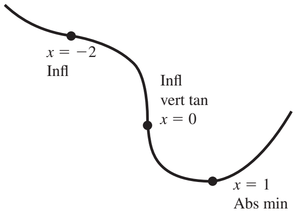
73. $y'' = -\sin t, 0 \leq t \leq 2\pi$



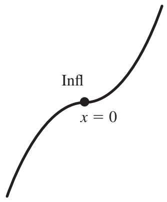
75. $y'' = -\frac{2}{3}(x + 1)^{-5/3}$



77. $y'' = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3}$



79. $y'' = \begin{cases} -2, & x < 0 \\ 2, & x > 0 \end{cases}$



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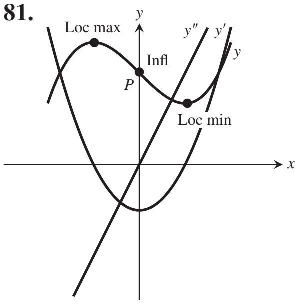

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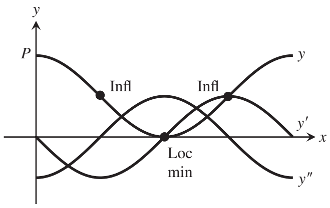

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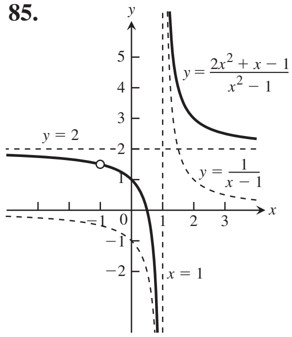

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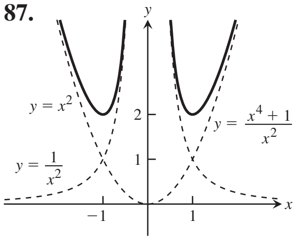

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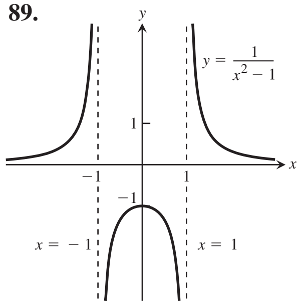

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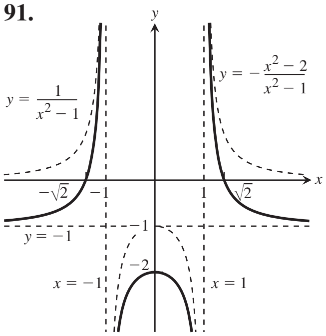

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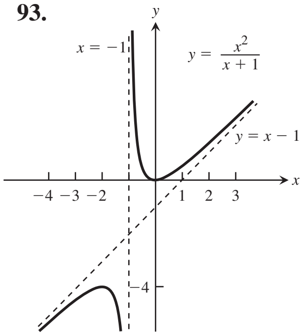

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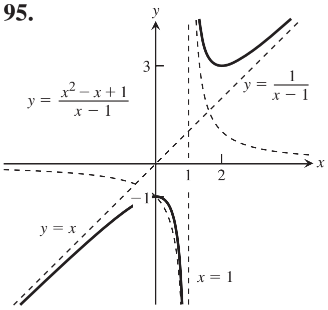

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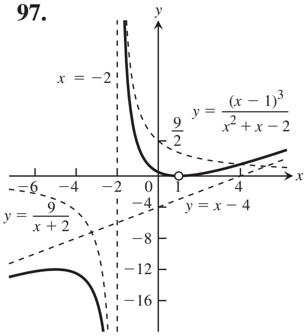

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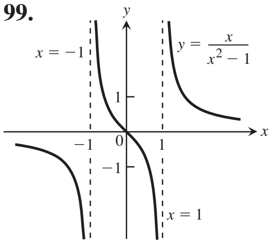
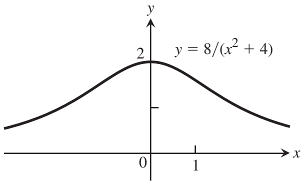


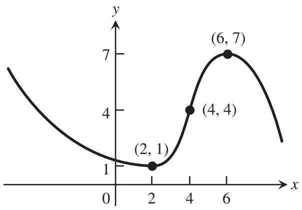
101.



103.

Point	y'	y''
P	$-$	$+$
Q	$+$	0
R	$+$	$-$
S	0	$-$
T	$-$	$-$

105.



107. **(a)** Towards origin: $0 \le t < 2$ and $6 \le t \le 10$; away from origin: $2 \le t \le 6$ and $10 \le t \le 15$
(b) $t = 2, t = 6, t = 10$
(c) $t = 5, t = 7, t = 13$
(d) Positive: $5 \le t \le 7$, $13 \le t \le 15$; negative: $0 \le t \le 5$, $7 \le t \le 13$

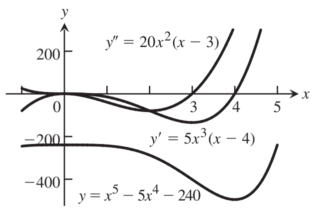
109. ≈ 60 thousand units

111. Local minimum at $x = 2$; inflection points at $x = 1$ and $x = 5/3$

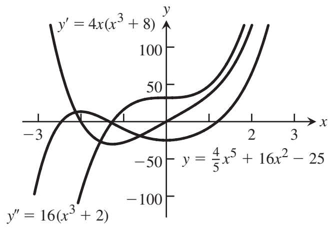
115. $b = -3$ **119.** $-1, 2$

121. $a = 1, b = 3, c = 9$

123. The zeros of $y' = 0$ and $y'' = 0$ are extrema and points of inflection, respectively. Inflection at $x = 3$, local maximum at $x = 0$, local minimum at $x = 4$.



125. The zeros of $y' = 0$ and $y'' = 0$ are extrema and points of inflection, respectively. Inflection at $x = -\sqrt[3]{2}$; local maximum at $x = -2$; local minimum at $x = 0$.

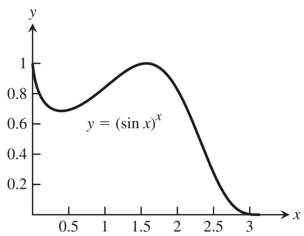


Section 4.5, pp. 262–263

1. $-1/4$ **3.** $5/7$ **5.** $1/2$ **7.** $1/4$ **9.** $-23/7$
11. $5/7$ **13.** 0 **15.** -16 **17.** -2 **19.** $1/4$
21. 2 **23.** 3 **25.** -1 **27.** $\ln 3$ **29.** $\frac{1}{\ln 2}$ **31.** $\ln 2$
33. 1 **35.** $1/2$ **37.** $\ln 2$ **39.** $-\infty$ **41.** $-1/2$
43. -1 **45.** 1 **47.** 0 **49.** 2 **51.** $1/e$ **53.** 1
55. $1/e$ **57.** $e^{1/2}$ **59.** 1 **61.** e^3 **63.** 0 **65.** 1
67. 3 **69.** 1 **71.** 0 **73.** ∞ **75.** (b) is correct.
77. (d) is correct. **79.** $c = \frac{27}{10}$ **81.** **(b)** $\frac{-1}{2}$ **83.** -1

87. **(a)** $y = 1$ **(b)** $y = 0, y = \frac{3}{2}$

89. **(a)** We should assign the value 1 to $f(x) = (\sin x)^x$ to make it continuous at $x = 0$.



(c) The maximum value of $f(x)$ is close to 1 near the point $x \approx 1.55$ (see the graph in part (a)).

Section 4.6, pp. 270–276

1. 16 in., 4 in. by 4 in.

3. **(a)** $(x, 1 - x)$ **(b)** $A(x) = 2x(1 - x)$
(c) $\frac{1}{2}$ square units, 1 by $\frac{1}{2}$

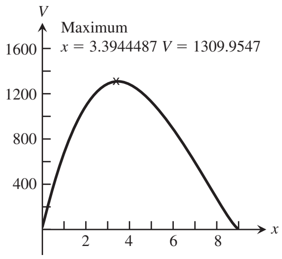
5. $\frac{14}{3} \times \frac{35}{3} \times \frac{5}{3}$ in., $\frac{2450}{27}$ in^3

7. 80,000 m^2; 400 m by 200 m

9. **(a)** The optimum dimensions of the tank are 10 ft on the base edges and 5 ft deep.
(b) Minimizing the surface area of the tank minimizes its weight for a given wall thickness. The thickness of the steel walls would likely be determined by other considerations such as structural requirements.

11. 9×18 in. **13.** $\frac{\pi}{2}$ **15.** $h:r = 8:\pi$

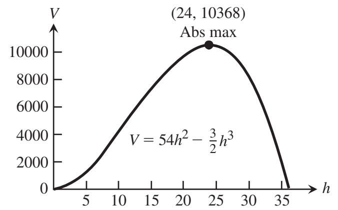
17. **(a)** $V(x) = 2x(24 - 2x)(18 - 2x)$ **(b)** Domain: $(0, 9)$



(c) Maximum volume $\approx 1309.95 \text{ in}^3$ when $x \approx 3.39$ in.
(d) $V'(x) = 24x^2 - 336x + 864$, so the critical point is at $x = 7 - \sqrt{13}$, which confirms the result in part (c).
(e) $x = 2$ in. or $x = 5$ in.

19. $\approx 2418.40 \text{ cm}^3$

21. **(a)** $h = 24, w = 18$
(b)



23. If r is the radius of the hemisphere, h the height of the cylinder, and V the volume, then $r = \left(\frac{3V}{8\pi}\right)^{1/3}$ and $h = \left(\frac{3V}{\pi}\right)^{1/3}$.

25. **(b)** $x = \frac{51}{8}$ **(c)** $L \approx 11$ in.

27. Radius $= \sqrt{2}$ m, height $= 1$ m, volume $= \frac{2\pi}{3}\text{m}^3$

29. 1 **31.** $\frac{9b}{9 + \sqrt{3}\pi}$ m, triangle; $\frac{b\sqrt{3}\pi}{9 + \sqrt{3}\pi}$ m, circle

33. $\frac{3}{2} \times 2$

35. **(a)** 16 **(b)** -1

37. **(a)** $v(0) = 96$ ft/sec
(b) 256 ft at $t = 3$ sec
(c) Velocity when $s = 0$ is $v(7) = -128$ ft/sec.

39. ≈ 46.87 ft **41.** **(a)** $6 \times 6\sqrt{3}$ in.

43. **(a)** $4\sqrt{3} \times 4\sqrt{6}$ in.

45. **(a)** $10\pi \approx 31.42$ cm/sec; when $t = 0.5$ sec, 1.5 sec, 2.5 sec, 3.5 sec; $s = 0$, acceleration is 0.
(b) 10 cm from rest position; speed is 0.

47. **(a)** $s = ((12 - 12t)^2 + 64t^2)^{1/2}$
(b) -12 knots, 8 knots
(c) No
(d) $4\sqrt{13}$. This limit is the square root of the sums of the squares of the individual speeds.

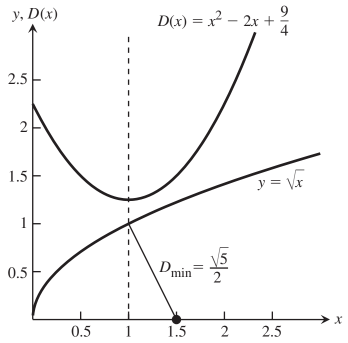
49. $x = \frac{a}{2}, v = \frac{ka^2}{4}$

51. $\frac{c}{2} + 50$

53. **(a)** $\sqrt{\frac{2km}{h}}$ **(b)** $\sqrt{\frac{2km}{h}}$

57. $4 \times 4 \times 3$ ft, \$288 **59.** $M = \frac{C}{2}$ **65.** **(a)** $y = -1$

67. **(a)** The minimum distance is $\frac{\sqrt{5}}{2}$.
(b) The minimum distance is from the point $(3/2, 0)$ to the point $(1, 1)$ on the graph of $y = \sqrt{x}$, and this occurs at the value $x = 1$, where $D(x)$, the distance squared, has its minimum value.

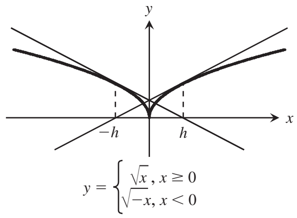


Section 4.7, pp. 279–280

1. $x_2 = -\frac{5}{3}, \frac{13}{21}$ **3.** $x_2 = -\frac{51}{31}, \frac{5763}{4945}$ **5.** $x_2 = \frac{2387}{2000}$

7. x_1, and all later approximations will equal x_0.

9.



11. The points of intersection of $y = x^3$ and $y = 3x + 1$ or $y = x^3 - 3x$ and $y = 1$ have the same x-values as the roots of part (i) or the solutions of part (iv). **13.** 1.165561185

15. **(a)** Two **(b)** 0.35003501505249 and -1.0261731615301

17. ± 1.3065629648764, ± 0.5411961001462 **19.** $x \approx 0.45$

21. 0.8192 **23.** 0, 0.53485 **25.** The root is 1.17951.

27. **(a)** For $x_0 = -2$ or $x_0 = -0.8$, $x_i \to -1$ as i gets large.
(b) For $x_0 = -0.5$ or $x_0 = 0.25$, $x_i \to 0$ as i gets large.
(c) For $x_0 = 0.8$ or $x_0 = 2$, $x_i \to 1$ as i gets large.
(d) For $x_0 = -\sqrt{21}/7$ or $x_0 = \sqrt{21}/7$, Newton's method does not converge. The values of x_i alternate between $-\sqrt{21}/7$ and $\sqrt{21}/7$ as i increases.

29. Answers will vary with machine speed.

Section 4.8, pp. 287–290

1. **(a)** x^2 **(b)** $\frac{x^3}{3}$ **(c)** $\frac{x^3}{3} - x^2 + x$

3. **(a)** x^{-3} **(b)** $-\frac{1}{3}x^{-3}$ **(c)** $-\frac{1}{3}x^{-3} + x^2 + 3x$

5. **(a)** $-\frac{1}{x}$ **(b)** $-\frac{5}{x}$ **(c)** $2x + \frac{5}{x}$

7. **(a)** $\sqrt{x^3}$ **(b)** $\sqrt{x}$ **(c)** $\frac{2\sqrt{x^3}}{3} + 2\sqrt{x}$

9. **(a)** $x^{2/3}$ **(b)** $x^{1/3}$ **(c)** $x^{-1/3}$

11. **(a)** $\ln x$ **(b)** $7 \ln x$ **(c)** $x - 5 \ln x$

13. **(a)** $\cos(\pi x)$ **(b)** $-3 \cos x$ **(c)** $-\frac{1}{\pi}\cos(\pi x) + \cos(3x)$

15. **(a)** $\tan x$ **(b)** $2 \tan\left(\frac{x}{3}\right)$ **(c)** $-\frac{2}{3}\tan\left(\frac{3x}{2}\right)$

17. **(a)** $-\csc x$ **(b)** $\frac{1}{5}\csc(5x)$ **(c)** $2\csc\left(\frac{\pi x}{2}\right)$

19. **(a)** $\frac{1}{3}e^{3x}$ **(b)** $-e^{-x}$ **(c)** $2e^{x/2}$

21. **(a)** $\frac{1}{\ln 3}3^x$ **(b)** $\frac{-1}{\ln 2}2^{-x}$ **(c)** $\frac{1}{\ln(5/3)}\left(\frac{5}{3}\right)^x$

23. **(a)** $2\sin^{-1}x$ **(b)** $\frac{1}{2}\tan^{-1}x$ **(c)** $\frac{1}{2}\tan^{-1}2x$

25. $\frac{x^2}{2}+x+C$ **27.** $t^3+\frac{t^2}{4}+C$ **29.** $\frac{x^4}{2}-\frac{5x^2}{2}+7x+C$

31. $-\frac{1}{x}-\frac{x^3}{3}-\frac{x}{3}+C$ **33.** $\frac{3}{2}x^{2/3}+C$

35. $\frac{2}{3}x^{3/2}+\frac{3}{4}x^{4/3}+C$ **37.** $4y^2-\frac{8}{3}y^{3/4}+C$

39. $x^2+\frac{2}{x}+C$ **41.** $2\sqrt{t}-\frac{2}{\sqrt{t}}+C$ **43.** $-2\sin t+C$

45. $-21\cos\frac{\theta}{3}+C$ **47.** $3\cot x+C$ **49.** $-\frac{1}{2}\csc\theta+C$

51. $\frac{1}{3}e^{3x}-5e^{-x}+C$ **53.** $-e^{-x}+\frac{4^x}{\ln 4}+C$

55. $4\sec x-2\tan x+C$ **57.** $-\frac{1}{2}\cos 2x+\cot x+C$

59. $\frac{t}{2}+\frac{\sin 4t}{8}+C$ **61.** $\ln|x|-5\tan^{-1}x+C$

63. $\frac{3x^{(\sqrt{3}+1)}}{\sqrt{3}+1}+C$ **65.** $\tan\theta+C$ **67.** $-\cot x-x+C$

69. $-\cos\theta+\theta+C$

83. **(a)** Wrong: $\frac{d}{dx}\left(\frac{x^2}{2}\sin x+C\right)=\frac{2x}{2}\sin x+\frac{x^2}{2}\cos x=$ $x\sin x+\frac{x^2}{2}\cos x$

(b) Wrong: $\frac{d}{dx}(-x\cos x+C)=-\cos x+x\sin x$

(c) Right: $\frac{d}{dx}(-x\cos x+\sin x+C)=-\cos x+x\sin x+\cos x=x\sin x$

85. **(a)** Wrong: $\frac{d}{dx}\left(\frac{(2x+1)^3}{3}+C\right)=\frac{3(2x+1)^2(2)}{3}=$ $2(2x+1)^2$

(b) Wrong: $\frac{d}{dx}((2x+1)^3+C)=3(2x+1)^2(2)=$ $6(2x+1)^2$

(c) Right: $\frac{d}{dx}((2x+1)^3+C)=6(2x+1)^2$

87. Right **89.** (b) **91.** $y=x^2-7x+10$

93. $y=-\frac{1}{x}+\frac{x^2}{2}-\frac{1}{2}$ **95.** $y=9x^{1/3}+4$

97. $s=t+\sin t+4$ **99.** $r=\cos(\pi\theta)-1$

101. $v=\frac{1}{2}\sec t+\frac{1}{2}$ **103.** $v=3\sec^{-1}t-\pi$

105. $y=x^2-x^3+4x+1$ **107.** $r=\frac{1}{t}+2t-2$

109. $y=x^3-4x^2+5$ **111.** $y=-\sin t+\cos t+t^3-1$

113. $y=2x^{3/2}-50$ **115.** $y=x-x^{4/3}+\frac{1}{2}$

117. $y=-\sin x-\cos x-2$

119. **(a)** **(i)** 33.2 units, **(ii)** 33.2 units, **(iii)** 33.2 units **(b)** True

121. $t=88/k$, $k=16$

123. **(a)** $v=10t^{3/2}-6t^{1/2}$ **(b)** $s=4t^{5/2}-4t^{3/2}$

127. **(a)** $-\sqrt{x}+C$ **(b)** $x+C$ **(c)** $\sqrt{x}+C$ **(d)** $-x+C$ **(e)** $x-\sqrt{x}+C$ **(f)** $-x-\sqrt{x}+C$

Practice Exercises, pp. 291–295

1. No **3.** No minimum; absolute maximum: $f(1)=16$; critical points: $x=1$ and $11/3$

5. Absolute minimum: $g(0)=1$; no absolute maximum; critical point: $x=0$

7. Absolute minimum: $2-2\ln 2$ at $x=2$; absolute maximum: 1 at $x=1$

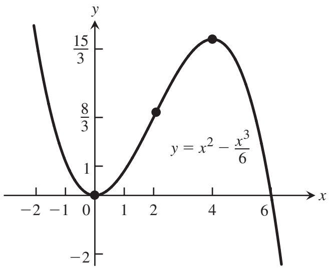
9. Yes, except at $x=0$ **11.** No **15.** **(b)** one

17. **(b)** 0.8555 99677 2

23. Global minimum value of $\frac{1}{2}$ at $x=2$

25. **(a)** $t=0, 6, 12$ **(b)** $t=3, 9$ **(c)** $6<t<12$ **(d)** $0<t<6$, $12<t<14$

27.



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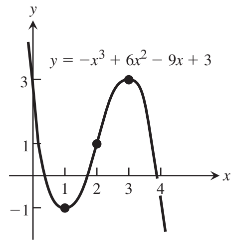

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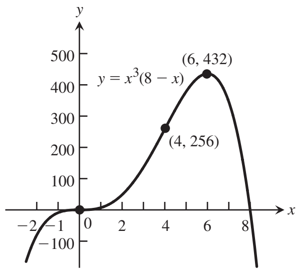

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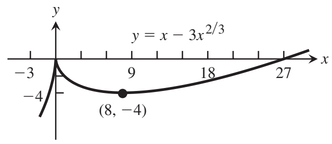

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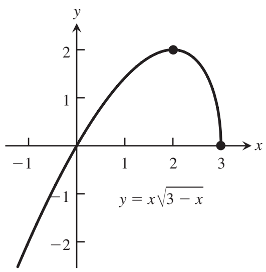

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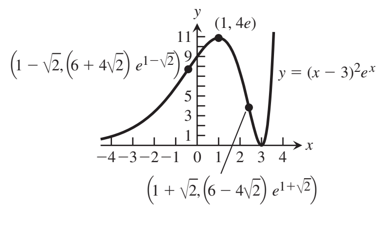

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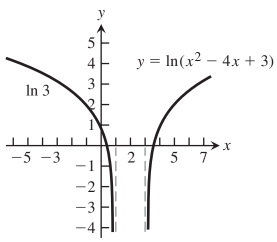

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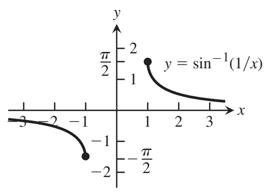
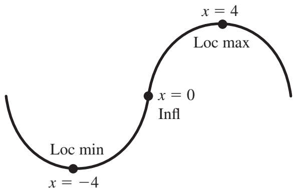


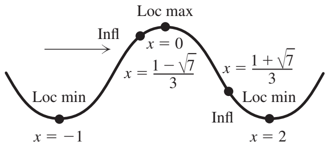
43. (a) Local maximum at $x = 4$, local minimum at $x = -4$, inflection point at $x = 0$

(b)



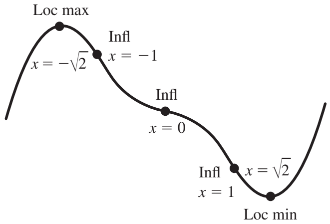
45. (a) Local maximum at $x = 0$, local minima at $x = -1$ and $x = 2$, inflection points at $x = (1 \pm \sqrt{7})/3$

(b)

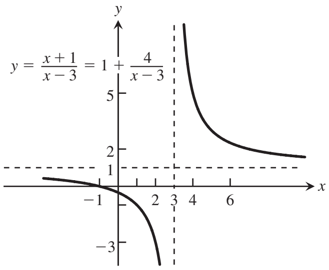


47. (a) Local maximum at $x = -\sqrt{2}$, local minimum at $x = \sqrt{2}$, inflection points at $x = \pm 1$ and 0

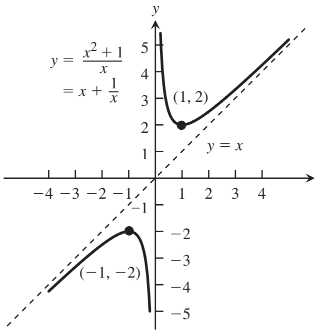
(b)



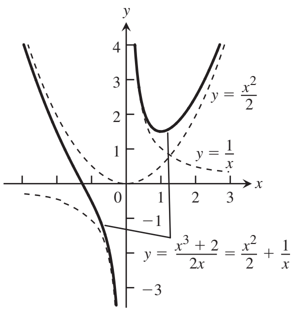
53.



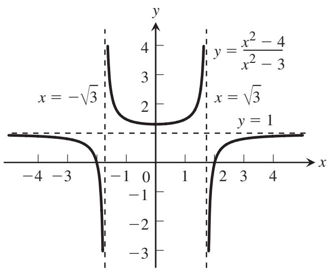
55.



57.



59.

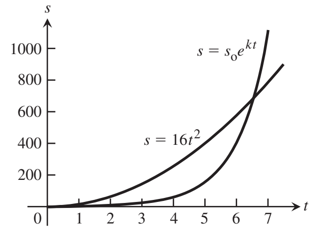


61. 5 **63.** 0 **65.** 1 **67.** 3/7 **69.** 0 **71.** 1
73. ln 10 **75.** ln 2 **77.** 5 **79.** $-\infty$ **81.** 1 **83.** e^{bk}
85. (a) 0, 36 **(b)** 18, 18 **87.** 54 square units
89. height $= 2$, radius $= \sqrt{2}$
91. $x = 5 - \sqrt{5}$ hundred ≈ 276 tires, $y = 2(5 - \sqrt{5})$ hundred ≈ 553 tires
93. Dimensions: base is 6 in. by 12 in., height $= 2$ in.; maximum volume $= 144 \text{ in}^3$
95. $x_5 = 2.1958\,23345$ **97.** $\frac{x^4}{4} + \frac{5}{2}x^2 - 7x + C$
99. $2t^{3/2} - \frac{4}{t} + C$ **101.** $-\frac{1}{r+5} + C$ **103.** $(\theta^2 + 1)^{3/2} + C$
105. $\frac{1}{3}(1 + x^4)^{3/4} + C$ **107.** $10 \tan \frac{s}{10} + C$
109. $-\frac{1}{\sqrt{2}} \csc \sqrt{2}\theta + C$ **111.** $\frac{1}{2}x - \sin \frac{x}{2} + C$
113. $3 \ln x - \frac{x^2}{2} + C$ **115.** $\frac{1}{2}e^t + e^{-t} + C$
117. $\frac{\theta^{2-\pi}}{2-\pi} + C$ **119.** $\frac{3}{2}\sec^{-1}|x| + C$
121. $y = x - \frac{1}{x} - 1$ **123.** $r = 4t^{5/2} + 4t^{3/2} - 8t$
125. Yes, $\sin^{-1}(x)$ and $-\cos^{-1}(x)$ differ by the constant $\pi/2$.
127. $1/\sqrt{2}$ units long by $1/\sqrt{e}$ units high, $A = 1/\sqrt{2e} \approx 0.43$ units^2
129. Absolute maximum $= 0$ at $x = e/2$, absolute minimum $= -0.5$ at $x = 0.5$
131. $x = \pm 1$ are the critical points; $y = 1$ is a horizontal asymptote in both directions; absolute minimum value of the function is $e^{-\sqrt{2}/2}$ at $x = -1$, and absolute maximum value is $e^{\sqrt{2}/2}$ at $x = 1$.
133. (a) Absolute maximum of $2/e$ at $x = e^2$, inflection point $(e^{8/3}, (8/3)e^{-4/3})$, concave up on $(e^{8/3}, \infty)$, concave down on $(0, e^{8/3})$
(b) Absolute maximum of 1 at $x = 0$, inflection points $(\pm 1/\sqrt{2}, 1/\sqrt{e})$, concave up on $(-\infty, -1/\sqrt{2}) \cup (1/\sqrt{2}, \infty)$, concave down on $(-1/\sqrt{2}, 1/\sqrt{2})$
(c) Absolute maximum of 1 at $x = 0$, inflection point $(1, 2/e)$, concave up on $(1, \infty)$, concave down on $(-\infty, 1)$

Additional and Advanced Exercises, pp. 295–298

1. The function is constant on the interval.
3. The extreme points will not be at the end of an open interval.
5. (a) A local minimum at $x = -1$, points of inflection at $x = 0$ and $x = 2$
(b) A local maximum at $x = 0$ and local minima at $x = -1$ and $x = 2$, points of inflection at $x = \frac{1 \pm \sqrt{7}}{3}$
9. No **11.** $a = 1, b = 0, c = 1$
13. Yes
15. Drill the hole at $y = h/2$.
17. $r = \frac{RH}{2(H - R)}$ for $H > 2R$, $r = R$ if $H \le 2R$
19. (a) $\frac{10}{3}$ **(b)** $\frac{5}{3}$ **(c)** $\frac{1}{2}$ **(d)** 0 **(e)** $-\frac{1}{2}$ **(f)** 1 **(g)** $\frac{1}{2}$ **(h)** 3
21. (a) $\frac{c - b}{2e}$ **(b)** $\frac{c + b}{2}$ **(c)** $\frac{b^2 - 2bc + c^2 + 4ae}{4e}$ **(d)** $\frac{c + b + t}{2}$
23. $m_0 = 1 - \frac{1}{q}$, $m_1 = \frac{1}{q}$

25. $s = ce^{kt}$



27. **(a)** $k = -38.72$ **(b)** 25 ft

29. Yes, $y = x + C$ **31.** $v_0 = \frac{2\sqrt{2}}{3} b^{3/4}$

Chapter 5

Section 5.1, pp. 307–309

1. **(a)** 0.125 **(b)** 0.21875 **(c)** 0.625 **(d)** 0.46875

3. **(a)** 1.066667 **(b)** 1.283333 **(c)** 2.666667 **(d)** 2.083333

5. 0.3125, 0.328125 **7.** 1.5, 1.574603

9. **(a)** 87 in. **(b)** 87 in. **11.** **(a)** 3490 ft **(b)** 3840 ft

13. **(a)** 74.65 ft/sec **(b)** 45.28 ft/sec **(c)** 146.59 ft

15. $\frac{31}{16}$ **17.** 1

19. **(a)** Upper = 758 gal, lower = 543 gal
(b) Upper = 2363 gal, lower = 1693 gal
(c) $\approx$ 31.4 h, $\approx$ 32.4 h

21. **(a)** 2 **(b)** $2\sqrt{2} \approx 2.828$

(c) $8\sin\left(\frac{\pi}{8}\right) \approx 3.061$

(d) Each area is less than the area of the circle, π. As n increases, the polygon area approaches π.

Section 5.2, pp. 315–316

1. $\frac{6(1)}{1+1} + \frac{6(2)}{2+1} = 7$

3. $\cos(1)\pi + \cos(2)\pi + \cos(3)\pi + \cos(4)\pi = 0$

5. $\sin\pi - \sin\frac{\pi}{2} + \sin\frac{\pi}{3} = \frac{\sqrt{3}-2}{2}$ **7.** All of them **9.** b

11. $\sum_{k=1}^{6} k$ **13.** $\sum_{k=1}^{4} \frac{1}{2^k}$ **15.** $\sum_{k=1}^{5} (-1)^{k+1}\frac{1}{k}$

17. **(a)** -15 **(b)** 1 **(c)** 1 **(d)** -11 **(e)** 16

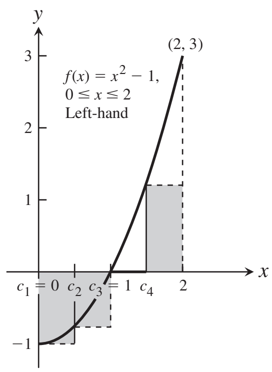
19. **(a)** 55 **(b)** 385 **(c)** 3025

21. -56 **23.** -73 **25.** 240 **27.** 3376

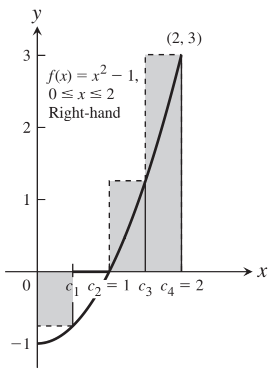
29. **(a)** 21 **(b)** 3500 **(c)** 2620

31. **(a)** $4n$ **(b)** cn **(c)** $(n^2 - n)/2$

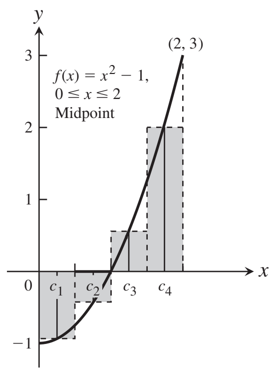
33. **(a)**



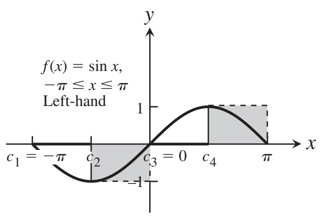
(b)



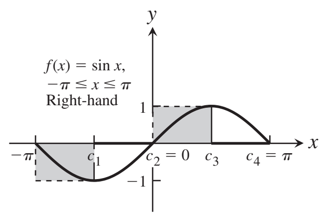
(c)



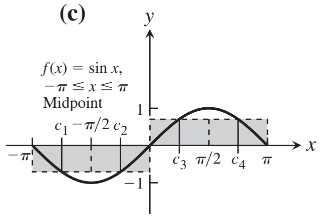
35. **(a)**



(b)



(c)



37. 1.2

39. $\frac{2}{3} - \frac{1}{2n} - \frac{1}{6n^2}$, $\frac{2}{3}$

41. $12 + \frac{27n + 9}{2n^2}$, 12

43. $\frac{5}{6} + \frac{6n + 1}{6n^2}$, $\frac{5}{6}$

45. $\frac{1}{2} + \frac{1}{n} + \frac{1}{2n^2}$, $\frac{1}{2}$

Section 5.3, pp. 324–328

1. $\int_0^2 x^2\,dx$ **3.** $\int_{-7}^{5} (x^2 - 3x)\,dx$ **5.** $\int_2^3 \frac{1}{1-x}\,dx$

7. $\int_{-\pi/4}^{0} \sec x\,dx$

9. **(a)** 0 **(b)** -8 **(c)** -12 **(d)** 10 **(e)** -2 **(f)** 16

11. **(a)** 5 **(b)** $5\sqrt{3}$ **(c)** -5 **(d)** -5

13. **(a)** 4 **(b)** -4 **15.** Area = 21 square units

17. Area = $9\pi/2$ square units **19.** Area = 2.5 square units

21. Area = 3 square units **23.** $b^2/4$ **25.** $b^2 - a^2$

27. **(a)** 2π **(b)** π **29.** 1/2 **31.** $3\pi^2/2$ **33.** 7/3

35. 1/24 **37.** $3a^2/2$ **39.** $b/3$ **41.** -14

43. -2 **45.** $-7/4$ **47.** 7 **49.** 0

51. Using n subintervals of length $\Delta x = b/n$ and right-endpoint values:

$$\text{Area} = \int_0^b 3x^2\,dx = b^3$$

53. Using n subintervals of length $\Delta x = b/n$ and right-endpoint values:

$$\text{Area} = \int_0^b 2x\,dx = b^2$$

55. $\text{av}(f) = 0$ **57.** $\text{av}(f) = -2$ **59.** $\text{av}(f) = 1$

61. **(a)** $\text{av}(g) = -1/2$ **(b)** $\text{av}(g) = 1$ **(c)** $\text{av}(g) = 1/4$

63. $c(b-a)$ **65.** $b^3/3 - a^3/3$ **67.** 9
69. $b^4/4 - a^4/4$ **71.** $a = 0$ and $b = 1$ maximize the integral.
73. Upper bound = 1, lower bound = 1/2

75. For example, $\int_0^1 \sin(x^2)\,dx \le \int_0^1 dx = 1$

77. $\int_a^b f(x)\,dx \ge \int_a^b 0\,dx = 0$ **79.** Upper bound = 1/2

Section 5.4, pp. 336–339

1. $-10/3$ **3.** $124/125$ **5.** $753/16$ **7.** 1 **9.** $2\sqrt{3}$
11. 0 **13.** $-\pi/4$ **15.** $1 - \frac{\pi}{4}$ **17.** $\frac{2-\sqrt{2}}{4}$ **19.** $-8/3$
21. $-3/4$ **23.** $\sqrt{2} - \sqrt[4]{8} + 1$ **25.** -1 **27.** 16
29. $7/3$ **31.** $2\pi/3$ **33.** $\frac{1}{\pi}(4^\pi - 2^\pi)$ **35.** $\frac{1}{2}(e-1)$
37. $\sqrt{26} - \sqrt{5}$ **39.** $(\cos\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)$ **41.** $4t^5$
43. $3x^2e^{-x^3}$ **45.** $\sqrt{1+x^2}$ **47.** $-\frac{1}{2}x^{-1/2}\sin x$ **49.** 0
51. 1 **53.** $2xe^{(1/2)x^2}$ **55.** 1 **57.** $28/3$ **59.** $1/2$
61. π **63.** $\frac{\sqrt{2}\pi}{2}$
65. d, since $y' = \frac{1}{x}$ and $y(\pi) = \int_\pi^\pi \frac{1}{t}dt - 3 = -3$
67. b, since $y' = \sec x$ and $y(0) = \int_0^0 \sec t\,dt + 4 = 4$
69. $y = \int_2^x \sec t\,dt + 3$ **71.** $\frac{2}{3}bh$ **73.** \$9.00
75. **(a)** $T(0) = 70°\text{F}$, $T(16) = 76°\text{F}$, $T(25) = 85°\text{F}$
(b) $\text{av}(T) = 75°\text{F}$
77. $2x - 2$ **79.** $-3x + 5$
81. **(a)** True. Since f is continuous, g is differentiable by Part 1 of the Fundamental Theorem of Calculus.
(b) True: g is continuous because it is differentiable.
(c) True, since $g'(1) = f(1) = 0$.
(d) False, since $g''(1) = f'(1) > 0$.
(e) True, since $g'(1) = 0$ and $g''(1) = f'(1) > 0$.
(f) False: $g''(x) = f'(x) > 0$, so g'' never changes sign.
(g) True, since $g'(1) = f(1) = 0$ and $g'(x) = f(x)$ is an increasing function of x (because $f'(x) > 0$).
83. **(a)** $v = \frac{ds}{dt} = \frac{d}{dt}\int_0^t f(x)\,dx = f(t) \Rightarrow v(5) = f(5) = 2$ m/sec
(b) $a = df/dt$ is negative, since the slope of the tangent line at $t = 5$ is negative.
(c) $s = \int_0^3 f(x)\,dx = \frac{1}{2}(3)(3) = \frac{9}{2}$m, since the integral is the area of the triangle formed by $y = f(x)$, the x-axis, and $x = 3$.
(d) $t = 6$, since after $t = 6$ to $t = 9$, the region lies below the x-axis.
(e) At $t = 4$ and $t = 7$, since there are horizontal tangents there.
(f) Toward the origin between $t = 6$ and $t = 9$, since the velocity is negative on this interval. Away from the origin between $t = 0$ and $t = 6$, since the velocity is positive there.
(g) Right or positive side, because the integral of f from 0 to 9 is positive, there being more area above the x-axis than below.

Section 5.5, pp. 345–346

1. $\frac{1}{6}(2x+4)^6 + C$ **3.** $-\frac{1}{3}(x^2+5)^{-3} + C$
5. $\frac{1}{10}(3x^2+4x)^5 + C$ **7.** $-\frac{1}{3}\cos 3x + C$
9. $\frac{1}{2}\sec 2t + C$ **11.** $-6(1-r^3)^{1/2} + C$
13. $\frac{1}{3}(x^{3/2} - 1) - \frac{1}{6}\sin(2x^{3/2} - 2) + C$
15. **(a)** $-\frac{1}{4}(\cot^2 2\theta) + C$ **(b)** $-\frac{1}{4}(\csc^2 2\theta) + C$
17. $-\frac{1}{3}(3-2s)^{3/2} + C$ **19.** $-\frac{2}{5}(1-\theta^2)^{5/4} + C$
21. $\left(-2/(1+\sqrt{x})\right) + C$ **23.** $\frac{1}{3}\tan(3x+2) + C$
25. $\frac{1}{2}\sin^6\left(\frac{x}{3}\right) + C$ **27.** $\left(\frac{r^3}{18} - 1\right)^6 + C$
29. $-\frac{2}{3}\cos(x^{3/2}+1) + C$ **31.** $\frac{1}{2\cos(2t+1)} + C$
33. $-\sin\left(\frac{1}{t} - 1\right) + C$ **35.** $-\frac{\sin^2(1/\theta)}{2} + C$
37. $\frac{2}{3}(1+x)^{3/2} - 2(1+x)^{1/2} + C$ **39.** $\frac{2}{3}\left(2 - \frac{1}{x}\right)^{3/2} + C$
41. $\frac{2}{27}\left(1 - \frac{3}{x^3}\right)^{3/2} + C$ **43.** $\frac{1}{12}(x-1)^{12} + \frac{1}{11}(x-1)^{11} + C$
45. $-\frac{1}{8}(1-x)^8 + \frac{4}{7}(1-x)^7 - \frac{2}{3}(1-x)^6 + C$
47. $\frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$ **49.** $\frac{-1}{4(x^2-4)^2} + C$
51. $e^{\sin x} + C$ **53.** $2\tan\left(e^{\sqrt{x}} + 1\right) + C$ **55.** $\ln|\ln x| + C$
57. $z - \ln(1 + e^z) + C$ **59.** $\frac{5}{6}\tan^{-1}\left(\frac{2r}{3}\right) + C$
61. $e^{\sin^{-1} x} + C$ **63.** $\frac{1}{3}(\sin^{-1} x)^3 + C$ **65.** $\ln|\tan^{-1} y| + C$
67. **(a)** $-\frac{6}{2+\tan^3 x} + C$ **(b)** $-\frac{6}{2+\tan^3 x} + C$
(c) $-\frac{6}{2+\tan^3 x} + C$
69. $\frac{1}{6}\sin\sqrt{3(2r-1)^2 + 6} + C$ **73.** $s = \frac{1}{2}(3t^2-1)^4 - 5$
75. $s = 4t - 2\sin\left(2t + \frac{\pi}{6}\right) + 9$
77. $s = \sin\left(2t - \frac{\pi}{2}\right) + 100t + 1$ **79.** 6 m

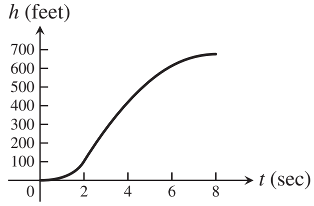
Section 5.6, pp. 353–356

1. **(a)** $14/3$ **(b)** $2/3$ **3.** **(a)** $1/2$ **(b)** $-1/2$
5. **(a)** $15/16$ **(b)** 0 **7.** **(a)** 0 **(b)** $1/8$ **9.** **(a)** 4 **(b)** 0
11. **(a)** $506/375$ **(b)** $86{,}744/375$ **13.** **(a)** 0 **(b)** 0
15. $2\sqrt{3}$ **17.** $3/4$ **19.** $3^{5/2} - 1$ **21.** 3 **23.** $\pi/3$
25. e **27.** $\ln 3$ **29.** $(\ln 2)^2$ **31.** $\frac{1}{\ln 4}$ **33.** $\ln 2$

35. $\ln\left(2+\sqrt{3}\right)-\frac{\sqrt{3}}{2}$ **37.** π **39.** $\pi/12$ **41.** $2\pi/3$
43. $\sqrt{3}-1$ **45.** $-\pi/12$ **47.** $16/3$ **49.** $2^{5/2}$
51. $\pi/2$ **53.** $128/15$ **55.** $4/3$ **57.** $5/6$ **59.** $38/3$
61. $49/6$ **63.** $32/3$ **65.** $48/5$ **67.** $8/3$ **69.** 8
71. $5/3$ (There are three intersection points.) **73.** 18
75. $243/8$ **77.** $8/3$ **79.** 2 **81.** $104/15$ **83.** $56/15$
85. 4 **87.** $\frac{4}{3}-\frac{4}{\pi}$ **89.** $\pi/2$ **91.** 2 **93.** $1/2$
95. 1 **97.** ln 16 **99.** 2 **101.** 2 ln 5
103. **(a)** $\left(\pm\sqrt{c}, c\right)$ **(b)** $c=4^{2/3}$ **(c)** $c=4^{2/3}$
105. $11/3$ **107.** $3/4$ **109.** Neither **111.** $F(6)-F(2)$
113. **(a)** -3 **(b)** 3 **115.** $I=a/2$

Practice Exercises, pp. 357–360

1. **(a)** About 680 ft **(b)**

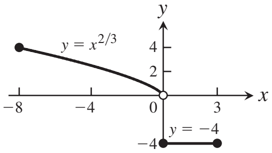


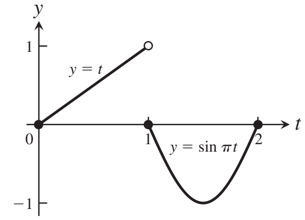
3. **(a)** $-1/2$ **(b)** 31 **(c)** 13 **(d)** 0
5. $\int_1^5 (2x-1)^{-1/2}\,dx=2$ **7.** $\int_{-\pi}^{0}\cos\frac{x}{2}\,dx=2$
9. **(a)** 4 **(b)** 2 **(c)** -2 **(d)** -2π **(e)** $8/5$
11. $8/3$ **13.** 62 **15.** 1 **17.** $1/6$ **19.** 18 **21.** $9/8$
23. $\frac{\pi^2}{32}+\frac{\sqrt{2}}{2}-1$ **25.** 4 **27.** $\frac{8\sqrt{2}-7}{6}$
29. Min: -4, max: 0, area: $27/4$ **31.** $6/5$ **33.** 1
37. $y=\int_5^x\left(\frac{\sin t}{t}\right)dt-3$ **39.** $y=\sin^{-1}x$
41. $y=\sec^{-1}x+\frac{2\pi}{3}, x>1$ **43.** $-4(\cos x)^{1/2}+C$
45. $\theta^2+\theta+\sin(2\theta+1)+C$ **47.** $\frac{t^3}{3}+\frac{4}{t}+C$
49. $-\frac{1}{3}\cos\left(2t^{3/2}\right)+C$ **51.** $\tan\left(e^x-7\right)+C$
53. $e^{\tan x}+C$ **55.** $\frac{-\ln 7}{3}$ **57.** $\ln(9/25)$
59. $-\frac{1}{2}(\ln x)^{-2}+C$ **61.** $\frac{1}{2\ln 3}\left(3^{x^2}\right)+C$
63. $\frac{3}{2}\sin^{-1}2(r-1)+C$ **65.** $\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right)+C$
67. $\frac{1}{4}\sec^{-1}\left|\frac{2x-1}{2}\right|+C$ **69.** $e^{\sin^{-1}\sqrt{x}}+C$
71. $2\sqrt{\tan^{-1}y}+C$ **73.** 16 **75.** 2 **77.** 1 **79.** 8
81. $27\sqrt{3}/160$ **83.** $\pi/2$ **85.** $\sqrt{3}$ **87.** $6\sqrt{3}-2\pi$
89. -1 **91.** 2 **93.** 1 **95.** $15/16+\ln 2$ **97.** $e-1$
99. $1/6$ **101.** $9/14$ **103.** $\frac{9\ln 2}{4}$ **105.** π
107. $\pi/\sqrt{3}$ **109.** $\pi/6$ **111.** $\pi/12$
113. **(a)** b **(b)** b

117. **(a)** $\frac{d}{dx}(x\ln x-x+C)=x\cdot\frac{1}{x}+\ln x-1+0=\ln x$
(b) $\frac{1}{e-1}$
119. 25°F **121.** $\sqrt{2+\cos^3 x}$ **123.** $\frac{-6}{3+x^4}$
125. $\frac{dy}{dx}=\frac{-2}{x}e^{\cos(2\ln x)}$ **127.** $\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}\sqrt{1-2(\sin^{-1}x)^2}}$
129. Yes **131.** $-\sqrt{1+x^2}$
133. Cost $\approx$ \$10,899 using a lower sum estimate

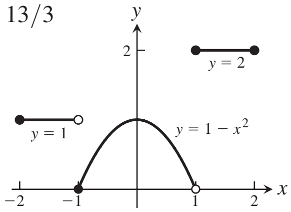
Additional and Advanced Exercises, pp. 361–364

1. **(a)** Yes **(b)** No **5.** **(a)** $1/4$ **(b)** $\sqrt[3]{12}$
7. $f(x)=\frac{x}{\sqrt{x^2+1}}$ **9.** $y=x^3+2x-4$
11. $36/5$ **13.** $\frac{1}{2}-\frac{2}{\pi}$





15. $13/3$



17. $1/2$ **19.** $\pi/2$
21. ln 2 **23.** $1/6$
25. $\int_0^1 f(x)\,dx$ **27.** **(b)** πr^2
29. **(a)** 0 **(b)** -1
(c) $-\pi$ **(d)** $x=1$
(e) $y=2x+2-\pi$
(f) $x=-1, x=2$
(g) $[-2\pi, 0]$
31. $2/x$ **33.** $\frac{\sin 4y}{\sqrt{y}}-\frac{\sin y}{2\sqrt{y}}$ **35.** $2x\ln|x|-x\ln\frac{|x|}{\sqrt{2}}$
37. $(\sin x)/x$ **39.** $x=1$ **41.** $\frac{1}{\ln 2}, \frac{1}{2\ln 2}, 2:1$
43. $2/17$

Chapter 6

Section 6.1, pp. 373–376

1. 16 **3.** $16/3$ **5.** **(a)** $2\sqrt{3}$ **(b)** 8 **7.** **(a)** 60 **(b)** 36
9. 8π **11.** 10 **13.** **(a)** s^2h **(b)** s^2h **15.** $\frac{2\pi}{3}$
17. $4-\pi$ **19.** $\frac{32\pi}{5}$ **21.** 36π **23.** π
25. $\frac{\pi}{2}\left(1-\frac{1}{e^2}\right)$ **27.** $\frac{\pi}{2}\ln 4$ **29.** $\pi\left(\frac{\pi}{2}+2\sqrt{2}-\frac{11}{3}\right)$
31. 2π **33.** 2π **35.** $4\pi\ln 4$ **37.** $\pi^2-2\pi$ **39.** $\frac{2\pi}{3}$
41. $\frac{117\pi}{5}$ **43.** $\pi(\pi-2)$ **45.** $\frac{4\pi}{3}$ **47.** 8π **49.** $\frac{7\pi}{6}$
51. **(a)** 8π **(b)** $\frac{32\pi}{5}$ **(c)** $\frac{8\pi}{3}$ **(d)** $\frac{224\pi}{15}$
53. **(a)** $\frac{16\pi}{15}$ **(b)** $\frac{56\pi}{15}$ **(c)** $\frac{64\pi}{15}$ **55.** $V=2a^2b\pi^2$

57. **(a)** $V = \dfrac{\pi h^2(3a - h)}{3}$ **(b)** $\dfrac{1}{120\pi}$ m/sec

61. $V = 3308 \text{ cm}^3$ **63.** $\dfrac{4 - b + a}{2}$

Section 6.2, pp. 381–383

1. 6π **3.** 2π **5.** $14\pi/3$ **7.** 8π **9.** $5\pi/6$

11. $\dfrac{7\pi}{15}$ **13.** **(b)** 4π **15.** $\dfrac{16\pi}{15}\left(3\sqrt{2} + 5\right)$

17. $\dfrac{8\pi}{3}$ **19.** $\dfrac{4\pi}{3}$ **21.** $\dfrac{16\pi}{3}$

23. **(a)** 16π **(b)** 32π **(c)** 28π
(d) 24π **(e)** 60π **(f)** 48π

25. **(a)** $\dfrac{27\pi}{2}$ **(b)** $\dfrac{27\pi}{2}$ **(c)** $\dfrac{72\pi}{5}$ **(d)** $\dfrac{108\pi}{5}$

27. **(a)** $\dfrac{6\pi}{5}$ **(b)** $\dfrac{4\pi}{5}$ **(c)** 2π **(d)** 2π

29. **(a)** About the x-axis: $V = \dfrac{2\pi}{15}$; about the y-axis: $V = \dfrac{\pi}{6}$

(b) About the x-axis: $V = \dfrac{2\pi}{15}$; about the y-axis: $V = \dfrac{\pi}{6}$

31. **(a)** $\dfrac{5\pi}{3}$ **(b)** $\dfrac{4\pi}{3}$ **(c)** 2π **(d)** $\dfrac{2\pi}{3}$

33. **(a)** $\dfrac{4\pi}{15}$ **(b)** $\dfrac{7\pi}{30}$

35. **(a)** $\dfrac{24\pi}{5}$ **(b)** $\dfrac{48\pi}{5}$

37. **(a)** $\dfrac{9\pi}{16}$ **(b)** $\dfrac{9\pi}{16}$

39. Disk: 2 integrals; washer: 2 integrals; shell: 1 integral

41. **(a)** $\dfrac{256\pi}{3}$ **(b)** $\dfrac{244\pi}{3}$

47. $\pi\left(1 - \dfrac{1}{e}\right)$

Section 6.3, pp. 388–389

1. 12 **3.** $\dfrac{53}{6}$ **5.** $\dfrac{123}{32}$ **7.** $\dfrac{99}{8}$ **9.** $\ln 2 + \dfrac{3}{8}$

11. $\dfrac{53}{6}$ **13.** 2

15. **(a)** $\displaystyle\int_{-1}^{2} \sqrt{1 + 4x^2}\, dx$ **(c)** ≈ 6.13

17. **(a)** $\displaystyle\int_{0}^{\pi} \sqrt{1 + \cos^2 y}\, dy$ **(c)** ≈ 3.82

19. **(a)** $\displaystyle\int_{-1}^{3} \sqrt{1 + (y + 1)^2}\, dy$ **(c)** ≈ 9.29

21. **(a)** $\displaystyle\int_{0}^{\pi/6} \sec x\, dx$ **(c)** ≈ 0.55

23. **(a)** $y = \sqrt{x}$ from (1, 1) to (4, 2)
(b) Only one. We know the derivative of the function and the value of the function at one value of x.

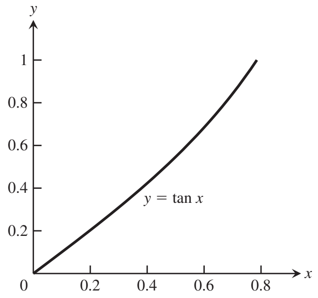
25. 1 **27.** Yes, $f(x) = \pm x + C$ where C is any real number.

35. $\displaystyle\int_{0}^{x} \sqrt{1 + 9t}\, dt, \frac{2}{27}(10^{3/2} - 1)$

Section 6.4, pp. 393–395

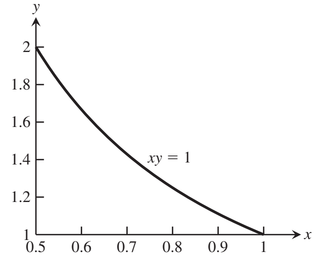
1. **(a)** $2\pi \displaystyle\int_{0}^{\pi/4} (\tan x)\sqrt{1 + \sec^4 x}\, dx$ **(c)** $S \approx 3.84$

(b)



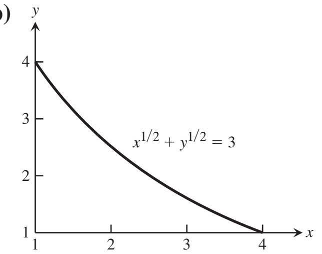
3. **(a)** $2\pi \displaystyle\int_{1}^{2} \frac{1}{y}\sqrt{1 + y^{-4}}\, dy$ **(c)** $S \approx 5.02$

(b)



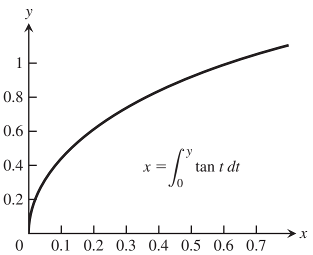
5. **(a)** $2\pi \displaystyle\int_{1}^{4} (3 - x^{1/2})^2\sqrt{1 + (1 - 3x^{-1/2})^2}\, dx$ **(c)** $S \approx 63.37$

(b)



7. **(a)** $2\pi \displaystyle\int_{0}^{\pi/3} \left(\int_{0}^{y} \tan t\, dt\right) \sec y\, dy$ **(c)** $S \approx 2.08$

(b)



9. $4\pi\sqrt{5}$ **11.** $3\pi\sqrt{5}$ **13.** $98\pi/81$ **15.** 2π

17. $\pi\left(\sqrt{8} - 1\right)/9$ **19.** $35\pi\sqrt{5}/3$ **21.** $\pi\left(\dfrac{15}{16} + \ln 2\right)$

23. $253\pi/20$ **27.** Order 226.2 liters of each color.

Section 6.5, pp. 400–404

1. 400 N/m **3.** 4 cm, 0.08 J

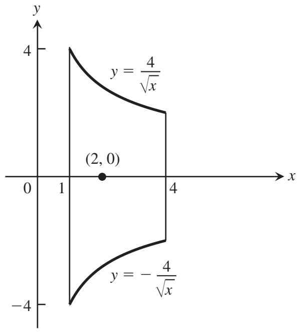
5. **(a)** 7238 lb/in. **(b)** 905 in.-lb, 2714 in.-lb

7. 780 J **9.** 72,900 ft-lb **11.** 160 ft-lb

13. **(a)** 1,497,600 ft-lb **(b)** 1 hr, 40 min
(d) At 62.26 lb/ft^3: a) 1,494,240 ft-lb b) 1 hr, 40 min
At 62.59 lb/ft^3: a) 1,502,160 ft-lb b) 1 hr, 40.1 min
15. 37,306 ft-lb **17.** 7,238,299.47 ft-lb
19. 2446.25 ft-lb **21.** 15,073,099.75 J
25. 85.1 ft-lb **27.** 98.35 ft-lb **29.** 91.32 in.-oz
31. 5.144×10^{10} J **33.** 1684.8 lb
35. **(a)** 6364.8 lb **(b)** 5990.4 lb **37.** 1164.8 lb **39.** 1309 lb
41. **(a)** 12,480 lb **(b)** 8580 lb **(c)** 9722.3 lb
43. **(a)** 93.33 lb **(b)** 3 ft **45.** $\frac{wb}{2}$
47. No. The tank will overflow because the movable end will have moved only $3\frac{1}{3}$ ft by the time the tank is full.

Section 6.6, pp. 413–415

1. $\bar{x} = 0, \bar{y} = 12/5$ **3.** $\bar{x} = 1, \bar{y} = -3/5$
5. $\bar{x} = 16/105, \bar{y} = 8/15$ **7.** $\bar{x} = 0, \bar{y} = \pi/8$
9. $\bar{x} \approx 1.44, \bar{y} \approx 0.36$
11. $\bar{x} = \frac{\ln 4}{\pi}, \bar{y} = 0$ **13.** $\bar{x} = 7, \bar{y} = \frac{\ln 16}{12}$
15. $\bar{x} = 3/2, \bar{y} = 1/2$
17. **(a)** $\frac{224\pi}{3}$ **(b)** $\bar{x} = 2, \bar{y} = 0$
(c)



21. $\bar{x} = \bar{y} = 1/3$ **23.** $\bar{x} = a/3, \bar{y} = b/3$ **25.** $13\delta/6$
27. $\bar{x} = 0, \bar{y} = \frac{a\pi}{4}$ **29.** $\bar{x} = 1/2, \bar{y} = 4$
31. $\bar{x} = 6/5, \bar{y} = 8/7$ **35.** $V = 32\pi, S = 32\sqrt{2}\pi$ **37.** $4\pi^2$
39. $\bar{x} = 0, \bar{y} = \frac{2a}{\pi}$ **41.** $\bar{x} = 0, \bar{y} = \frac{4b}{3\pi}$
43. $\sqrt{2}\pi a^3(4 + 3\pi)/6$ **45.** $\bar{x} = \frac{a}{3}, \bar{y} = \frac{b}{3}$

Practice Exercises, pp. 416–417

1. $\frac{9\pi}{280}$ **3.** π^2 **5.** $\frac{72\pi}{35}$
7. **(a)** 2π **(b)** π **(c)** $12\pi/5$ **(d)** $26\pi/5$
9. **(a)** 8π **(b)** $1088\pi/15$ **(c)** $512\pi/15$
11. $\pi(3\sqrt{3} - \pi)/3$ **13.** π **15.** $\frac{28\pi}{3}$ ft^3 **17.** $\frac{10}{3}$
19. $3 + \frac{1}{8}\ln 2$ **21.** $28\pi\sqrt{2}/3$ **23.** 4π **25.** 4640 J
27. $\frac{w}{2}(2ar - a^2)$ **29.** 418,208.81 ft-lb
31. $22{,}500\pi$ ft-lb, 257 sec **33.** $\bar{x} = 0, \bar{y} = 8/5$
35. $\bar{x} = 3/2, \bar{y} = 12/5$ **37.** $\bar{x} = 9/5, \bar{y} = 11/10$
39. 332.8 lb **41.** 2196.48 lb

Additional and Advanced Exercises, pp. 417–418

1. $f(x) = \sqrt{\frac{2x - a}{\pi}}$ **3.** $f(x) = \sqrt{C^2 - 1}\,x + a$, where $C \ge 1$
5. $\frac{\pi}{30\sqrt{2}}$ **7.** $28/3$ **9.** $\frac{4h\sqrt{3mh}}{3}$
11. $\bar{x} = 0, \bar{y} = \frac{n}{2n + 1}, (0, 1/2)$
15. **(a)** $\bar{x} = \bar{y} = 4(a^2 + ab + b^2)/(3\pi(a + b))$
(b) $(2a/\pi, 2a/\pi)$
17. ≈ 2329.6 lb

CHAPTER 7

Section 7.1, pp. 428–430

1. $\ln\left(\frac{2}{3}\right)$ **3.** $\ln|y^2 - 25| + C$ **5.** $\ln|6 + 3\tan t| + C$
7. $\ln\left(1 + \sqrt{x}\right) + C$ **9.** 1 **11.** $2(\ln 2)^4$ **13.** 2
15. $2e^{\sqrt{r}} + C$ **17.** $-e^{-t^2} + C$ **19.** $-e^{1/x} + C$
21. $\frac{1}{\pi}e^{\sec \pi t} + C$ **23.** 1 **25.** $\ln(1 + e^r) + C$ **27.** $\frac{1}{2\ln 2}$
29. $\frac{1}{\ln 2}$ **31.** $\frac{6}{\ln 7}$ **33.** 32760 **35.** $3^{\sqrt{2}+1}$
37. $\frac{1}{\ln 10}\left(\frac{(\ln x)^2}{2}\right) + C$ **39.** $2(\ln 2)^2$ **41.** $\frac{3\ln 2}{2}$ **43.** $\ln 10$
45. $(\ln 10)\ln|\ln x| + C$ **47.** $y = 1 - \cos(e^t - 2)$
49. $y = 2(e^{-x} + x) - 1$ **51.** $y = x + \ln|x| + 2$
53. $\pi \ln 16$ **55.** $6 + \ln 2$ **57.** **(b)** 0.00469
69. **(a)** 1.89279 **(b)** −0.35621 **(c)** 0.94575 **(d)** −2.80735
(e) 5.29595 **(f)** 0.97041 **(g)** −1.03972 **(h)** −1.61181

Section 7.2, pp. 437–447

9. $\frac{2}{3}y^{3/2} - x^{1/2} = C$ **11.** $e^y - e^x = C$
13. $-x + 2\tan\sqrt{y} = C$ **15.** $e^{-y} + 2e^{\sqrt{x}} = C$
17. $y = \sin(x^2 + C)$ **19.** $\frac{1}{3}\ln|y^3 - 2| = x^3 + C$
21. $4\ln\left(\sqrt{y} + 2\right) = e^{x^2} + C$
23. **(a)** −0.00001 **(b)** 10,536 years **(c)** 82%
25. 54.88 g **27.** 59.8 ft **29.** 2.8147498×10^{14}
31. **(a)** 8 years **(b)** 32.02 years **33.** Yes, $y(20) < 1$
35. 15.28 years **37.** 56,562 years
41. **(a)** 17.5 min **(b)** 13.26 min
43. −3°C **45.** About 6693 years **47.** 54.62% **49.** ≈15,683 years

Section 7.3, pp. 445–447

1. $\cosh x = 5/4, \tanh x = -3/5, \coth x = -5/3$, $\operatorname{sech} x = 4/5, \operatorname{csch} x = -4/3$
3. $\sinh x = 8/15, \tanh x = 8/17, \coth x = 17/8, \operatorname{sech} x = 15/17$, $\operatorname{csch} x = 15/8$
5. $x + \frac{1}{x}$ **7.** e^{5x} **9.** e^{4x} **13.** $2\cosh\frac{x}{3}$
15. $\operatorname{sech}^2\sqrt{t} + \frac{\tanh\sqrt{t}}{\sqrt{t}}$ **17.** $\coth z$
19. $(\ln \operatorname{sech}\theta)(\operatorname{sech}\theta\tanh\theta)$ **21.** $\tanh^3 v$ **23.** 2
25. $\frac{1}{2\sqrt{x(1 + x)}}$ **27.** $\frac{1}{1 + \theta} - \tanh^{-1}\theta$

29. $\frac{1}{2\sqrt{t}} - \coth^{-1}\sqrt{t}$ **31.** $-\operatorname{sech}^{-1} x$ **33.** $\frac{\ln 2}{\sqrt{1 + \left(\frac{1}{2}\right)^{2\theta}}}$

35. $|\sec x|$ **41.** $\frac{\cosh 2x}{2} + C$

43. $12\sinh\left(\frac{x}{2} - \ln 3\right) + C$ **45.** $7\ln|e^{x/7} + e^{-x/7}| + C$

47. $\tanh\left(x - \frac{1}{2}\right) + C$ **49.** $-2\operatorname{sech}\sqrt{t} + C$ **51.** $\ln\frac{5}{2}$

53. $\frac{3}{32} + \ln 2$ **55.** $e - e^{-1}$ **57.** $3/4$ **59.** $\frac{3}{8} + \ln\sqrt{2}$

61. $\ln(2/3)$ **63.** $\frac{-\ln 3}{2}$ **65.** $\ln 3$

67. **(a)** $\sinh^{-1}(\sqrt{3})$ **(b)** $\ln(\sqrt{3} + 2)$

69. **(a)** $\coth^{-1}(2) - \coth^{-1}(5/4)$ **(b)** $\left(\frac{1}{2}\right)\ln\left(\frac{1}{3}\right)$

71. **(a)** $-\operatorname{sech}^{-1}\left(\frac{12}{13}\right) + \operatorname{sech}^{-1}\left(\frac{4}{5}\right)$

(b) $-\ln\left(\frac{1 + \sqrt{1 - (12/13)^2}}{(12/13)}\right) + \ln\left(\frac{1 + \sqrt{1 - (4/5)^2}}{(4/5)}\right)$

$= -\ln\left(\frac{3}{2}\right) + \ln(2) = \ln(4/3)$

73. **(a)** 0 **(b)** 0

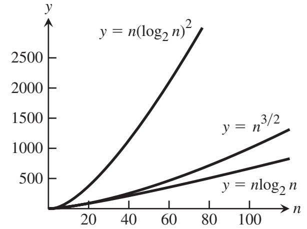
77. **(a)** $\sqrt{\frac{mg}{k}}$ **(b)** $80\sqrt{5} \approx 178.89$ ft/sec

79. 2π **81.** $\frac{6}{5}$

Section 7.4, pp. 452–453

1. **(a)** Slower **(b)** Slower **(c)** Slower **(d)** Faster **(e)** Slower **(f)** Slower **(g)** Same **(h)** Slower
3. **(a)** Same **(b)** Faster **(c)** Same **(d)** Same **(e)** Slower **(f)** Faster **(g)** Slower **(h)** Same
5. **(a)** Same **(b)** Same **(c)** Same **(d)** Faster **(e)** Faster **(f)** Same **(g)** Slower **(h)** Faster **7.** d, a, c, b
9. **(a)** False **(b)** False **(c)** True **(d)** True **(e)** True **(f)** True **(g)** False **(h)** True
13. When the degree of f is less than or equal to the degree of g.
15. 1, 1
21. **(b)** $\ln(e^{17000000}) = 17{,}000{,}000 < (e^{17\times 10^6})^{1/10^6}$
$= e^{17} \approx 24{,}154{,}952.75$
(c) $x \approx 3.4306311 \times 10^{15}$
(d) They cross at $x \approx 3.4306311 \times 10^{15}$.
23. **(a)** The algorithm that takes $O(n\log_2 n)$ steps

(b)



25. It could take one million for a sequential search; at most 20 steps for a binary search.

Practice Exercises, pp. 453–454

1. $-\cos e^x + C$ **3.** $\ln 8$ **5.** $2\ln 2$ **7.** $\frac{1}{2}(\ln(x - 5))^2 + C$

9. $3\ln 7$ **11.** $2(\sqrt{2} - 1)$ **13.** $y = \frac{\ln 2}{\ln(3/2)}$

15. $y = \ln x - \ln 3$ **17.** $y = \frac{1}{1 - e^x}$

19. **(a)** Same rate **(b)** Same rate **(c)** Faster **(d)** Faster **(e)** Same rate **(f)** Same rate
21. **(a)** True **(b)** False **(c)** False **(d)** True **(e)** True **(f)** True
23. $1/3$ **25.** $1/e$ m/sec **27.** $\ln 5x - \ln 3x = \ln(5/3)$

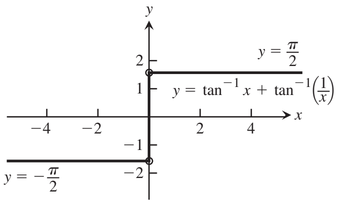
29. $1/2$ **31.** $y = \left(\tan^{-1}\left(\frac{x + C}{2}\right)\right)^2$

33. $y^2 = \sin^{-1}(2\tan x + C)$
35. $y = -2 + \ln(2 - e^{-x})$ **37.** $y = 4x - 4\sqrt{x} + 1$
39. 19,035 years

Additional and Advanced Exercises, p. 455

1. **(a)** 1 **(b)** $\pi/2$ **(c)** π

3. $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)$ is a constant and the constant is $\frac{\pi}{2}$ for $x > 0$; it is $-\frac{\pi}{2}$ for $x < 0$.



7. $\bar{x} = \frac{\ln 4}{\pi}$, $\bar{y} = 0$

Chapter 8

Section 8.1, pp. 460–461

1. $\ln 5$ **3.** $2\tan x - 2\sec x - x + C$

5. $\sin^{-1}x + \sqrt{1 - x^2} + C$ **7.** $e^{-\cot z} + C$
9. $\tan^{-1}(e^z) + C$ **11.** π **13.** $t + \cot t + \csc t + C$

15. $\sqrt{2}$ **17.** $\frac{1}{8}\ln(1 + 4\ln^2 y) + C$

19. $\ln|1 + \sin\theta| + C$ **21.** $2t^2 - t + 2\tan^{-1}\left(\frac{t}{2}\right) + C$

23. $2(\sqrt{2} - 1) \approx 0.82843$ **25.** $\sec^{-1}(e^y) + C$
27. $\sin^{-1}(2\ln x) + C$ **29.** $\ln|\sin x| + \ln|\cos x| + C$
31. $7 + \ln 8$ **33.** $\left(\sin^{-1}y - \sqrt{1 - y^2}\right]_{-1}^{0} = \frac{\pi}{2} - 1$

35. $\sec^{-1}\left|\frac{x - 1}{7}\right| + C$ **37.** $\frac{\theta^3}{3} - \frac{\theta^2}{2} + \theta + \frac{5}{2}\ln|2\theta - 5| + C$

39. $x - \ln(1 + e^x) + C$ **41.** $2\sqrt{2} - \ln(3 + 2\sqrt{2})$

43. $\ln(2 + \sqrt{3})$ **45.** $\bar{x} = 0$, $\bar{y} = \frac{1}{\ln(3 + 2\sqrt{2})}$

47. $xe^{x^3} + C$ **49.** $\frac{1}{30}(x^4 + 1)^{3/2}(3x^4 - 2) + C$

Section 8.2, pp. 467–469

1. $-2x\cos(x/2) + 4\sin(x/2) + C$
3. $t^2\sin t + 2t\cos t - 2\sin t + C$
5. $\ln 4 - \frac{3}{4}$ **7.** $xe^x - e^x + C$
9. $-(x^2 + 2x + 2)e^{-x} + C$
11. $y\tan^{-1}(y) - \ln\sqrt{1 + y^2} + C$
13. $x\tan x + \ln|\cos x| + C$
15. $(x^3 - 3x^2 + 6x - 6)e^x + C$ **17.** $(x^2 - 7x + 7)e^x + C$
19. $(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120)e^x + C$
21. $\frac{1}{2}(-e^\theta\cos\theta + e^\theta\sin\theta) + C$
23. $\frac{e^{2x}}{13}(3\sin 3x + 2\cos 3x) + C$
25. $\frac{2}{3}\left(\sqrt{3s+9}\,e^{\sqrt{3s+9}} - e^{\sqrt{3s+9}}\right) + C$
27. $\frac{\pi\sqrt{3}}{3} - \ln(2) - \frac{\pi^2}{18}$
29. $\frac{1}{2}[-x\cos(\ln x) + x\sin(\ln x)] + C$
31. $\frac{1}{2}\ln|\sec x^2 + \tan x^2| + C$
33. $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2 + C$
35. $-\frac{1}{x}\ln x - \frac{1}{x} + C$ **37.** $\frac{1}{4}e^{x^4} + C$
39. $\frac{1}{3}x^2(x^2 + 1)^{3/2} - \frac{2}{15}(x^2 + 1)^{5/2} + C$
41. $-\frac{2}{5}\sin 3x\sin 2x - \frac{3}{5}\cos 3x\cos 2x + C$
43. $\frac{2}{9}x^{3/2}(3\ln x - 2) + C$
45. $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$
47. $\frac{\pi^2 - 4}{8}$ **49.** $\frac{5\pi - 3\sqrt{3}}{9}$
51. $\frac{1}{2}(x^2 + 1)\tan^{-1}x - \frac{x}{2} + C$
53. **(a)** π **(b)** 3π **(c)** 5π **(d)** $(2n + 1)\pi$
55. $2\pi(1 - \ln 2)$ **57.** **(a)** $\pi(\pi - 2)$ **(b)** 2π
59. **(a)** 1 **(b)** $(e - 2)\pi$ **(c)** $\frac{\pi}{2}(e^2 + 9)$
(d) $\bar{x} = \frac{1}{4}(e^2 + 1), \bar{y} = \frac{1}{2}(e - 2)$
61. $\frac{1}{2\pi}(1 - e^{-2\pi})$ **63.** $u = x^n, dv = \cos x\,dx$
65. $u = x^n, dv = e^{ax}\,dx$ **71.** $x\sin^{-1}x + \cos(\sin^{-1}x) + C$
73. $x\sec^{-1}x - \ln|x + \sqrt{x^2 - 1}| + C$ **75.** Yes
77. **(a)** $x\sinh^{-1}x - \cosh(\sinh^{-1}x) + C$
(b) $x\sinh^{-1}x - (1 + x^2)^{1/2} + C$

Section 8.3, pp. 474–475

1. $\frac{1}{2}\sin 2x + C$ **3.** $-\frac{1}{4}\cos^4 x + C$
5. $\frac{1}{3}\cos^3 x - \cos x + C$
7. $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$
9. $\sin x - \frac{1}{3}\sin^3 x + C$ **11.** $\frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + C$
13. $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$ **15.** $16/35$ **17.** 3π
19. $-4\sin x\cos^3 x + 2\cos x\sin x + 2x + C$
21. $-\cos^4 2\theta + C$ **23.** 4 **25.** 2
27. $\sqrt{\frac{3}{2}} - \frac{2}{3}$ **29.** $\frac{4}{5}\left(\frac{3}{2}\right)^{5/2} - \frac{18}{35} - \frac{2}{7}\left(\frac{3}{2}\right)^{7/2}$ **31.** $\sqrt{2}$
33. $\frac{1}{2}\tan^2 x + C$ **35.** $\frac{1}{3}\sec^3 x + C$ **37.** $\frac{1}{3}\tan^3 x + C$
39. $2\sqrt{3} + \ln(2 + \sqrt{3})$ **41.** $\frac{2}{3}\tan\theta + \frac{1}{3}\sec^2\theta\tan\theta + C$
43. $4/3$ **45.** $2\tan^2 x - 2\ln(1 + \tan^2 x) + C$
47. $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \ln|\sec x| + C$ **49.** $\frac{4}{3} - \ln\sqrt{3}$
51. $-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C$ **53.** π
55. $\frac{1}{2}\sin x + \frac{1}{14}\sin 7x + C$
57. $\frac{1}{6}\sin 3\theta - \frac{1}{4}\sin\theta - \frac{1}{20}\sin 5\theta + C$
59. $-\frac{2}{5}\cos^5\theta + C$ **61.** $\frac{1}{4}\cos\theta - \frac{1}{20}\cos 5\theta + C$
63. $\sec x - \ln|\csc x + \cot x| + C$ **65.** $\cos x + \sec x + C$
67. $\frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + C$ **69.** $\ln(1 + \sqrt{2})$
71. $\pi^2/2$ **73.** $\bar{x} = \frac{4\pi}{3}, \bar{y} = \frac{8\pi^2 + 3}{12\pi}$

Section 8.4, pp. 479–480

1. $\ln|\sqrt{9 + x^2} + x| + C$ **3.** $\pi/4$ **5.** $\pi/6$
7. $\frac{25}{2}\sin^{-1}\left(\frac{t}{5}\right) + \frac{t\sqrt{25 - t^2}}{2} + C$
9. $\frac{1}{2}\ln\left|\frac{2x}{7} + \frac{\sqrt{4x^2 - 49}}{7}\right| + C$
11. $7\left[\frac{\sqrt{y^2 - 49}}{7} - \sec^{-1}\left(\frac{y}{7}\right)\right] + C$ **13.** $\frac{\sqrt{x^2 - 1}}{x} + C$
15. $-\sqrt{9 - x^2} + C$ **17.** $\frac{1}{3}(x^2 + 4)^{3/2} - 4\sqrt{x^2 + 4} + C$
19. $\frac{-2\sqrt{4 - w^2}}{w} + C$ **21.** $\sin^{-1}x - \sqrt{1 - x^2} + C$
23. $4\sqrt{3} - \frac{4\pi}{3}$ **25.** $-\frac{x}{\sqrt{x^2 - 1}} + C$
27. $-\frac{1}{5}\left(\frac{\sqrt{1 - x^2}}{x}\right)^5 + C$ **29.** $2\tan^{-1}2x + \frac{4x}{(4x^2 + 1)} + C$
31. $\frac{1}{2}x^2 + \frac{1}{2}\ln|x^2 - 1| + C$ **33.** $\frac{1}{3}\left(\frac{v}{\sqrt{1 - v^2}}\right)^3 + C$
35. $\ln 9 - \ln(1 + \sqrt{10})$ **37.** $\pi/6$ **39.** $\sec^{-1}|x| + C$
41. $\sqrt{x^2 - 1} + C$ **43.** $\frac{1}{2}\ln|\sqrt{1 + x^4} + x^2| + C$
45. $4\sin^{-1}\frac{\sqrt{x}}{2} + \sqrt{x}\sqrt{4 - x} + C$
47. $\frac{1}{4}\sin^{-1}\sqrt{x} - \frac{1}{4}\sqrt{x}\sqrt{1 - x}(1 - 2x) + C$
49. $y = 2\left[\frac{\sqrt{x^2 - 4}}{2} - \sec^{-1}\left(\frac{x}{2}\right)\right]$
51. $y = \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) - \frac{3\pi}{8}$ **53.** $3\pi/4$

55. **(a)** $\frac{1}{12}(\pi + 6\sqrt{3} - 12)$

(b) $\bar{x} = \frac{3\sqrt{3} - \pi}{4(\pi + 6\sqrt{3} - 12)}, \bar{y} = \frac{\pi^2 + 12\sqrt{3}\pi - 72}{12(\pi + 6\sqrt{3} - 12)}$

57. **(a)** $-\frac{1}{3}x^2(1 - x^2)^{3/2} - \frac{2}{15}(1 - x^2)^{5/2} + C$

(b) $-\frac{1}{3}(1 - x^2)^{3/2} + \frac{1}{5}(1 - x^2)^{5/2} + C$

(c) $\frac{1}{5}(1 - x^2)^{5/2} - \frac{1}{3}(1 - x^2)^{3/2} + C$

Section 8.5, pp. 487–488

1. $\frac{2}{x - 3} + \frac{3}{x - 2}$ **3.** $\frac{1}{x + 1} + \frac{3}{(x + 1)^2}$

5. $\frac{-2}{z} + \frac{-1}{z^2} + \frac{2}{z - 1}$ **7.** $1 + \frac{17}{t - 3} + \frac{-12}{t - 2}$

9. $\frac{1}{2}[\ln|1 + x| - \ln|1 - x|] + C$

11. $\frac{1}{7}\ln|(x + 6)^2(x - 1)^5| + C$ **13.** $(\ln 15)/2$

15. $-\frac{1}{2}\ln|t| + \frac{1}{6}\ln|t + 2| + \frac{1}{3}\ln|t - 1| + C$ **17.** $3\ln 2 - 2$

19. $\frac{1}{4}\ln\left|\frac{x + 1}{x - 1}\right| - \frac{x}{2(x^2 - 1)} + C$ **21.** $(\pi + 2\ln 2)/8$

23. $\tan^{-1}y - \frac{1}{y^2 + 1} + C$

25. $-(s - 1)^{-2} + (s - 1)^{-1} + \tan^{-1}s + C$

27. $\frac{2}{3}\ln|x - 1| + \frac{1}{6}\ln|x^2 + x + 1| - \sqrt{3}\tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right) + C$

29. $\frac{1}{4}\ln\left|\frac{x - 1}{x + 1}\right| + \frac{1}{2}\tan^{-1}x + C$

31. $\frac{-1}{\theta^2 + 2\theta + 2} + \ln(\theta^2 + 2\theta + 2) - \tan^{-1}(\theta + 1) + C$

33. $x^2 + \ln\left|\frac{x - 1}{x}\right| + C$

35. $9x + 2\ln|x| + \frac{1}{x} + 7\ln|x - 1| + C$

37. $\frac{y^2}{2} - \ln|y| + \frac{1}{2}\ln(1 + y^2) + C$ **39.** $\ln\left(\frac{e^t + 1}{e^t + 2}\right) + C$

41. $\frac{1}{5}\ln\left|\frac{\sin y - 2}{\sin y + 3}\right| + C$

43. $\frac{(\tan^{-1}2x)^2}{4} - 3\ln|x - 2| + \frac{6}{x - 2} + C$

45. $\ln\left|\frac{\sqrt{x} - 1}{\sqrt{x} + 1}\right| + C$

47. $2\sqrt{1 + x} + \ln\left|\frac{\sqrt{x + 1} - 1}{\sqrt{x + 1} + 1}\right| + C$

49. $\frac{1}{4}\ln\left|\frac{x^4}{x^4 + 1}\right| + C$ **51.** $x = \ln|t - 2| - \ln|t - 1| + \ln 2$

53. $x = \frac{6t}{t + 2} - 1$ **55.** $3\pi\ln 25$ **57.** 1.10

59. **(a)** $x = \frac{1000e^{4t}}{499 + e^{4t}}$ **(b)** 1.55 days

Section 8.6, pp. 493–494

1. $\frac{2}{\sqrt{3}}\left(\tan^{-1}\sqrt{\frac{x - 3}{3}}\right) + C$

3. $\sqrt{x - 2}\left(\frac{2(x - 2)}{3} + 4\right) + C$ **5.** $\frac{(2x - 3)^{3/2}(x + 1)}{5} + C$

7. $\frac{-\sqrt{9 - 4x}}{x} - \frac{2}{3}\ln\left|\frac{\sqrt{9 - 4x} - 3}{\sqrt{9 - 4x} + 3}\right| + C$

9. $\frac{(x + 2)(2x - 6)\sqrt{4x - x^2}}{6} + 4\sin^{-1}\left(\frac{x - 2}{2}\right) + C$

11. $-\frac{1}{\sqrt{7}}\ln\left|\frac{\sqrt{7} + \sqrt{7 + x^2}}{x}\right| + C$

13. $\sqrt{4 - x^2} - 2\ln\left|\frac{2 + \sqrt{4 - x^2}}{x}\right| + C$

15. $\frac{e^{2t}}{13}(2\cos 3t + 3\sin 3t) + C$

17. $\frac{x^2}{2}\cos^{-1}x + \frac{1}{4}\sin^{-1}x - \frac{1}{4}x\sqrt{1 - x^2} + C$

19. $\frac{x^3}{3}\tan^{-1}x - \frac{x^2}{6} + \frac{1}{6}\ln(1 + x^2) + C$

21. $-\frac{\cos 5x}{10} - \frac{\cos x}{2} + C$

23. $8\left[\frac{\sin(7t/2)}{7} - \frac{\sin(9t/2)}{9}\right] + C$

25. $6\sin(\theta/12) + \frac{6}{7}\sin(7\theta/12) + C$

27. $\frac{1}{2}\ln(x^2 + 1) + \frac{x}{2(1 + x^2)} + \frac{1}{2}\tan^{-1}x + C$

29. $\left(x - \frac{1}{2}\right)\sin^{-1}\sqrt{x} + \frac{1}{2}\sqrt{x - x^2} + C$

31. $\sin^{-1}\sqrt{x} - \sqrt{x - x^2} + C$

33. $\sqrt{1 - \sin^2 t} - \ln\left|\frac{1 + \sqrt{1 - \sin^2 t}}{\sin t}\right| + C$

35. $\ln|\ln y + \sqrt{3 + (\ln y)^2}| + C$

37. $\ln|x + 1 + \sqrt{x^2 + 2x + 5}| + C$

39. $\frac{x + 2}{2}\sqrt{5 - 4x - x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x + 2}{3}\right) + C$

41. $-\frac{\sin^4 2x\cos 2x}{10} - \frac{2\sin^2 2x\cos 2x}{15} - \frac{4\cos 2x}{15} + C$

43. $\frac{\sin^3 2\theta\cos^2 2\theta}{10} + \frac{\sin^3 2\theta}{15} + C$

45. $\tan^2 2x - 2\ln|\sec 2x| + C$

47. $\frac{(\sec\pi x)(\tan\pi x)}{\pi} + \frac{1}{\pi}\ln|\sec\pi x + \tan\pi x| + C$

49. $\frac{-\csc^3 x\cot x}{4} - \frac{3\csc x\cot x}{8} - \frac{3}{8}\ln|\csc x + \cot x| + C$

51. $\frac{1}{2}[\sec(e^t - 1)\tan(e^t - 1) + \ln|\sec(e^t - 1) + \tan(e^t - 1)|] + C$

53. $\sqrt{2} + \ln(\sqrt{2} + 1)$ **55.** $\pi/3$

57. $2\pi\sqrt{3} + \pi\sqrt{2}\ln(\sqrt{2} + \sqrt{3})$ **59.** $\bar{x} = 4/3, \bar{y} = \ln\sqrt{2}$

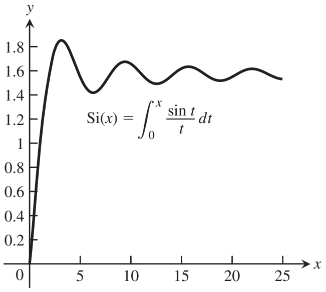
61. 7.62 **63.** $\pi/8$ **67.** $\pi/4$

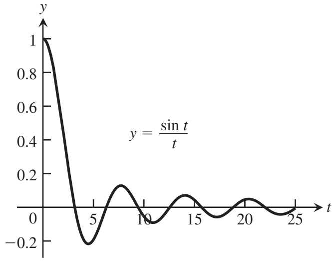
Section 8.7, pp. 501–504

1. I: (a) 1.5, 0 **(b)** 1.5, 0 **(c)** 0%
II: (a) 1.5, 0 **(b)** 1.5, 0 **(c)** 0%
3. I: (a) 2.75, 0.08 **(b)** 2.67, 0.08 **(c)** 0.0312 ≈ 3%
II: (a) 2.67, 0 **(b)** 2.67, 0 **(c)** 0%
5. I: (a) 6.25, 0.5 **(b)** 6, 0.25 **(c)** 0.0417 ≈ 4%
II: (a) 6, 0 **(b)** 6, 0 **(c)** 0%
7. I: (a) 0.509, 0.03125 **(b)** 0.5, 0.009 **(c)** 0.018 ≈ 2%
II: (a) 0.5, 0.002604 **(b)** 0.5, 0.4794 **(c)** 0%
9. I: (a) 1.8961, 0.161 **(b)** 2, 0.1039 **(c)** 0.052 ≈ 5%
II: (a) 2.0045, 0.0066 **(b)** 2, 0.00454 **(c)** 0.2%
11. (a) 1 **(b)** 2 **13. (a)** 116 **(b)** 2
15. (a) 283 **(b)** 2 **17. (a)** 71 **(b)** 10
19. (a) 76 **(b)** 12 **21. (a)** 82 **(b)** 8
23. 15,990 ft^3 **25.** ≈10.63 ft
27. (a) ≈0.00021 **(b)** ≈1.37079 **(c)** ≈0.015%
31. (a) ≈5.870 **(b)** $|E_T| \le 0.0032$ **33.** 21.07 in. **35.** 14.4
39. ≈28.7 mg

Section 8.8, pp. 513–515

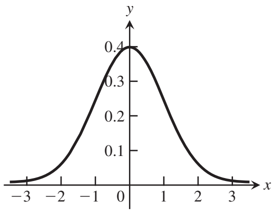
1. $\pi/2$ **3.** 2 **5.** 6 **7.** $\pi/2$ **9.** ln 3 **11.** ln 4
13. 0 **15.** $\sqrt{3}$ **17.** π **19.** $\ln\left(1 + \frac{\pi}{2}\right)$
21. −1 **23.** 1 **25.** −1/4 **27.** $\pi/2$ **29.** $\pi/3$
31. 6 **33.** ln 2 **35.** Diverges **37.** Diverges
39. Converges **41.** Converges **43.** Diverges
45. Converges **47.** Converges **49.** Diverges
51. Converges **53.** Converges **55.** Diverges
57. Converges **59.** Diverges **61.** Converges
63. Converges
65. (a) Converges when $p < 1$ **(b)** Converges when $p > 1$
67. 1 **69.** 2π **71.** ln 2
73. (a) $\pi/2$ **(b)** π **75. (b)** ≈0.88621
77. (a)





(b) $\pi/2$

79. (a)



(b) ≈0.683, ≈0.954, ≈0.997
85. ≈0.16462

Section 8.9, pp. 526–528

1. No **3.** Yes **5.** Yes **7.** Yes **11.** ≈0.537
13. ≈0.688 **15.** ≈0.0502 **17.** $\sqrt{21}$ **19.** $\frac{1}{2}\ln 2$
21. $\frac{1}{\pi}, \frac{1}{\pi}\left(\tan^{-1}2 - \frac{\pi}{4}\right) \approx 0.10242$
25. mean $= \frac{8}{3} \approx 2.67$, median $= \sqrt{8} \approx 2.83$
27. mean $= 2$, median $= \sqrt{2} \approx 1.41$
29. $P\left(X < \frac{1}{2}\right) \approx 0.3935$
31. (a) ≈0.57, so about 57 in every 100 bulbs will fail.
(b) ≈832 hr
33. ≈60 hydra **35. (a)** ≈0.393 **(b)** ≈0.135 **(c)** 0
(d) The probability that any customer waits longer than 3 minutes is $1 - (0.997521)^{200} \approx 0.391 < 1/2$. So the most likely outcome is that all 200 would be served within 3 minutes.
37. \$10, 256 **39.** ≈323, ≈262 **41.** ≈0.89435
43. (a) ≈16% **(b)** ≈0.23832 **45.** ≈618 females
47. ≈61 adults **49.** ≈289 shafts
51. (a) ≈0.977 **(b)** ≈0.159 **(c)** ≈0.838
55. (a) {LLL, LLD, LDL, DLL, LLU, LUL, ULL, LDD, LDU, LUD, LUU, DLD, DLU, ULD, ULU, DDL, DUL, UDL, UUL, DDD, DDU, DUD, UDD, DUU, UDU, UUD, UUU}
(c) 7/27 ≈ 0.26 **(d)** 20/27 ≈ 0.74

Practice Exercises, pp. 529–531

1. $(x + 1)(\ln(x + 1)) - (x + 1) + C$
3. $x\tan^{-1}(3x) - \frac{1}{6}\ln(1 + 9x^2) + C$
5. $(x + 1)^2 e^x - 2(x + 1)e^x + 2e^x + C$
7. $\frac{2e^x \sin 2x}{5} + \frac{e^x \cos 2x}{5} + C$
9. $2\ln|x - 2| - \ln|x - 1| + C$
11. $\ln|x| - \ln|x + 1| + \frac{1}{x + 1} + C$
13. $-\frac{1}{3}\ln\left|\frac{\cos\theta - 1}{\cos\theta + 2}\right| + C$
15. $4\ln|x| - \frac{1}{2}\ln(x^2 + 1) + 4\tan^{-1}x + C$
17. $\frac{1}{16}\ln\left|\frac{(v - 2)^5(v + 2)}{v^6}\right| + C$
19. $\frac{1}{2}\tan^{-1}t - \frac{\sqrt{3}}{6}\tan^{-1}\frac{t}{\sqrt{3}} + C$
21. $\frac{x^2}{2} + \frac{4}{3}\ln|x + 2| + \frac{2}{3}\ln|x - 1| + C$
23. $\frac{x^2}{2} - \frac{9}{2}\ln|x + 3| + \frac{3}{2}\ln|x + 1| + C$
25. $\frac{1}{3}\ln\left|\frac{\sqrt{x + 1} - 1}{\sqrt{x + 1} + 1}\right| + C$ **27.** $\ln|1 - e^{-s}| + C$
29. $-\sqrt{16 - y^2} + C$ **31.** $-\frac{1}{2}\ln|4 - x^2| + C$
33. $\ln\frac{1}{\sqrt{9 - x^2}} + C$ **35.** $\frac{1}{6}\ln\left|\frac{x + 3}{x - 3}\right| + C$
37. $-\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$ **39.** $\frac{\tan^5 x}{5} + C$
41. $\frac{\cos\theta}{2} - \frac{\cos 11\theta}{22} + C$ **43.** $4\sqrt{1 - \cos(t/2)} + C$
45. At least 16 **47.** $T = \pi, S = \pi$ **49.** 25°F
51. (a) ≈2.42 gal **(b)** ≈24.83 mi/gal

53. $\pi/2$ **55.** 6 **57.** ln 3 **59.** 2 **61.** $\pi/6$
63. Diverges **65.** Diverges **67.** Converges

69. $\frac{2x^{3/2}}{3} - x + 2\sqrt{x} - 2\ln\left(\sqrt{x} + 1\right) + C$

71. $\frac{1}{2}\sin^{-1}(x-1) + \frac{1}{2}(x-1)\sqrt{2x - x^2} + C$

73. $-2\cot x - \ln|\csc x + \cot x| + \csc x + C$

75. $\frac{1}{12}\ln\left|\frac{3+v}{3-v}\right| + \frac{1}{6}\tan^{-1}\frac{v}{3} + C$

77. $\frac{\theta\sin(2\theta+1)}{2} + \frac{\cos(2\theta+1)}{4} + C$ **79.** $\frac{1}{4}\sec^2\theta + C$

81. $2\left(\frac{\left(\sqrt{2-x}\right)^3}{3} - 2\sqrt{2-x}\right) + C$

83. $\tan^{-1}(y-1) + C$

85. $\frac{1}{4}\ln|z| - \frac{1}{4z} - \frac{1}{4}\left[\frac{1}{2}\ln(z^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{z}{2}\right)\right] + C$

87. $-\frac{1}{4}\sqrt{9 - 4t^2} + C$ **89.** $\ln\left(\frac{e^t + 1}{e^t + 2}\right) + C$ **91.** $1/4$

93. $\frac{2}{3}x^{3/2} + C$ **95.** $-\frac{1}{5}\tan^{-1}(\cos 5t) + C$

97. $2\sqrt{r} - 2\ln\left(1 + \sqrt{r}\right) + C$

99. $\frac{1}{2}x^2 - \frac{1}{2}\ln(x^2+1) + C$

101. $\frac{2}{3}\ln|x+1| + \frac{1}{6}\ln|x^2 - x + 1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

103. $\frac{4}{7}\left(1+\sqrt{x}\right)^{7/2} - \frac{8}{5}\left(1+\sqrt{x}\right)^{5/2} + \frac{4}{3}\left(1+\sqrt{x}\right)^{3/2} + C$

105. $2\ln\left|\sqrt{x} + \sqrt{1+x}\right| + C$

107. $\ln x - \ln|1 + \ln x| + C$

109. $\frac{1}{2}x^{\ln x} + C$ **111.** $\frac{1}{2}\ln\left|\frac{1 - \sqrt{1 - x^4}}{x^2}\right| + C$

113. **(b)** $\frac{\pi}{4}$ **115.** $x - \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right) + C$

Additional and Advanced Exercises, pp. 531–535

1. $x(\sin^{-1}x)^2 + 2(\sin^{-1}x)\sqrt{1-x^2} - 2x + C$

3. $\frac{x^2\sin^{-1}x}{2} + \frac{x\sqrt{1-x^2} - \sin^{-1}x}{4} + C$

5. $\frac{1}{2}\left(\ln\left(t - \sqrt{1-t^2}\right) - \sin^{-1}t\right) + C$ **7.** 0

9. $\ln(4) - 1$ **11.** 1 **13.** $32\pi/35$ **15.** 2π

17. **(a)** π **(b)** $\pi(2e - 5)$

19. **(b)** $\pi\left(\frac{8(\ln 2)^2}{3} - \frac{16(\ln 2)}{9} + \frac{16}{27}\right)$

21. $\left(\frac{e^2+1}{4}, \frac{e-2}{2}\right)$

23. $\sqrt{1+e^2} - \ln\left(\frac{\sqrt{1+e^2}}{e} + \frac{1}{e}\right) - \sqrt{2} + \ln\left(1 + \sqrt{2}\right)$

25. $\frac{12\pi}{5}$ **27.** $a = \frac{1}{2}, -\frac{\ln 2}{4}$ **29.** $\frac{1}{2} < p \le 1$

33. $\frac{e^{2x}}{13}(3\sin 3x + 2\cos 3x) + C$

35. $\frac{\cos x\sin 3x - 3\sin x\cos 3x}{8} + C$

37. $\frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx) + C$

39. $x\ln(ax) - x + C$ **41.** $\frac{2}{1 - \tan(x/2)} + C$ **43.** 1

45. $\frac{\sqrt{3}\pi}{9}$ **47.** $\frac{1}{\sqrt{2}}\ln\left|\frac{\tan(t/2) + 1 - \sqrt{2}}{\tan(t/2) + 1 + \sqrt{2}}\right| + C$

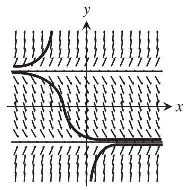
49. $\ln\left|\frac{1 + \tan(\theta/2)}{1 - \tan(\theta/2)}\right| + C$

Chapter 9

Section 9.1, pp. 542–544

1. (d) **3.** (a)

5.



7. $y' = x - y;\ y(1) = -1$ **9.** $y' = -(1+y)\sin x;\ y(0) = 2$

11. $y(\text{exact}) = \frac{x}{2} - \frac{4}{x}$, $y_1 = -0.25$, $y_2 = 0.3$, $y_3 = 0.75$

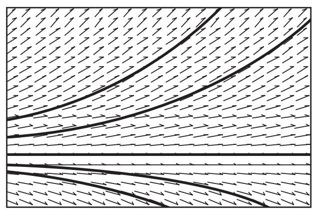
13. $y(\text{exact}) = 3e^{x(x+2)}$, $y_1 = 4.2$, $y_2 = 6.216$, $y_3 = 9.697$

15. $y(\text{exact}) = e^{x^2} + 1$, $y_1 = 2.0$, $y_2 = 2.0202$, $y_3 = 2.0618$

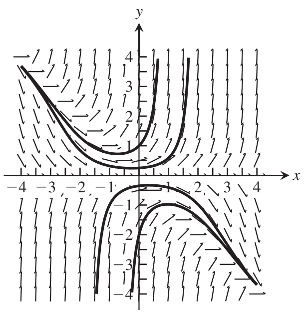
17. $y \approx 2.48832$, exact value is e.

19. $y \approx -0.2272$, exact value is $1/\left(1 - 2\sqrt{5}\right) \approx -0.2880$.

23.



25.



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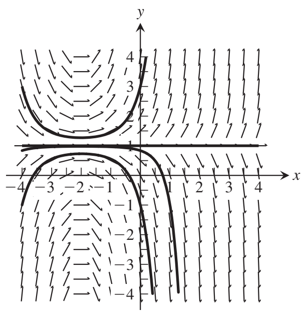


35. Euler's method gives $y \approx 3.45835$; the exact solution is $y = 1 + e \approx 3.71828$.

37. $y \approx 1.5000$; exact value is 1.5275.

Section 9.2, pp. 548–550

1. $y = \dfrac{e^x + C}{x}, \quad x > 0$ **3.** $y = \dfrac{C - \cos x}{x^3}, \quad x > 0$

5. $y = \dfrac{1}{2} - \dfrac{1}{x} + \dfrac{C}{x^2}, \quad x > 0$ **7.** $y = \dfrac{1}{2}xe^{x/2} + Ce^{x/2}$

9. $y = x(\ln x)^2 + Cx$

11. $s = \dfrac{t^3}{3(t-1)^4} - \dfrac{t}{(t-1)^4} + \dfrac{C}{(t-1)^4}$

13. $r = (\csc\theta)(\ln|\sec\theta| + C), \quad 0 < \theta < \pi/2$

15. $y = \dfrac{3}{2} - \dfrac{1}{2}e^{-2t}$ **17.** $y = -\dfrac{1}{\theta}\cos\theta + \dfrac{\pi}{2\theta}$

19. $y = 6e^{x^2} - \dfrac{e^{x^2}}{x+1}$ **21.** $y = y_0 e^{kt}$

23. (b) is correct, but (a) is not. **25.** $t = \dfrac{L}{R}\ln 2$ sec

27. (a) $i = \dfrac{V}{R} - \dfrac{V}{R}e^{-3} = \dfrac{V}{R}(1 - e^{-3}) \approx 0.95\dfrac{V}{R}$ amp **(b)** 86%

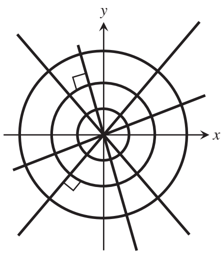
29. $y = \dfrac{1}{1 + Ce^{-x}}$ **31.** $y^3 = 1 + Cx^{-3}$

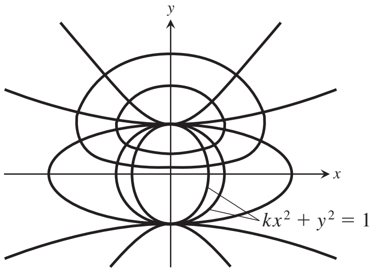
Section 9.3, pp. 555–556

1. (a) 168.5 m **(b)** 41.13 sec

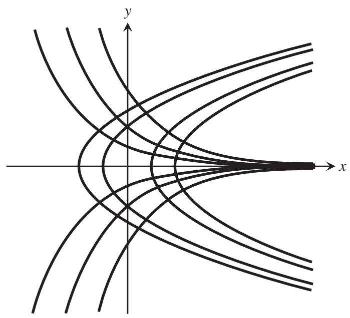
3. $s(t) = 4.91\left(1 - e^{-(22.36/39.92)t}\right)$

5. $x^2 + y^2 = C$ **7.** $\ln|y| - \dfrac{1}{2}y^2 = \dfrac{1}{2}x^2 + C$





9. $y = \pm\sqrt{2x + C}$



13. (a) 10 lb/min **(b)** $(100 + t)$ gal **(c)** $4\left(\dfrac{y}{100 + t}\right)$ lb/min

(d) $\dfrac{dy}{dt} = 10 - \dfrac{4y}{100 + t}, \quad y(0) = 50,$

$$y = 2(100 + t) - \frac{150}{\left(1 + \dfrac{t}{100}\right)^4}$$

(e) Concentration $= \dfrac{y(25)}{\text{amt. brine in tank}} = \dfrac{188.6}{125} \approx 1.5$ lb/gal

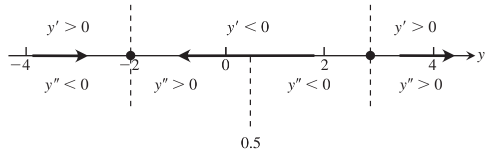
15. $y(27.8) \approx 14.8$ lb, $t \approx 27.8$ min

Section 9.4, pp. 562–563

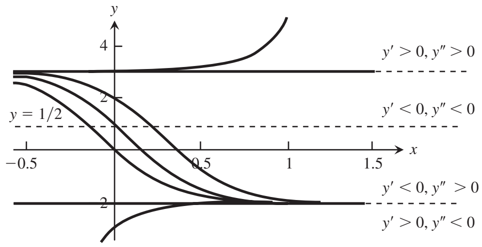
1. $y' = (y + 2)(y - 3)$

(a) $y = -2$ is a stable equilibrium value and $y = 3$ is an unstable equilibrium.

(b) $y'' = 2(y + 2)\left(y - \dfrac{1}{2}\right)(y - 3)$



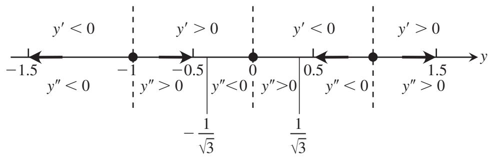
(c)



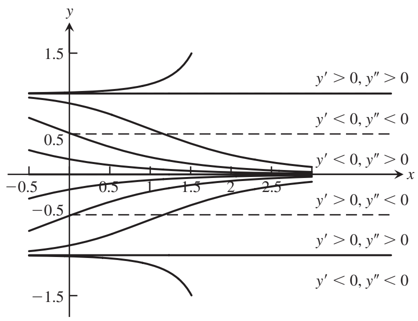
3. $y' = y^3 - y = (y + 1)y(y - 1)$

(a) $y = -1$ and $y = 1$ are unstable equilibria and $y = 0$ is a stable equilibrium.

(b) $y'' = (3y^2 - 1)y'$
$= 3(y + 1)\left(y + 1/\sqrt{3}\right)y\left(y - 1/\sqrt{3}\right)(y - 1)$



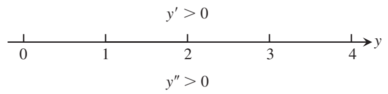
(c)



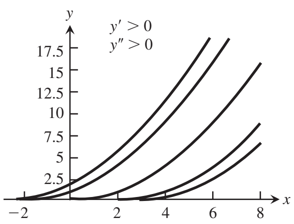
5. $y' = \sqrt{y}, \; y > 0$

(a) There are no equilibrium values.

(b) $y'' = \dfrac{1}{2}$



(c)

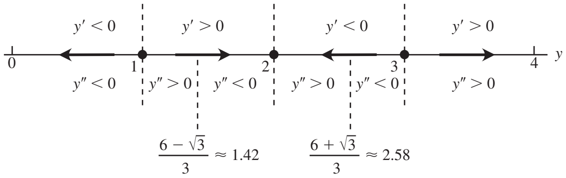


7. $y' = (y - 1)(y - 2)(y - 3)$

(a) $y = 1$ and $y = 3$ are unstable equilibria and $y = 2$ is a stable equilibrium.

(b) $y'' = (3y^2 - 12y + 11)(y - 1)(y - 2)(y - 3) =$

$$3(y-1)\left(y - \frac{6-\sqrt{3}}{3}\right)(y-2)\left(y - \frac{6+\sqrt{3}}{3}\right)(y-3)$$



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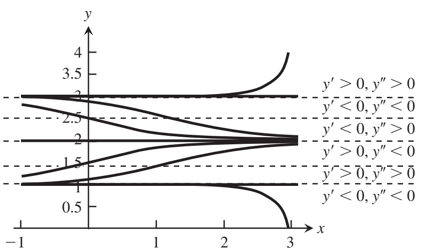
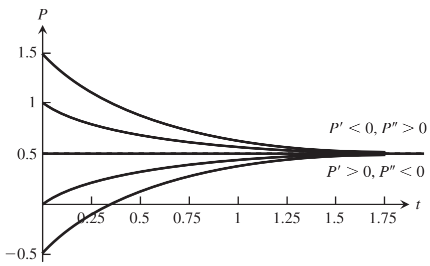
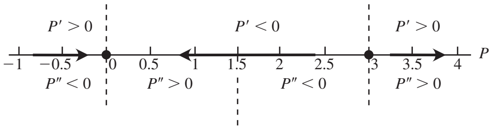
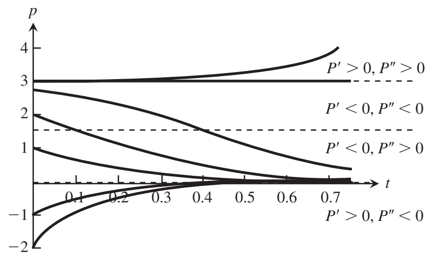


9. $\dfrac{dP}{dt} = 1 - 2P$ has a stable equilibrium at $P = \dfrac{1}{2}$; $\dfrac{d^2P}{dt^2} = -2\dfrac{dP}{dt} = -2(1 - 2P)$.

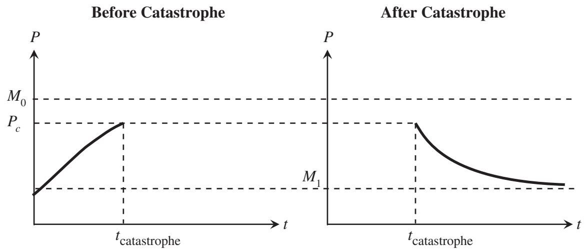


11. $\dfrac{dP}{dt} = 2P(P - 3)$ has a stable equilibrium at $P = 0$ and an unstable equilibrium at $P = 3$; $\dfrac{d^2P}{dt^2} = 2(2P - 3)\dfrac{dP}{dt} = 4P(2P - 3)(P - 3)$.





13. Before the catastrophe, the population exhibits logistic growth and $P(t)$ increases toward M_0, the stable equilibrium. After the catastrophe, the population declines logistically and $P(t)$ decreases toward M_1, the new stable equilibrium.

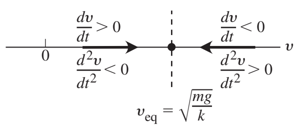


15. $\dfrac{dv}{dt} = g - \dfrac{k}{m}v^2, \quad g, k, m > 0$ and $v(t) \ge 0$

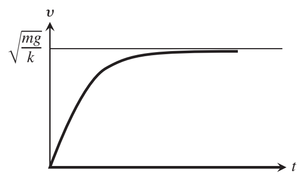
Equilibrium: $\dfrac{dv}{dt} = g - \dfrac{k}{m}v^2 = 0 \Rightarrow v = \sqrt{\dfrac{mg}{k}}$

Concavity: $\dfrac{d^2v}{dt^2} = -2\left(\dfrac{k}{m}v\right)\dfrac{dv}{dt} = -2\left(\dfrac{k}{m}v\right)\left(g - \dfrac{k}{m}v^2\right)$

(a)



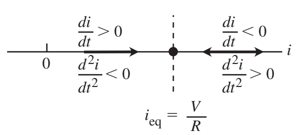
(b)



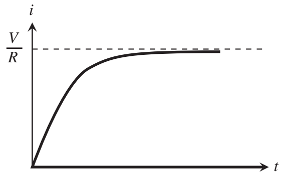
(c) $v_{\text{terminal}} = \sqrt{\dfrac{160}{0.005}} = 178.9$ ft/sec $= 122$ mph

17. $F = F_p - F_r$; $ma = 50 - 5|v|$; $\dfrac{dv}{dt} = \dfrac{1}{m}(50 - 5|v|)$. The maximum velocity occurs when $\dfrac{dv}{dt} = 0$ or $v = 10$ ft/sec.

19. Phase line:



If the switch is closed at $t = 0$, then $i(0) = 0$, and the graph of the solution looks like this:



As $t \to \infty$, $i(t) \to i_{\text{steady state}} = \dfrac{V}{R}$.

Section 9.5, pp. 567–569

1. Seasonal variations, nonconformity of the environments, effects of other interactions, unexpected disasters, etc.

3. This model assumes that the number of interactions is proportional to the product of x and y:

$$\frac{dx}{dt} = (a - by)x, \quad a < 0,$$

$$\frac{dy}{dt} = m\left(1 - \frac{y}{M}\right)y - nxy = y\left(m - \frac{m}{M}y - nx\right).$$

Rest points are (0, 0), unstable, and (0, M), stable.

5. (a) Logistic growth occurs in the absence of the competitor, and involves a simple interaction between the species: Growth dominates the competition when either population is small, so it is difficult to drive either species to extinction.

(b) a: per capita growth rate for trout
m: per capita growth rate for bass
b: intensity of competition to the trout
n: intensity of competition to the bass
k_1: environmental carrying capacity for the trout
k_2: environmental carrying capacity for the bass
$\dfrac{a}{b}$: growth versus competition or net growth of trout
$\dfrac{m}{n}$: relative survival of bass

(c) $\dfrac{dx}{dt} = 0$ when $x = 0$ or $y = \dfrac{a}{b} - \dfrac{a}{bk_1}x$,

$\dfrac{dy}{dt} = 0$ when $y = 0$ or $y = k_2 - \dfrac{k_2 n}{m}x$.

By picking $a/b > k_2$ and $m/n > k_1$, we ensure that an equilibrium point exists inside the first quadrant.

Practice Exercises, pp. 569–570

1. $y = -\ln\left(C - \frac{2}{5}(x-2)^{5/2} - \frac{4}{3}(x-2)^{3/2}\right)$

3. $\tan y = -x\sin x - \cos x + C$ **5.** $(y+1)e^{-y} = -\ln|x| + C$

7. $y = C\dfrac{x-1}{x}$ **9.** $y = \dfrac{x^2}{4}e^{x/2} + Ce^{x/2}$

11. $y = \dfrac{x^2 - 2x + C}{2x^2}$ **13.** $y = \dfrac{e^{-x} + C}{1 + e^x}$ **15.** $xy + y^3 = C$

17. $y = \dfrac{2x^3 + 3x^2 + 6}{6(x+1)^2}$ **19.** $y = \dfrac{1}{3}\left(1 - 4e^{-x^3}\right)$

21. $y = e^{-x}(3x^3 - 3x^2)$

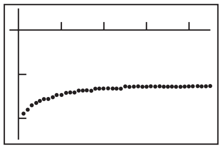
23.

x	y
0	0
0.1	0.1000
0.2	0.2095
0.3	0.3285
0.4	0.4568
0.5	0.5946
0.6	0.7418
0.7	0.8986
0.8	1.0649
0.9	1.2411
1.0	1.4273

x	y
1.1	1.6241
1.2	1.8319
1.3	2.0513
1.4	2.2832
1.5	2.5285
1.6	2.7884
1.7	3.0643
1.8	3.3579
1.9	3.6709
2.0	4.0057

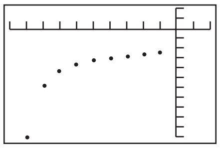
25. $y(3) \approx 0.9131$

27. (a)



[−0.2, 4.5] by [−2.5, 0.5]

(b) Note that we choose a small interval of x-values because the y-values decrease very rapidly and our calculator cannot handle the calculations for $x \le -1$. (This occurs because the analytic solution is $y = -2 + \ln(2 - e^{-x})$, which has an asymptote at $x = -\ln 2 \approx -0.69$. Obviously, the Euler approximations are misleading for $x \le -0.7$.)



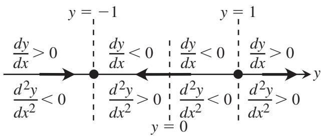
[−1, 0.2] by [−10, 2]

29. $y(\text{exact}) = \frac{1}{2}x^2 - \frac{3}{2}$; $y(2) \approx 0.4$; exact value is $\frac{1}{2}$.

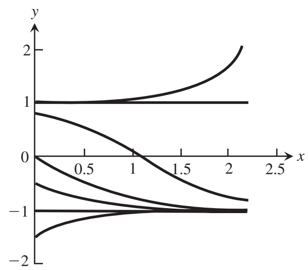
31. $y(\text{exact}) = -e^{(x^2-1)/2}$; $y(2) \approx -3.4192$; exact value is $-e^{3/2} \approx -4.4817$.

33. (a) $y = -1$ is stable and $y = 1$ is unstable.

(b) $\dfrac{d^2y}{dx^2} = 2y\dfrac{dy}{dx} = 2y(y^2 - 1)$



(c)



Additional and Advanced Exercises, pp. 570–571

1. (a) $y = c + (y_0 - c)e^{-k(A/V)t}$
(b) Steady-state solution: $y_\infty = c$
5. $x^2(x^2 + 2y^2) = C$
7. $\ln|x| + e^{-y/x} = C$
9. $\ln|x| - \ln|\sec(y/x - 1) + \tan(y/x - 1)| = C$

Chapter 10

Section 10.1, pp. 581–584

1. $a_1 = 0, a_2 = -1/4, a_3 = -2/9, a_4 = -3/16$
3. $a_1 = 1, a_2 = -1/3, a_3 = 1/5, a_4 = -1/7$
5. $a_1 = 1/2, a_2 = 1/2, a_3 = 1/2, a_4 = 1/2$
7. $1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \frac{63}{32}, \frac{127}{64}, \frac{255}{128}, \frac{511}{256}, \frac{1023}{512}$
9. $2, 1, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, -\frac{1}{64}, \frac{1}{128}, \frac{1}{256}$
11. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55
13. $a_n = (-1)^{n+1}, n \geq 1$
15. $a_n = (-1)^{n+1}(n)^2, n \geq 1$ **17.** $a_n = \frac{2^{n-1}}{3(n+2)}, n \geq 1$
19. $a_n = n^2 - 1, n \geq 1$ **21.** $a_n = 4n - 3, n \geq 1$
23. $a_n = \frac{3n+2}{n!}, n \geq 1$ **25.** $a_n = \frac{1 + (-1)^{n+1}}{2}, n \geq 1$
27. Converges, 2 **29.** Converges, -1 **31.** Converges, -5
33. Diverges **35.** Diverges **37.** Converges, $1/2$
39. Converges, 0 **41.** Converges, $\sqrt{2}$ **43.** Converges, 1
45. Converges, 0 **47.** Converges, 0 **49.** Converges, 0
51. Converges, 1 **53.** Converges, e^7 **55.** Converges, 1
57. Converges, 1 **59.** Diverges **61.** Converges, 4
63. Converges, 0 **65.** Diverges **67.** Converges, e^{-1}
69. Converges, $e^{2/3}$ **71.** Converges, x $(x > 0)$
73. Converges, 0 **75.** Converges, 1 **77.** Converges, $1/2$
79. Converges, 1 **81.** Converges, $\pi/2$ **83.** Converges, 0
85. Converges, 0 **87.** Converges, $1/2$ **89.** Converges, 0
91. 8 **93.** 4 **95.** 5 **97.** $1 + \sqrt{2}$ **99.** $x_n = 2^{n-2}$
101. (a) $f(x) = x^2 - 2$, $1.414213562 \approx \sqrt{2}$
(b) $f(x) = \tan(x) - 1$, $0.7853981635 \approx \pi/4$
(c) $f(x) = e^x$, diverges
103. (b) 1 **111.** Nondecreasing, bounded
113. Not nondecreasing, bounded
115. Converges, nondecreasing sequence theorem
117. Converges, nondecreasing sequence theorem
119. Diverges, definition of divergence **121.** Converges
123. Converges **135. (b)** $\sqrt{3}$

Section 10.2, pp. 591–593

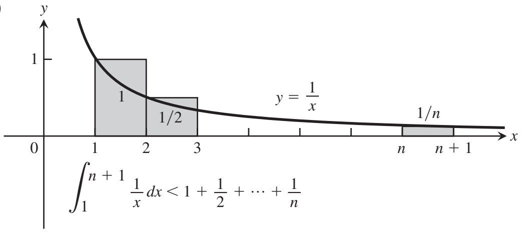
1. $s_n = \frac{2(1 - (1/3)^n)}{1 - (1/3)}, 3$ **3.** $s_n = \frac{1 - (-1/2)^n}{1 - (-1/2)}, 2/3$
5. $s_n = \frac{1}{2} - \frac{1}{n+2}, \frac{1}{2}$ **7.** $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \cdots, \frac{4}{5}$
9. $-\frac{3}{4} + \frac{9}{16} + \frac{57}{64} + \frac{249}{256} + \cdots$, diverges.
11. $(5 + 1) + \left(\frac{5}{2} + \frac{1}{3}\right) + \left(\frac{5}{4} + \frac{1}{9}\right) + \left(\frac{5}{8} + \frac{1}{27}\right) + \cdots, \frac{23}{2}$
13. $(1 + 1) + \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{4} + \frac{1}{25}\right) + \left(\frac{1}{8} - \frac{1}{125}\right) + \cdots, \frac{17}{6}$
15. Converges, $5/3$ **17.** Converges, $1/7$ **19.** $23/99$
21. $7/9$ **23.** $1/15$ **25.** $41333/33300$ **27.** Diverges
29. Inconclusive **31.** Diverges **33.** Diverges
35. $s_n = 1 - \frac{1}{n+1}$; converges, 1 **37.** $s_n = \ln\sqrt{n+1}$; diverges
39. $s_n = \frac{\pi}{3} - \cos^{-1}\left(\frac{1}{n+2}\right)$; converges, $-\frac{\pi}{6}$
41. 1 **43.** 5 **45.** 1 **47.** $-\frac{1}{\ln 2}$ **49.** Converges, $2 + \sqrt{2}$
51. Converges, 1 **53.** Diverges **55.** Converges, $\frac{e^2}{e^2 - 1}$
57. Converges, $2/9$ **59.** Converges, $3/2$ **61.** Diverges
63. Converges, 4 **65.** Diverges **67.** Converges, $\frac{\pi}{\pi - e}$
69. $a = 1, r = -x$; converges to $1/(1 + x)$ for $|x| < 1$
71. $a = 3, r = (x - 1)/2$; converges to $6/(3 - x)$ for x in $(-1, 3)$
73. $|x| < \frac{1}{2}, \frac{1}{1 - 2x}$ **75.** $-2 < x < 0, \frac{1}{2 + x}$
77. $x \neq (2k + 1)\frac{\pi}{2}$, k an integer; $\frac{1}{1 - \sin x}$
79. (a) $\sum_{n=-2}^{\infty} \frac{1}{(n+4)(n+5)}$ **(b)** $\sum_{n=0}^{\infty} \frac{1}{(n+2)(n+3)}$
(c) $\sum_{n=5}^{\infty} \frac{1}{(n-3)(n-2)}$
89. (a) $r = 3/5$ **(b)** $r = -3/10$ **91.** $|r| < 1, \frac{1 + 2r}{1 - r^2}$
93. (a) 16.84 mg, 17.79 mg **(b)** 17.84 mg
95. (a) $0, \frac{1}{27}, \frac{2}{27}, \frac{1}{9}, \frac{2}{9}, \frac{7}{27}, \frac{8}{27}, \frac{1}{3}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, 1$
(b) $\sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{2}{3}\right)^{n-1} = 1$

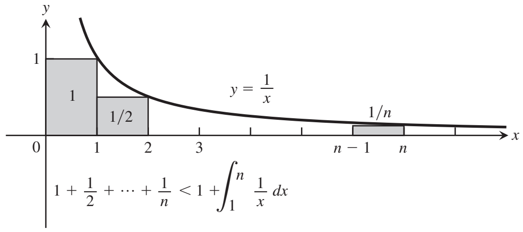
Section 10.3, pp. 598–599

1. Converges **3.** Converges **5.** Converges **7.** Diverges
9. Converges **11.** Converges; geometric series, $r = \frac{1}{10} < 1$
13. Diverges; $\lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$
15. Diverges; p-series, $p < 1$
17. Converges; geometric series, $r = \frac{1}{8} < 1$
19. Diverges; Integral Test
21. Converges; geometric series, $r = 2/3 < 1$
23. Diverges; Integral Test
25. Diverges; $\lim_{n\to\infty} \frac{2^n}{n+1} \neq 0$
27. Diverges; $\lim_{n\to\infty}\left(\sqrt{n}/\ln n\right) \neq 0$
29. Diverges; geometric series, $r = \frac{1}{\ln 2} > 1$
31. Converges; Integral Test **33.** Diverges; nth-Term Test
35. Converges; Integral Test **37.** Converges; Integral Test

39. Converges; Integral Test **41.** $a = 1$

43. (a)





(b) ≈ 41.55

45. True **47. (b)** $n \geq 251{,}415$

49. $s_8 = \sum_{n=1}^{8} \frac{1}{n^3} \approx 1.195$ **51.** 10^{60}

59. (a) $1.20166 \leq S \leq 1.20253$ **(b)** $S \approx 1.2021$, error < 0.0005

61. $\left(\frac{\pi^2}{6} - 1\right) \approx 0.64493$

Section 10.4, pp. 603–604

1. Converges; compare with $\sum(1/n^2)$

3. Diverges; compare with $\sum\left(1/\sqrt{n}\right)$

5. Converges; compare with $\sum\left(1/n^{3/2}\right)$

7. Converges; compare with $\sum\sqrt{\frac{n+4n}{n^4+0}} = \sqrt{5}\,\sum\frac{1}{n^{3/2}}$

9. Converges **11.** Diverges; limit comparison with $\sum(1/n)$

13. Diverges; limit comparison with $\sum\left(1/\sqrt{n}\right)$

15. Diverges **17.** Diverges; limit comparison with $\sum\left(1/\sqrt{n}\right)$

19. Converges; compare with $\sum(1/2^n)$

21. Diverges; nth-Term Test

23. Converges; compare with $\sum(1/n^2)$

25. Converges; $\left(\frac{n}{3n+1}\right)^n < \left(\frac{n}{3n}\right)^n = \left(\frac{1}{3}\right)^n$

27. Diverges; direct comparison with $\sum(1/n)$

29. Diverges; limit comparison with $\sum(1/n)$

31. Diverges; limit comparison with $\sum(1/n)$

33. Converges; compare with $\sum\left(1/n^{3/2}\right)$

35. Converges; $\frac{1}{n2^n} \leq \frac{1}{2^n}$

37. Converges; $\frac{1}{3^{n-1}+1} < \frac{1}{3^{n-1}}$

39. Converges; comparison with $\sum\left(1/5n^2\right)$

41. Diverges; comparison with $\sum(1/n)$

43. Converges; comparison with $\sum\frac{1}{n(n-1)}$ or limit comparison with $\sum(1/n^2)$

45. Diverges; limit comparison with $\sum(1/n)$

47. Converges; $\frac{\tan^{-1} n}{n^{1.1}} < \frac{\pi/2}{n^{1.1}}$

49. Converges; compare with $\sum(1/n^2)$

51. Diverges; limit comparison with $\sum(1/n)$

53. Converges; limit comparison with $\sum(1/n^2)$

65. Converges **67.** Converges **69.** Converges

Section 10.5, pp. 609–610

1. Converges **3.** Diverges **5.** Converges **7.** Converges

9. Converges **11.** Diverges **13.** Converges **15.** Converges

17. Converges; Ratio Test **19.** Diverges; Ratio Test

21. Converges; Ratio Test

23. Converges; compare with $\sum(3/(1.25)^n)$

25. Diverges; $\lim_{n\to\infty}\left(1 - \frac{3}{n}\right)^n = e^{-3} \neq 0$

27. Converges; compare with $\sum(1/n^2)$

29. Diverges; compare with $\sum(1/(2n))$

31. Diverges; $a_n \not\to 0$ **33.** Converges; Ratio Test

35. Converges; Ratio Test **37.** Converges; Ratio Test

39. Converges; Root Test **41.** Converges; compare with $\sum(1/n^2)$

43. Converges; Ratio Test **45.** Converges; Ratio Test

47. Diverges; Ratio Test **49.** Converges; Ratio Test

51. Converges; Ratio Test **53.** Diverges; $a_n = \left(\frac{1}{3}\right)^{(1/n!)} \to 1$

55. Converges; Ratio Test **57.** Diverges; Root Test

59. Converges; Root Test **61.** Converges; Ratio Test **65.** Yes

Section 10.6, pp. 615–616

1. Converges by Alternating Series Test

3. Converges; Alternating Series Test

5. Converges; Alternating Series Test

7. Diverges; $a_n \not\to 0$

9. Diverges; $a_n \not\to 0$

11. Converges; Alternating Series Test

13. Converges by Alternating Series Test

15. Converges absolutely. Series of absolute values is a convergent geometric series.

17. Converges conditionally; $1/\sqrt{n} \to 0$ but $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ diverges.

19. Converges absolutely; compare with $\sum_{n=1}^{\infty}(1/n^2)$.

21. Converges conditionally; $1/(n+3) \to 0$ but $\sum_{n=1}^{\infty}\frac{1}{n+3}$ diverges (compare with $\sum_{n=1}^{\infty}(1/n)$).

23. Diverges; $\frac{3+n}{5+n} \to 1$

25. Converges conditionally; $\left(\frac{1}{n^2} + \frac{1}{n}\right) \to 0$ but $(1+n)/n^2 > 1/n$

27. Converges absolutely; Ratio Test

29. Converges absolutely by Integral Test

31. Diverges; $a_n \not\to 0$ **33.** Converges absolutely by Ratio Test

35. Converges absolutely, since $\left|\frac{\cos n\pi}{n\sqrt{n}}\right| = \left|\frac{(-1)^{n+1}}{n^{3/2}}\right| = \frac{1}{n^{3/2}}$ (convergent p-series)

37. Converges absolutely by Root Test **39.** Diverges; $a_n \to \infty$

41. Converges conditionally; $\sqrt{n+1} - \sqrt{n} = 1/(\sqrt{n} + \sqrt{n+1}) \to 0$, but series of absolute values diverges (compare with $\sum(1/\sqrt{n})$).

43. Diverges, $a_n \to 1/2 \neq 0$

45. Converges absolutely; $\operatorname{sech} n = \frac{2}{e^n + e^{-n}} = \frac{2e^n}{e^{2n}+1} < \frac{2e^n}{e^{2n}} = \frac{2}{e^n}$, a term from a convergent geometric series.

47. Converges conditionally; $\sum(-1)^{n+1}\frac{1}{2(n+1)}$ converges by Alternating Series Test; $\sum\frac{1}{2(n+1)}$ diverges by limit comparison with $\sum(1/n)$.

49. $|\text{Error}| < 0.2$ **51.** $|\text{Error}| < 2 \times 10^{-11}$

53. $n \geq 31$ **55.** $n \geq 4$ **57.** 0.54030

59. **(a)** $a_n \geq a_{n+1}$ **(b)** $-1/2$

Section 10.7, pp. 624–626

1. **(a)** $1, -1 < x < 1$ **(b)** $-1 < x < 1$ **(c)** none

3. **(a)** $1/4, -1/2 < x < 0$ **(b)** $-1/2 < x < 0$ **(c)** none

5. **(a)** $10, -8 < x < 12$ **(b)** $-8 < x < 12$ **(c)** none

7. **(a)** $1, -1 < x < 1$ **(b)** $-1 < x < 1$ **(c)** none

9. **(a)** $3, -3 \leq x \leq 3$ **(b)** $-3 \leq x \leq 3$ **(c)** none

11. **(a)** ∞, for all x **(b)** for all x **(c)** none

13. **(a)** $1/2, -1/2 < x < 1/2$ **(b)** $-1/2 < x < 1/2$ **(c)** none

15. **(a)** $1, -1 \leq x < 1$ **(b)** $-1 < x < 1$ **(c)** $x = -1$

17. **(a)** $5, -8 < x < 2$ **(b)** $-8 < x < 2$ **(c)** none

19. **(a)** $3, -3 < x < 3$ **(b)** $-3 < x < 3$ **(c)** none

21. **(a)** $1, -2 < x < 0$ **(b)** $-2 < x < 0$ **(c)** none

23. **(a)** $1, -1 < x < 1$ **(b)** $-1 < x < 1$ **(c)** none

25. **(a)** $0, x = 0$ **(b)** $x = 0$ **(c)** none

27. **(a)** $2, -4 < x \leq 0$ **(b)** $-4 < x < 0$ **(c)** $x = 0$

29. **(a)** $1, -1 \leq x \leq 1$ **(b)** $-1 \leq x \leq 1$ **(c)** none

31. **(a)** $1/4, 1 \leq x \leq 3/2$ **(b)** $1 \leq x \leq 3/2$ **(c)** none

33. **(a)** ∞, for all x **(b)** for all x **(c)** none

35. **(a)** $1, -1 \leq x < 1$ **(b)** $-1 < x < 1$ **(c)** -1

37. 3 **39.** 8 **41.** $-1/3 < x < 1/3, 1/(1-3x)$

43. $-1 < x < 3, 4/(3 + 2x - x^2)$

45. $0 < x < 16, 2/(4 - \sqrt{x})$

47. $-\sqrt{2} < x < \sqrt{2}, 3/(2 - x^2)$

49. $\frac{2}{x} = \sum_{n=0}^{\infty} 2(-1)^n(x-1)^n, 0 < x < 2$

51. $\sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n(x-5)^n, 2 < x < 8$

53. $1 < x < 5, 2/(x-1), \sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^n n(x-3)^{n-1}$,
$1 < x < 5, -2/(x-1)^2$

55. **(a)** $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$; converges for all x

(b) Same answer as part (c)

(c) $2x - \frac{2^3x^3}{3!} + \frac{2^5x^5}{5!} - \frac{2^7x^7}{7!} + \frac{2^9x^9}{9!} - \frac{2^{11}x^{11}}{11!} + \cdots$

57. **(a)** $\frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \frac{17x^8}{2520} + \frac{31x^{10}}{14175}, -\frac{\pi}{2} < x < \frac{\pi}{2}$

(b) $1 + x^2 + \frac{2x^4}{3} + \frac{17x^6}{45} + \frac{62x^8}{315} + \cdots, -\frac{\pi}{2} < x < \frac{\pi}{2}$

Section 10.8, pp. 630–631

1. $P_0(x) = 1, P_1(x) = 1 + 2x, P_2(x) = 1 + 2x + 2x^2$,
$P_3(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3$

3. $P_0(x) = 0, P_1(x) = x - 1, P_2(x) = (x-1) - \frac{1}{2}(x-1)^2$,
$P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

5. $P_0(x) = \frac{1}{2}, P_1(x) = \frac{1}{2} - \frac{1}{4}(x-2)$,
$P_2(x) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2$,
$P_3(x) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$

7. $P_0(x) = \frac{\sqrt{2}}{2}, P_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$,
$P_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$,
$P_3(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3$

9. $P_0(x) = 2, P_1(x) = 2 + \frac{1}{4}(x-4)$,
$P_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$,
$P_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$

11. $\sum_{n=0}^{\infty}\frac{(-x)^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$

13. $\sum_{n=0}^{\infty}(-1)^n x^n = 1 - x + x^2 - x^3 + \cdots$

15. $\sum_{n=0}^{\infty}\frac{(-1)^n 3^{2n+1}x^{2n+1}}{(2n+1)!}$ **17.** $7\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$ **19.** $\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$

21. $x^4 - 2x^3 - 5x + 4$

23. $8 + 10(x-2) + 6(x-2)^2 + (x-2)^3$

25. $21 - 36(x+2) + 25(x+2)^2 - 8(x+2)^3 + (x+2)^4$

27. $\sum_{n=0}^{\infty}(-1)^n(n+1)(x-1)^n$ **29.** $\sum_{n=0}^{\infty}\frac{e^2}{n!}(x-2)^n$

31. $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{2^{2n}}{(2n)!}\left(x - \frac{\pi}{4}\right)^{2n}$

33. $-1 - 2x - \frac{5}{2}x^2 - \cdots, -1 < x < 1$

35. $x^2 - \frac{1}{2}x^3 + \frac{1}{6}x^4 + \cdots, -1 < x < 1$

41. $L(x) = 0, Q(x) = -x^2/2$ **43.** $L(x) = 1, Q(x) = 1 + x^2/2$

45. $L(x) = x, Q(x) = x$

Section 10.9, pp. 637–638

1. $\sum_{n=0}^{\infty}\frac{(-5x)^n}{n!} = 1 - 5x + \frac{5^2x^2}{2!} - \frac{5^3x^3}{3!} + \cdots$

3. $\sum_{n=0}^{\infty}\frac{5(-1)^n(-x)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty}\frac{5(-1)^{n+1}x^{2n+1}}{(2n+1)!}$
$= -5x + \frac{5x^3}{3!} - \frac{5x^5}{5!} + \frac{5x^7}{7!} + \cdots$

5. $\sum_{n=0}^{\infty} \frac{(-1)^n(5x^2)^{2n}}{(2n)!} = 1 - \frac{25x^4}{2!} + \frac{625x^8}{4!} - \cdots$

7. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n} = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots$

9. $\sum_{n=0}^{\infty} (-1)^n \left(\frac{3}{4}\right)^n x^{3n} = 1 - \frac{3}{4}x^3 + \frac{3^2}{4^2}x^6 - \frac{3^3}{4^3}x^9 + \cdots$

11. $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!} = x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \cdots$

13. $\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$

15. $x - \frac{\pi^2 x^3}{2!} + \frac{\pi^4 x^5}{4!} - \frac{\pi^6 x^7}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n} x^{2n+1}}{(2n)!}$

17. $1 + \sum_{n=1}^{\infty} \frac{(-1)^n (2x)^{2n}}{2 \cdot (2n)!} =$
$1 - \frac{(2x)^2}{2 \cdot 2!} + \frac{(2x)^4}{2 \cdot 4!} - \frac{(2x)^6}{2 \cdot 6!} + \frac{(2x)^8}{2 \cdot 8!} - \cdots$

19. $x^2 \sum_{n=0}^{\infty} (2x)^n = x^2 + 2x^3 + 4x^4 + \cdots$

21. $\sum_{n=1}^{\infty} nx^{n-1} = 1 + 2x + 3x^2 + 4x^3 + \cdots$

23. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{4n-1}}{2n-1} = x^3 - \frac{x^7}{3} + \frac{x^{11}}{5} - \frac{x^{15}}{7} + \cdots$

25. $\sum_{n=0}^{\infty} \left(\frac{1}{n!} + (-1)^n\right) x^n = 2 + \frac{3}{2}x^2 - \frac{5}{6}x^3 + \frac{25}{24}x^4 - \cdots$

27. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n+1}}{3n} = \frac{x^3}{3} - \frac{x^5}{6} + \frac{x^7}{9} - \cdots$

29. $x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} + \cdots$

31. $x^2 - \frac{2}{3}x^4 + \frac{23}{45}x^6 - \frac{44}{105}x^8 + \cdots$

33. $1 + x + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \cdots$

35. $|\text{Error}| \le \frac{1}{10^4 \cdot 4!} < 4.2 \times 10^{-6}$

37. $|x| < (0.06)^{1/5} < 0.56968$

39. $|\text{Error}| < (10^{-3})^3/6 < 1.67 \times 10^{-10}, \quad -10^{-3} < x < 0$

41. $|\text{Error}| < (3^{0.1})(0.1)^3/6 < 1.87 \times 10^{-4}$

49. **(a)** $Q(x) = 1 + kx + \frac{k(k-1)}{2}x^2$ **(b)** $0 \le x < 100^{-1/3}$

Section 10.10, pp. 645–647

1. $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$ **3.** $1 + 3x + 6x^2 + 10x^3$

5. $1 - x + \frac{3x^2}{4} - \frac{x^3}{2}$ **7.** $1 - \frac{x^3}{2} + \frac{3x^6}{8} - \frac{5x^9}{16}$

9. $1 + \frac{1}{2x} - \frac{1}{8x^2} + \frac{1}{16x^3}$

11. $(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$

13. $(1 - 2x)^3 = 1 - 6x + 12x^2 - 8x^3$

15. 0.0713362 **17.** 0.4969536 **19.** 0.0999445 **21.** 0.10000

23. $\frac{1}{13 \cdot 6!} \approx 0.00011$ **25.** $\frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!}$

27. **(a)** $\frac{x^2}{2} - \frac{x^4}{12}$

(b) $\frac{x^2}{2} - \frac{x^4}{3 \cdot 4} + \frac{x^6}{5 \cdot 6} - \frac{x^8}{7 \cdot 8} + \cdots + (-1)^{15} \frac{x^{32}}{31 \cdot 32}$

29. $1/2$ **31.** $-1/24$ **33.** $1/3$ **35.** -1 **37.** 2

39. $3/2$ **41.** e **43.** $\cos\frac{3}{4}$ **45.** $\frac{\sqrt{3}}{2}$ **47.** $\frac{x^3}{1-x}$

49. $\frac{x^3}{1+x^2}$ **51.** $\frac{-1}{(1+x)^2}$ **55.** 500 terms **57.** 4 terms

59. **(a)** $x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112}$, radius of convergence $= 1$

(b) $\frac{\pi}{2} - x - \frac{x^3}{6} - \frac{3x^5}{40} - \frac{5x^7}{112}$

61. $1 - 2x + 3x^2 - 4x^3 + \cdots$

67. **(a)** -1 **(b)** $(1/\sqrt{2})(1 + i)$ **(c)** $-i$

71. $x + x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5 + \cdots$, for all x

Practice Exercises, pp. 648–649

1. Converges to 1 **3.** Converges to -1 **5.** Diverges
7. Converges to 0 **9.** Converges to 1 **11.** Converges to e^{-5}
13. Converges to 3 **15.** Converges to ln 2 **17.** Diverges
19. $1/6$ **21.** $3/2$ **23.** $e/(e-1)$ **25.** Diverges
27. Converges conditionally **29.** Converges conditionally
31. Converges absolutely **33.** Converges absolutely
35. Converges absolutely **37.** Converges absolutely
39. Converges absolutely
41. **(a)** $3, -7 \le x < -1$ **(b)** $-7 < x < -1$ **(c)** $x = -7$
43. **(a)** $1/3, 0 \le x \le 2/3$ **(b)** $0 \le x \le 2/3$ **(c)** None
45. **(a)** ∞, for all x **(b)** For all x **(c)** None
47. **(a)** $\sqrt{3}, -\sqrt{3} < x < \sqrt{3}$ **(b)** $-\sqrt{3} < x < \sqrt{3}$ **(c)** None
49. **(a)** $e, -e < x < e$ **(b)** $-e < x < e$ **(c)** Empty set

51. $\frac{1}{1+x}, \frac{1}{4}, \frac{4}{5}$ **53.** $\sin x, \pi, 0$ **55.** e^x, ln 2, 2 **57.** $\sum_{n=0}^{\infty} 2^n x^n$

59. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1} x^{2n+1}}{(2n+1)!}$ **61.** $\sum_{n=0}^{\infty} \frac{(-1)^n x^{10n/3}}{(2n)!}$ **63.** $\sum_{n=0}^{\infty} \frac{((\pi x)/2)^n}{n!}$

65. $2 - \frac{(x+1)}{2 \cdot 1!} + \frac{3(x+1)^2}{2^3 \cdot 2!} + \frac{9(x+1)^3}{2^5 \cdot 3!} + \cdots$

67. $\frac{1}{4} - \frac{1}{4^2}(x-3) + \frac{1}{4^3}(x-3)^2 - \frac{1}{4^4}(x-3)^3$

69. 0.4849171431 **71.** 0.4872223583 **73.** $7/2$ **75.** $1/12$

77. -2 **79.** $r = -3, s = 9/2$ **81.** $2/3$

83. $\ln\left(\frac{n+1}{2n}\right)$; the series converges to $\ln\left(\frac{1}{2}\right)$.

85. **(a)** ∞ **(b)** $a = 1, b = 0$ **87.** It converges.

Additional and Advanced Exercises, pp. 650–652

1. Converges; Comparison Test **3.** Diverges; nth-Term Test
5. Converges; Comparison Test **7.** Diverges; nth-Term Test

9. With $a = \pi/3$, $\cos x = \frac{1}{2} - \frac{\sqrt{3}}{2}(x - \pi/3) - \frac{1}{4}(x - \pi/3)^2 + \frac{\sqrt{3}}{12}(x - \pi/3)^3 + \cdots$

11. With $a = 0$, $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

13. With $a = 22\pi$, $\cos x = 1 - \frac{1}{2}(x - 22\pi)^2 + \frac{1}{4!}(x - 22\pi)^4 - \frac{1}{6!}(x - 22\pi)^6 + \cdots$

15. Converges, limit $= b$ **17.** $\pi/2$ **21.** $b = \pm\frac{1}{5}$

23. $a = 2, L = -7/6$ **27. (b)** Yes

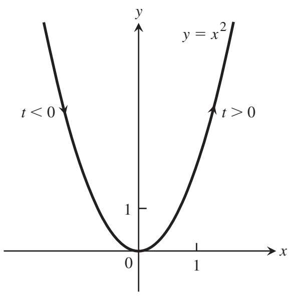
31. (a) $\sum_{n=1}^{\infty} nx^{n-1}$ **(b)** 6 **(c)** $\frac{1}{q}$

33. (a) $R_n = C_0 e^{-kt_0}(1 - e^{-nkt_0})/(1 - e^{-kt_0})$,
$R = C_0(e^{-kt_0})/(1 - e^{-kt_0}) = C_0/(e^{kt_0} - 1)$
(b) $R_1 = 1/e \approx 0.368$,
$R_{10} = R(1 - e^{-10}) \approx R(0.9999546) \approx 0.58195$;
$R \approx 0.58198$; $0 < (R - R_{10})/R < 0.0001$
(c) 7

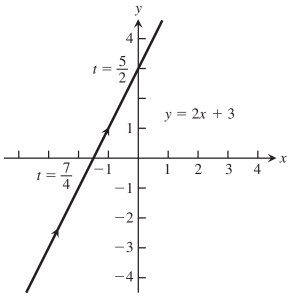
Chapter 11

Section 11.1, pp. 659–661

1.



3.



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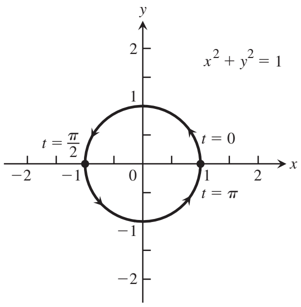

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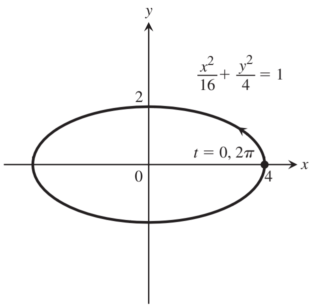

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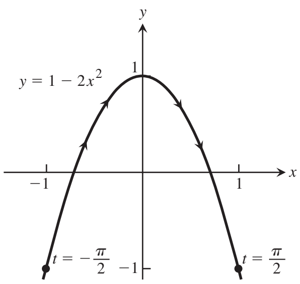

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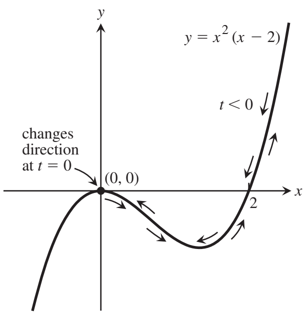

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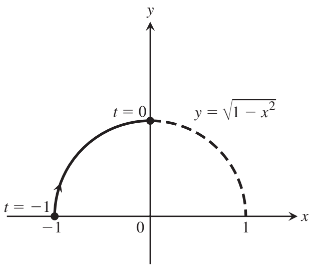

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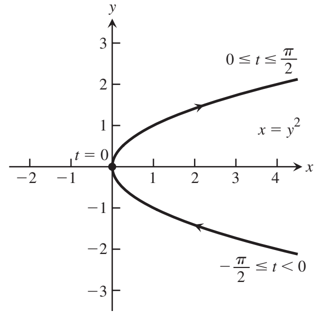

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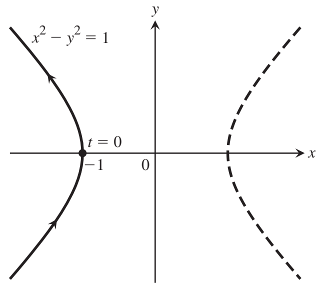


19. (a) $x = a\cos t, \quad y = -a\sin t, \quad 0 \le t \le 2\pi$
(b) $x = a\cos t, \quad y = a\sin t, \quad 0 \le t \le 2\pi$
(c) $x = a\cos t, \quad y = -a\sin t, \quad 0 \le t \le 4\pi$
(d) $x = a\cos t, \quad y = a\sin t, \quad 0 \le t \le 4\pi$

21. Possible answer: $x = -1 + 5t, \quad y = -3 + 4t, \quad 0 \le t \le 1$

23. Possible answer: $x = t^2 + 1, \quad y = t, \quad t \le 0$

25. Possible answer: $x = 2 - 3t, \quad y = 3 - 4t, \quad t \ge 0$

27. Possible answer: $x = 2\cos t, \quad y = 2\,|\sin t|, \quad 0 \le t \le 4\pi$

29. Possible answer: $x = \frac{-at}{\sqrt{1 + t^2}}, \quad y = \frac{a}{\sqrt{1 + t^2}}, -\infty < t < \infty$

31. Possible answer: $x = \frac{4}{1 + 2\tan\theta}, \quad y = \frac{4\tan\theta}{1 + 2\tan\theta}$,
$0 \le \theta < \pi/2$ and $x = 0, y = 2$ if $\theta = \pi/2$

33. Possible answer: $x = 2 - \cos t, \quad y = \sin t, \quad 0 \le t \le 2\pi$

35. $x = 2\cot t, \quad y = 2\sin^2 t, \quad 0 < t < \pi$

37. $x = a\sin^2 t\tan t, \quad y = a\sin^2 t, \quad 0 \le t < \pi/2$ **39.** (1, 1)

Section 11.2, pp. 669–671

1. $y = -x + 2\sqrt{2}, \quad \frac{d^2y}{dx^2} = -\sqrt{2}$

3. $y = -\frac{1}{2}x + 2\sqrt{2}, \quad \frac{d^2y}{dx^2} = -\frac{\sqrt{2}}{4}$

5. $y = x + \frac{1}{4}, \quad \frac{d^2y}{dx^2} = -2$ **7.** $y = 2x - \sqrt{3}, \quad \frac{d^2y}{dx^2} = -3\sqrt{3}$

9. $y = x - 4, \quad \frac{d^2y}{dx^2} = \frac{1}{2}$

11. $y = \sqrt{3}x - \frac{\pi\sqrt{3}}{3} + 2, \quad \frac{d^2y}{dx^2} = -4$

13. $y = 9x - 1, \quad \frac{d^2y}{dx^2} = 108$ **15.** $-\frac{3}{16}$ **17.** -6

19. 1 **21.** $3a^2\pi$ **23.** $|ab|\pi$ **25.** 4 **27.** 12

29. π^2 **31.** $8\pi^2$ **33.** $\frac{52\pi}{3}$ **35.** $3\pi\sqrt{5}$

37. $(\bar{x}, \bar{y}) = \left(\frac{12}{\pi} - \frac{24}{\pi^2}, \frac{24}{\pi^2} - 2\right)$

39. $(\bar{x}, \bar{y}) = \left(\frac{1}{3}, \pi - \frac{4}{3}\right)$ **41. (a)** π **(b)** π

43. (a) $x = 1, \quad y = 0, \quad \frac{dy}{dx} = \frac{1}{2}$ **(b)** $x = 0, \quad y = 3, \quad \frac{dy}{dx} = 0$

(c) $x = \frac{\sqrt{3} - 1}{2}, \quad y = \frac{3 - \sqrt{3}}{2}, \quad \frac{dy}{dx} = \frac{2\sqrt{3} - 1}{\sqrt{3} - 2}$

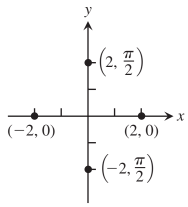
45. $\left(\frac{\sqrt{2}}{2}, 1\right)$, $y = 2x$ at $t = 0$, $y = -2x$ at $t = \pi$

47. (a) 8a **(b)** $\frac{64\pi}{3}$

Section 11.3, pp. 674–675

1. a, e; b, g; c, h; d, f

3.



(a) $\left(2, \frac{\pi}{2} + 2n\pi\right)$ and $\left(-2, \frac{\pi}{2} + (2n + 1)\pi\right)$, n an integer

(b) $(2, 2n\pi)$ and $(-2, (2n + 1)\pi)$, n an integer

(c) $\left(2, \frac{3\pi}{2} + 2n\pi\right)$ and $\left(-2, \frac{3\pi}{2} + (2n + 1)\pi\right)$, n an integer

(d) $(2, (2n + 1)\pi)$ and $(-2, 2n\pi)$, n an integer

5. (a) $(3, 0)$ **(b)** $(-3, 0)$ **(c)** $(-1, \sqrt{3})$ **(d)** $(1, \sqrt{3})$
(e) $(3, 0)$ **(f)** $(1, \sqrt{3})$ **(g)** $(-3, 0)$ **(h)** $(-1, \sqrt{3})$

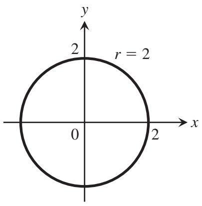
7. (a) $\left(\sqrt{2}, \frac{\pi}{4}\right)$ **(b)** $(3, \pi)$ **(c)** $\left(2, \frac{11\pi}{6}\right)$

(d) $\left(5, \pi - \tan^{-1}\frac{4}{3}\right)$

9. (a) $\left(-3\sqrt{2}, \frac{5\pi}{4}\right)$ **(b)** $(-1, 0)$ **(c)** $\left(-2, \frac{5\pi}{3}\right)$

(d) $\left(-5, \pi - \tan^{-1}\frac{3}{4}\right)$

11.



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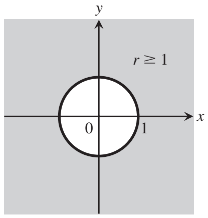

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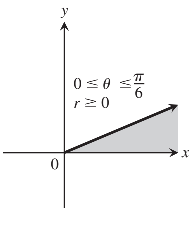

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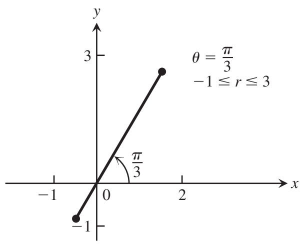

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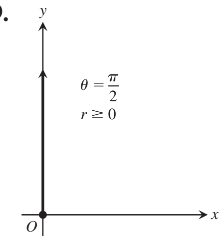

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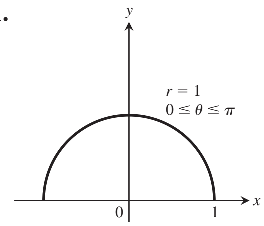

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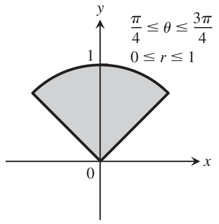

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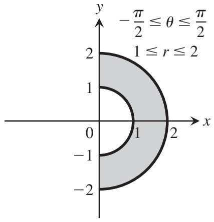
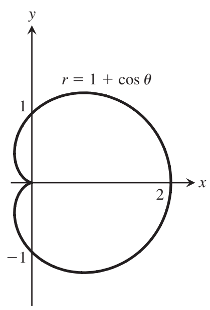


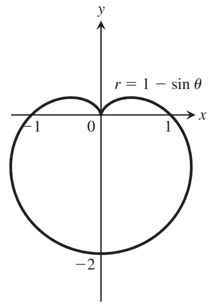
27. $x = 2$, vertical line through $(2, 0)$ **29.** $y = 0$, the x-axis
31. $y = 4$, horizontal line through $(0, 4)$
33. $x + y = 1$, line, $m = -1, b = 1$
35. $x^2 + y^2 = 1$, circle, $C(0, 0)$, radius 1
37. $y - 2x = 5$, line, $m = 2, b = 5$
39. $y^2 = x$, parabola, vertex $(0, 0)$, opens right
41. $y = e^x$, graph of natural exponential function
43. $x + y = \pm 1$, two straight lines of slope -1, y-intercepts $b = \pm 1$
45. $(x + 2)^2 + y^2 = 4$, circle, $C(-2, 0)$, radius 2
47. $x^2 + (y - 4)^2 = 16$, circle, $C(0, 4)$, radius 4
49. $(x - 1)^2 + (y - 1)^2 = 2$, circle, $C(1, 1)$, radius $\sqrt{2}$
51. $\sqrt{3}y + x = 4$ **53.** $r\cos\theta = 7$ **55.** $\theta = \pi/4$
57. $r = 2$ or $r = -2$ **59.** $4r^2\cos^2\theta + 9r^2\sin^2\theta = 36$
61. $r\sin^2\theta = 4\cos\theta$ **63.** $r = 4\sin\theta$
65. $r^2 = 6r\cos\theta - 2r\sin\theta - 6$
67. $(0, \theta)$, where θ is any angle

Section 11.4, pp. 678–679

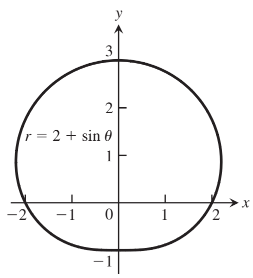
1. x-axis



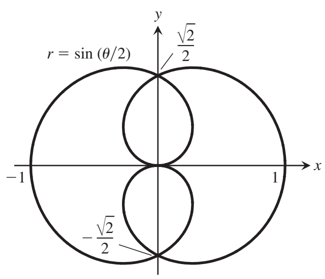
3. y-axis



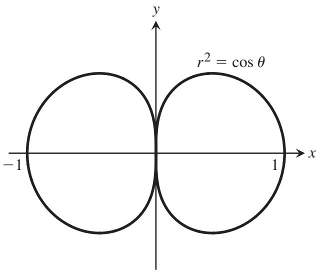
5. y-axis



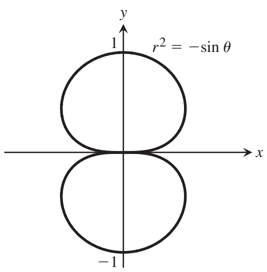
7. x-axis, y-axis, origin



9. x-axis, y-axis, origin

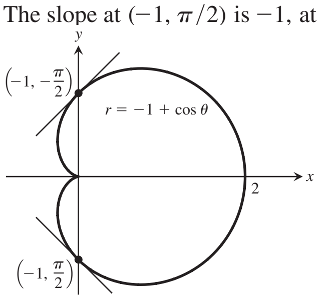


11. y-axis, x-axis, origin

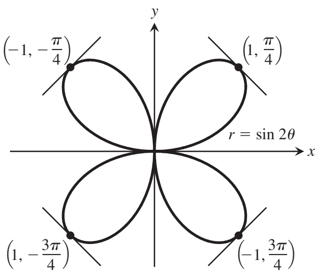


13. x-axis, y-axis, origin **15.** Origin

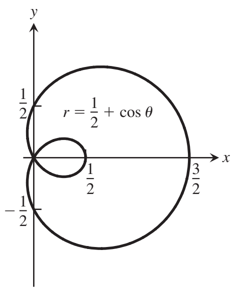
17. The slope at $(-1, \pi/2)$ is -1, at $(-1, -\pi/2)$ is 1.



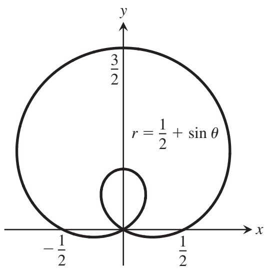
19. The slope at $(1, \pi/4)$ is -1, at $(-1, -\pi/4)$ is 1, at $(-1, 3\pi/4)$ is 1, at $(1, -3\pi/4)$ is -1.



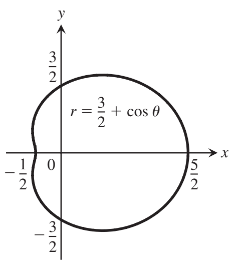
21. (a)



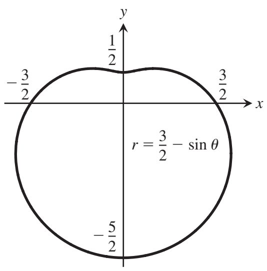
(b)



23. (a)



(b)



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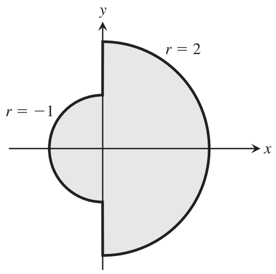
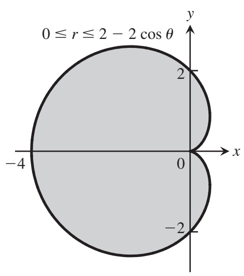


27.



29. Equation (a)

Section 11.5, pp. 682–683

1. $\frac{1}{6}\pi^3$ **3.** 18π **5.** $\frac{\pi}{8}$ **7.** 2 **9.** $\frac{\pi}{2} - 1$

11. $5\pi - 8$ **13.** $3\sqrt{3} - \pi$ **15.** $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$

17. $\frac{8\pi}{3} + \sqrt{3}$ **19. (a)** $\frac{3}{2} - \frac{\pi}{4}$ **21.** $19/3$ **23.** 8

25. $3\left(\sqrt{2} + \ln\left(1 + \sqrt{2}\right)\right)$ **27.** $\frac{\pi}{8} + \frac{3}{8}$

31. (a) a **(b)** a **(c)** $2a/\pi$

Section 11.6, pp. 689–692

1. $y^2 = 8x$, $F(2, 0)$, directrix: $x = -2$

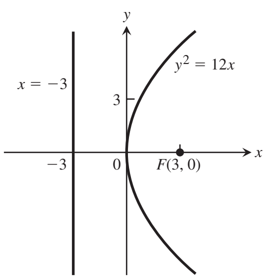
3. $x^2 = -6y$, $F(0, -3/2)$, directrix: $y = 3/2$

5. $\frac{x^2}{4} - \frac{y^2}{9} = 1$, $F\left(\pm\sqrt{13}, 0\right)$, $V(\pm 2, 0)$,

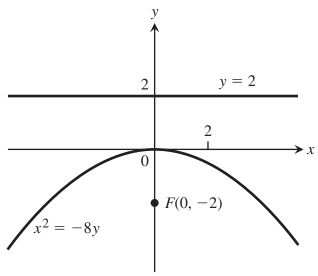
asymptotes: $y = \pm\frac{3}{2}x$

7. $\frac{x^2}{2} + y^2 = 1$, $F(\pm 1, 0)$, $V\left(\pm\sqrt{2}, 0\right)$

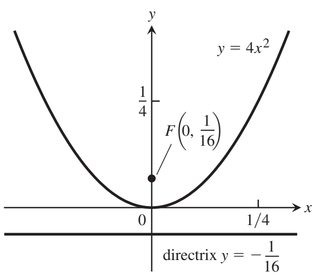
9.



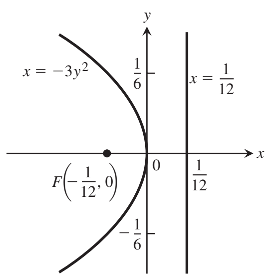
11.



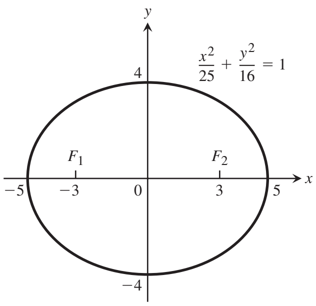
13.



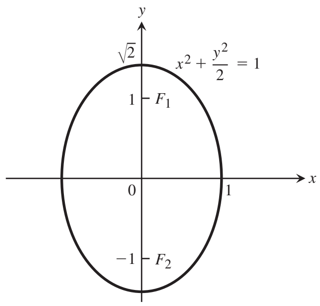
15.



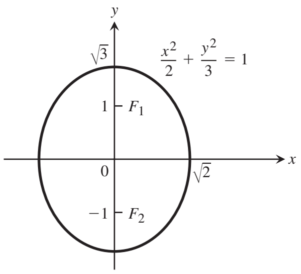
17.



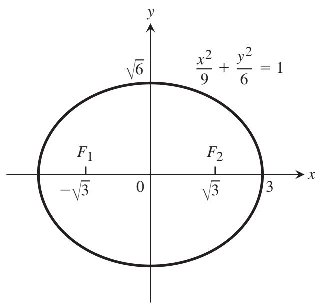
19.



21.

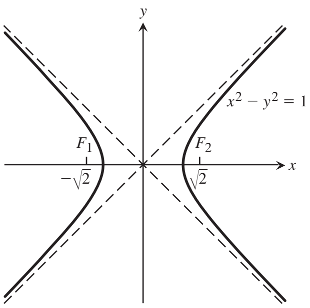


23.

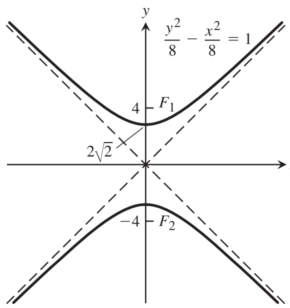


25. $\dfrac{x^2}{4}+\dfrac{y^2}{2}=1$

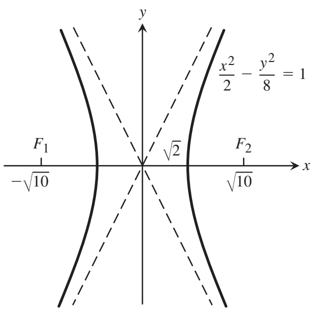
27. Asymptotes: $y=\pm x$



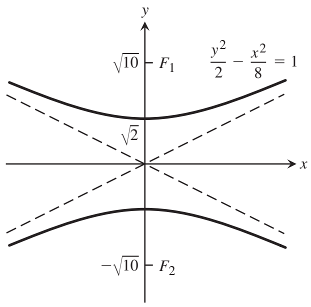
29. Asymptotes: $y=\pm x$



31. Asymptotes: $y=\pm 2x$



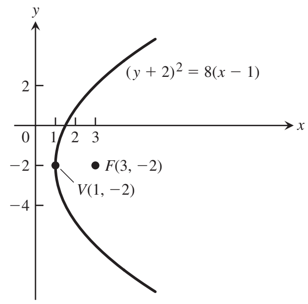
33. Asymptotes: $y=\pm x/2$



35. $y^2-x^2=1$ **37.** $\dfrac{x^2}{9}-\dfrac{y^2}{16}=1$

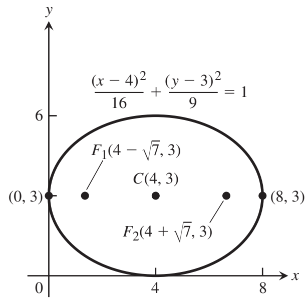
39. **(a)** Vertex: $(1,-2)$; focus: $(3,-2)$; directrix: $x=-1$

(b)



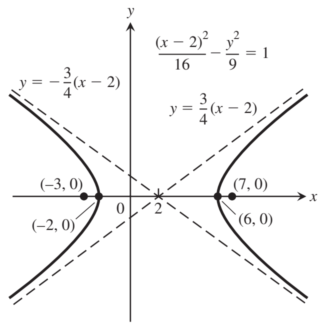
41. **(a)** Foci: $\left(4\pm\sqrt{7},3\right)$; vertices: $(8,3)$ and $(0,3)$; center: $(4,3)$

(b)



43. **(a)** Center: $(2,0)$; foci: $(7,0)$ and $(-3,0)$; vertices: $(6,0)$ and $(-2,0)$; asymptotes: $y=\pm\dfrac{3}{4}(x-2)$

(b)



45. $(y+3)^2=4(x+2)$, $V(-2,-3)$, $F(-1,-3)$, directrix: $x=-3$

47. $(x-1)^2=8(y+7)$, $V(1,-7)$, $F(1,-5)$, directrix: $y=-9$

49. $\dfrac{(x+2)^2}{6}+\dfrac{(y+1)^2}{9}=1$, $F\left(-2,\pm\sqrt{3}-1\right)$, $V(-2,\pm 3-1)$, $C(-2,-1)$

51. $\dfrac{(x-2)^2}{3}+\dfrac{(y-3)^2}{2}=1$, $F(3,3)$ and $F(1,3)$, $V\left(\pm\sqrt{3}+2,3\right)$, $C(2,3)$

53. $\dfrac{(x-2)^2}{4}-\dfrac{(y-2)^2}{5}=1$, $C(2,2)$, $F(5,2)$ and $F(-1,2)$, $V(4,2)$ and $V(0,2)$; asymptotes: $(y-2)=\pm\dfrac{\sqrt{5}}{2}(x-2)$

55. $(y+1)^2-(x+1)^2=1$, $C(-1,-1)$, $F\left(-1,\sqrt{2}-1\right)$ and $F\left(-1,-\sqrt{2}-1\right)$, $V(-1,0)$ and $V(-1,-2)$; asymptotes $(y+1)=\pm(x+1)$

57. $C(-2,0)$, $a=4$ **59.** $V(-1,1)$, $F(-1,0)$

61. Ellipse: $\dfrac{(x+2)^2}{5}+y^2=1$, $C(-2,0)$, $F(0,0)$ and $F(-4,0)$, $V\left(\sqrt{5}-2,0\right)$ and $V\left(-\sqrt{5}-2,0\right)$

63. Ellipse: $\dfrac{(x-1)^2}{2}+(y-1)^2=1$, $C(1,1)$, $F(2,1)$ and $F(0,1)$, $V\left(\sqrt{2}+1,1\right)$ and $V\left(-\sqrt{2}+1,1\right)$

65. Hyperbola: $(x-1)^2-(y-2)^2=1$, $C(1,2)$, $F\left(1+\sqrt{2},2\right)$ and $F\left(1-\sqrt{2},2\right)$, $V(2,2)$ and $V(0,2)$; asymptotes: $(y-2)=\pm(x-1)$

67. Hyperbola: $\dfrac{(y-3)^2}{6}-\dfrac{x^2}{3}=1$, $C(0,3)$, $F(0,6)$ and $F(0,0)$, $V\left(0,\sqrt{6}+3\right)$ and $V\left(0,-\sqrt{6}+3\right)$; asymptotes: $y=\sqrt{2}x+3$ or $y=-\sqrt{2}x+3$

69. **(b)** 1 : 1 **73.** Length $=2\sqrt{2}$, width $=\sqrt{2}$, area $=4$

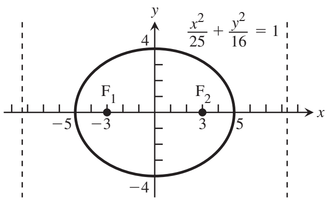
75. 24π

77. $x=0, y=0$: $y=-2x$; $x=0, y=2$: $y=2x+2$; $x=4, y=0$: $y=2x-8$

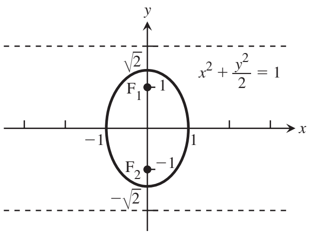
79. $\bar{x}=0$, $\bar{y}=\dfrac{16}{3\pi}$

Section 11.7, pp. 697–698

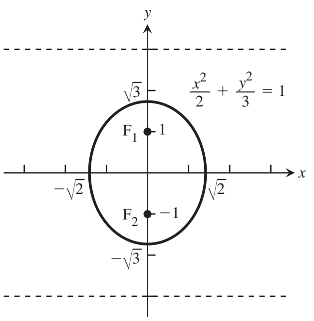
1. $e=\dfrac{3}{5}$, $F(\pm 3,0)$; directrices are $x=\pm\dfrac{25}{3}$.



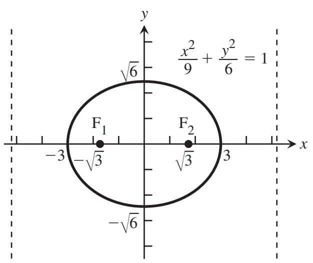
3. $e=\dfrac{1}{\sqrt{2}}$; $F(0,\pm 1)$; directrices are $y=\pm 2$.



5. $e=\dfrac{1}{\sqrt{3}}$; $F(0,\pm 1)$; directrices are $y=\pm 3$.



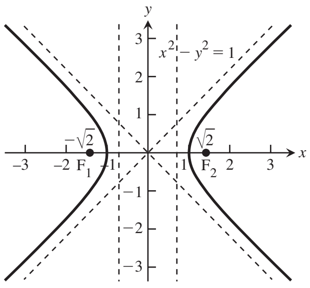
7. $e=\dfrac{\sqrt{3}}{3}$; $F\left(\pm\sqrt{3},0\right)$; directrices are $x=\pm 3\sqrt{3}$.



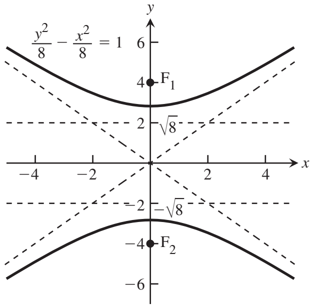
9. $\dfrac{x^2}{27}+\dfrac{y^2}{36}=1$ **11.** $\dfrac{x^2}{4851}+\dfrac{y^2}{4900}=1$

13. $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$ **15.** $\dfrac{x^2}{64}+\dfrac{y^2}{48}=1$

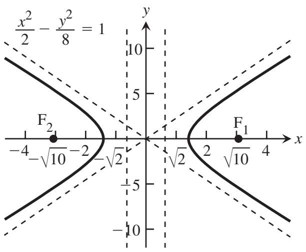
17. $e=\sqrt{2}$; $F\left(\pm\sqrt{2},0\right)$; directrices are $x=\pm\dfrac{1}{\sqrt{2}}$.



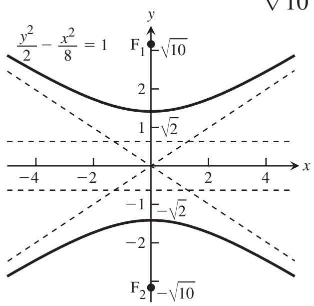
19. $e=\sqrt{2}$; $F(0,\pm 4)$; directrices are $y=\pm 2$.



21. $e=\sqrt{5}$; $F\left(\pm\sqrt{10},0\right)$; directrices are $x=\pm\dfrac{2}{\sqrt{10}}$.



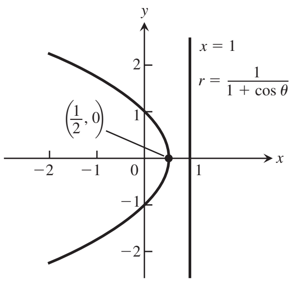
23. $e=\sqrt{5}$; $F\left(0,\pm\sqrt{10}\right)$; directrices are $y=\pm\dfrac{2}{\sqrt{10}}$.



25. $y^2-\dfrac{x^2}{8}=1$ **27.** $x^2-\dfrac{y^2}{8}=1$ **29.** $r=\dfrac{2}{1+\cos\theta}$

31. $r=\dfrac{30}{1-5\sin\theta}$ **33.** $r=\dfrac{1}{2+\cos\theta}$ **35.** $r=\dfrac{10}{5-\sin\theta}$

37.



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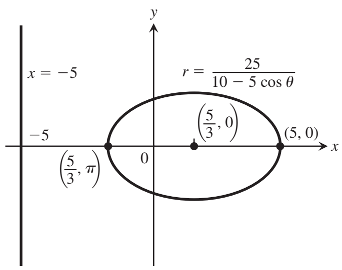

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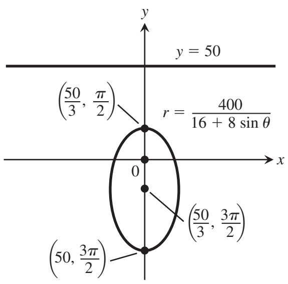

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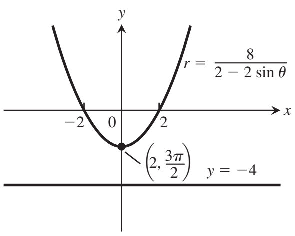
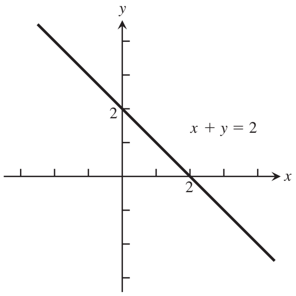
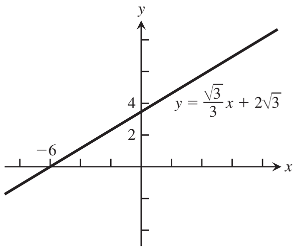


45. $y = 2 - x$

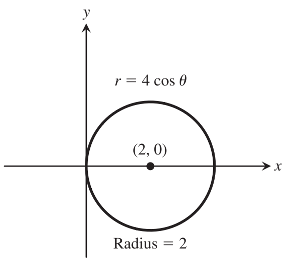


47. $y = \frac{\sqrt{3}}{3}x + 2\sqrt{3}$

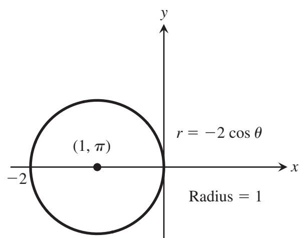


49. $r\cos\left(\theta - \frac{\pi}{4}\right) = 3$ **51.** $r\cos\left(\theta + \frac{\pi}{2}\right) = 5$

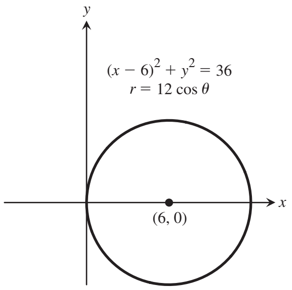
53.



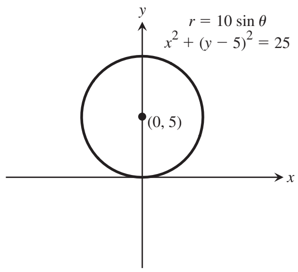
55.



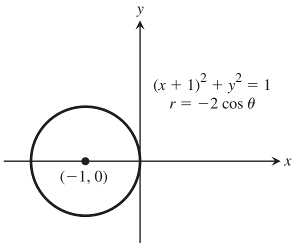
57. $r = 12\cos\theta$



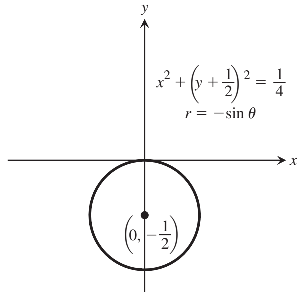
59. $r = 10\sin\theta$



61. $r = -2\cos\theta$



63. $r = -\sin\theta$



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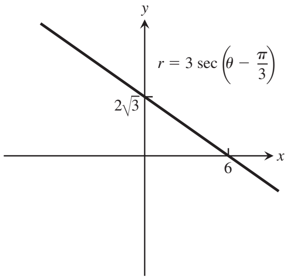

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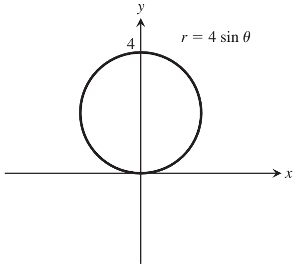

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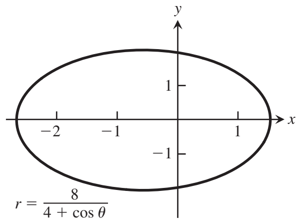

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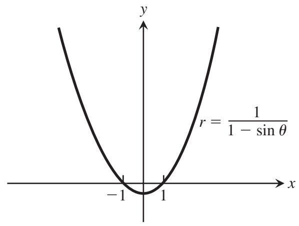
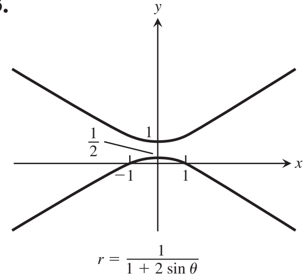


73.

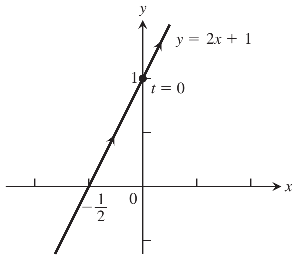


75. (b)

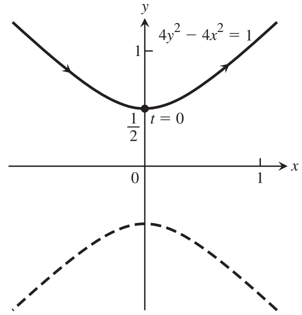
Planet	Perihelion	Aphelion
Mercury	0.3075 AU	0.4667 AU
Venus	0.7184 AU	0.7282 AU
Earth	0.9833 AU	1.0167 AU
Mars	1.3817 AU	1.6663 AU
Jupiter	4.9512 AU	5.4548 AU
Saturn	9.0210 AU	10.0570 AU
Uranus	18.2977 AU	20.0623 AU
Neptune	29.8135 AU	30.3065 AU

Practice Exercises, pp. 699–701

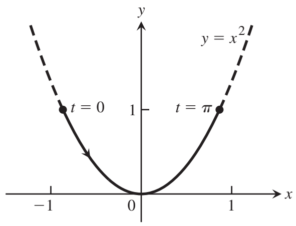
1.



3.



5.



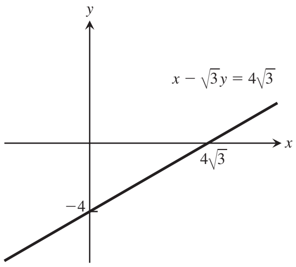
7. $x = 3\cos t, \quad y = 4\sin t, \quad 0 \le t \le 2\pi$

9. $y = \frac{\sqrt{3}}{2}x + \frac{1}{4}, \frac{1}{4}$

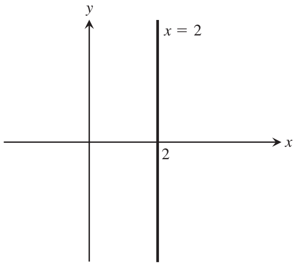
11. (a) $y = \dfrac{\pm |x|^{3/2}}{8} - 1$ **(b)** $y = \dfrac{\pm\sqrt{1 - x^2}}{x}$

13. $\dfrac{10}{3}$ **15.** $\dfrac{285}{8}$ **17.** 10 **19.** $\dfrac{9\pi}{2}$ **21.** $\dfrac{76\pi}{3}$

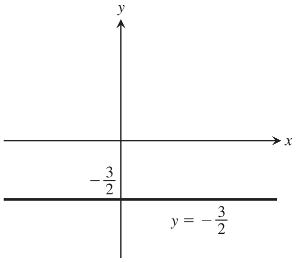
23. $y = \dfrac{\sqrt{3}}{3}x - 4$



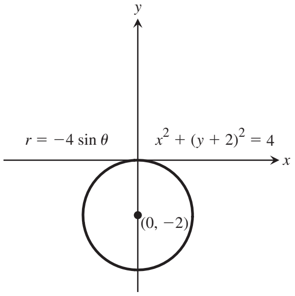
25. $x = 2$



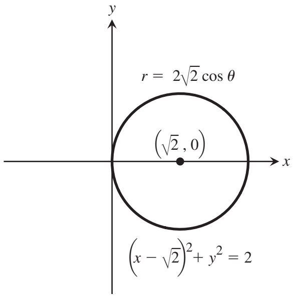
27. $y = -\dfrac{3}{2}$



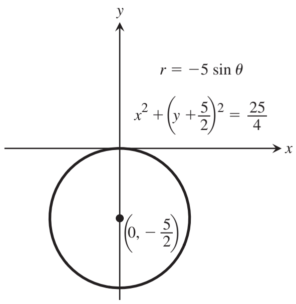
29. $x^2 + (y + 2)^2 = 4$



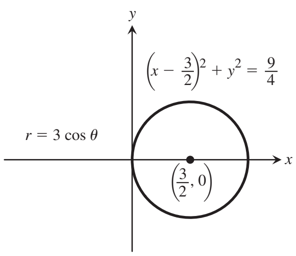
31. $\left(x - \sqrt{2}\right)^2 + y^2 = 2$



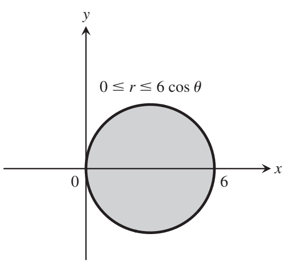
33. $r = -5\sin\theta$



35. $r = 3\cos\theta$



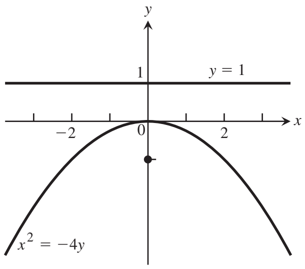
37.



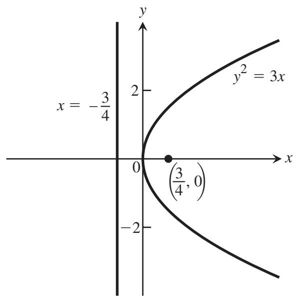
39. d **41.** l **43.** k **45.** i **47.** $\dfrac{9}{2}\pi$ **49.** $2 + \dfrac{\pi}{4}$

51. 8 **53.** $\pi - 3$

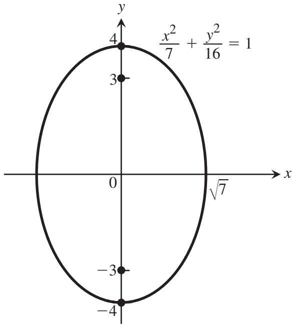
55. Focus is $(0, -1)$, directrix is $y = 1$.



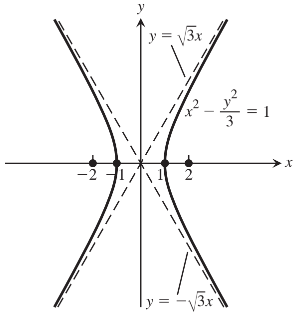
57. Focus is $\left(\dfrac{3}{4}, 0\right)$, directrix is $x = -\dfrac{3}{4}$.



59. $e = \dfrac{3}{4}$



61. $e = 2$; the asymptotes are $y = \pm\sqrt{3}\,x$.



63. $(x - 2)^2 = -12(y - 3)$, $V(2, 3)$, $F(2, 0)$, directrix is $y = 6$.

65. $\dfrac{(x + 3)^2}{9} + \dfrac{(y + 5)^2}{25} = 1$, $C(-3, -5)$, $F(-3, -1)$ and $F(-3, -9)$, $V(-3, -10)$ and $V(-3, 0)$.

67. $\dfrac{\left(y - 2\sqrt{2}\right)^2}{8} - \dfrac{(x - 2)^2}{2} = 1$, $C\left(2, 2\sqrt{2}\right)$, $F\left(2, 2\sqrt{2} \pm \sqrt{10}\right)$, $V\left(2, 4\sqrt{2}\right)$ and $V(2, 0)$, the asymptotes are $y = 2x - 4 + 2\sqrt{2}$ and $y = -2x + 4 + 2\sqrt{2}$.

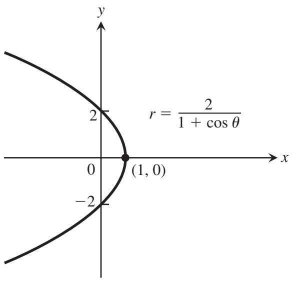
69. Hyperbola: $C(2, 0)$, $V(0, 0)$ and $V(4, 0)$, the foci are $F\left(2 \pm \sqrt{5}, 0\right)$, and the asymptotes are $y = \pm\dfrac{x - 2}{2}$.

71. Parabola: $V(-3, 1)$, $F(-7, 1)$, and the directrix is $x = 1$.

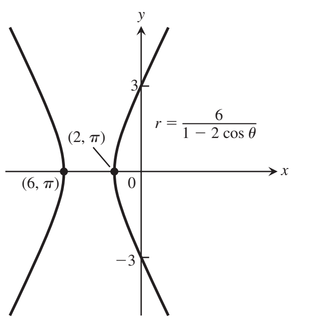
73. Ellipse: $C(-3, 2)$, $F\left(-3 \pm \sqrt{7}, 2\right)$, $V(1, 2)$ and $V(-7, 2)$

75. Circle: $C(1, 1)$ and radius $= \sqrt{2}$

77. $V(1, 0)$



79. $V(2, \pi)$ and $V(6, \pi)$

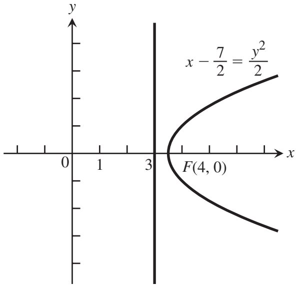


81. $r = \dfrac{4}{1 + 2\cos\theta}$ **83.** $r = \dfrac{2}{2 + \sin\theta}$

85. (a) 24π **(b)** 16π

Additional and Advanced Exercises, pp. 701–703

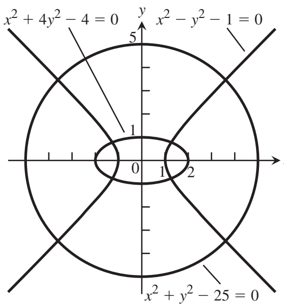
1. $x - \dfrac{7}{2} = \dfrac{y^2}{2}$



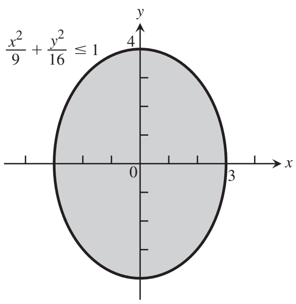
3. $3x^2 + 3y^2 - 8y + 4 = 0$ **5.** $F(0, \pm 1)$

7. (a) $\dfrac{(y-1)^2}{16} - \dfrac{x^2}{48} = 1$ **(b)** $\dfrac{\left(y + \frac{3}{4}\right)^2}{\left(\frac{25}{16}\right)} - \dfrac{x^2}{\left(\frac{75}{2}\right)} = 1$

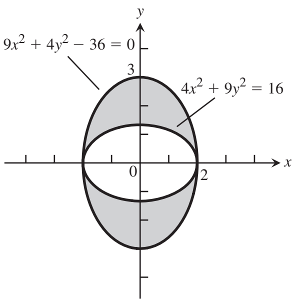
11.



13.



15.



17. (a) $r = e^{2\theta}$ **(b)** $\dfrac{\sqrt{5}}{2}(e^{4\pi} - 1)$

19. $r = \dfrac{4}{1 + 2\cos\theta}$ **21.** $r = \dfrac{2}{2 + \sin\theta}$

23. $x = (a + b)\cos\theta - b\cos\left(\dfrac{a+b}{b}\theta\right),$

$y = (a + b)\sin\theta - b\sin\left(\dfrac{a+b}{b}\theta\right)$

27. $\dfrac{\pi}{2}$

Chapter 12

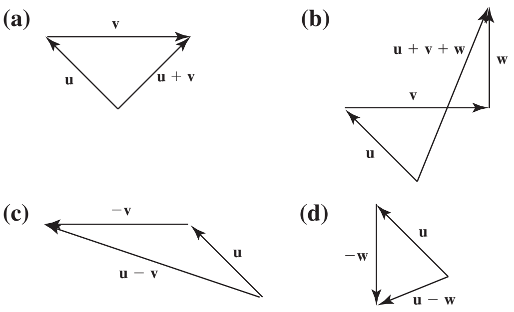
Section 12.1, pp. 707–708

1. The line through the point (2, 3, 0) parallel to the z-axis
3. The x-axis
5. The circle $x^2 + y^2 = 4$ in the xy-plane
7. The circle $x^2 + z^2 = 4$ in the xz-plane
9. The circle $y^2 + z^2 = 1$ in the yz-plane
11. The circle $x^2 + y^2 = 16$ in the xy-plane
13. The ellipse formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z = y$
15. The parabola $y = x^2$ in the xy-plane
17. (a) The first quadrant of the xy-plane
(b) The fourth quadrant of the xy-plane
19. (a) The ball of radius 1 centered at the origin
(b) All points more than 1 unit from the origin
21. (a) The ball of radius 2 centered at the origin with the interior of the ball of radius 1 centered at the origin removed
(b) The solid upper hemisphere of radius 1 centered at the origin
23. (a) The region on or inside the parabola $y = x^2$ in the xy-plane and all points above this region
(b) The region on or to the left of the parabola $x = y^2$ in the xy-plane and all points above it that are 2 units or less away from the xy-plane
25. (a) $x = 3$ **(b)** $y = -1$ **(c)** $z = -2$
27. (a) $z = 1$ **(b)** $x = 3$ **(c)** $y = -1$
29. (a) $x^2 + (y - 2)^2 = 4, z = 0$
(b) $(y - 2)^2 + z^2 = 4, x = 0$ **(c)** $x^2 + z^2 = 4, y = 2$
31. (a) $y = 3, z = -1$ **(b)** $x = 1, z = -1$
(c) $x = 1, y = 3$
33. $x^2 + y^2 + z^2 = 25, z = 3$ **35.** $0 \le z \le 1$ **37.** $z \le 0$
39. (a) $(x - 1)^2 + (y - 1)^2 + (z - 1)^2 < 1$
(b) $(x - 1)^2 + (y - 1)^2 + (z - 1)^2 > 1$
41. 3 **43.** 7 **45.** $2\sqrt{3}$ **47.** $C(-2, 0, 2), a = 2\sqrt{2}$
49. $C\left(\sqrt{2}, \sqrt{2}, -\sqrt{2}\right), a = \sqrt{2}$
51. $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 14$
53. $(x + 1)^2 + \left(y - \dfrac{1}{2}\right)^2 + \left(z + \dfrac{2}{3}\right)^2 = \dfrac{16}{81}$
55. $C(-2, 0, 2), a = \sqrt{8}$ **57.** $C\left(-\dfrac{1}{4}, -\dfrac{1}{4}, -\dfrac{1}{4}\right), a = \dfrac{5\sqrt{3}}{4}$
59. (a) $\sqrt{y^2 + z^2}$ **(b)** $\sqrt{x^2 + z^2}$ **(c)** $\sqrt{x^2 + y^2}$
61. $\sqrt{17} + \sqrt{33} + 6$ **63.** $y = 1$
65. (a) $(0, 3, -3)$ **(b)** $(0, 5, -5)$

Section 12.2, pp. 716–718

1. (a) $\langle 9, -6\rangle$ **(b)** $3\sqrt{13}$ **3. (a)** $\langle 1, 3\rangle$ **(b)** $\sqrt{10}$
5. (a) $\langle 12, -19\rangle$ **(b)** $\sqrt{505}$
7. (a) $\left\langle \dfrac{1}{5}, \dfrac{14}{5}\right\rangle$ **(b)** $\dfrac{\sqrt{197}}{5}$ **9.** $\langle 1, -4\rangle$
11. $\langle -2, -3\rangle$ **13.** $\left\langle -\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right\rangle$ **15.** $\left\langle -\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right\rangle$
17. $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ **19.** $-3\mathbf{i} + 16\mathbf{j}$
21. $3\mathbf{i} + 5\mathbf{j} - 8\mathbf{k}$

23. The vector **v** is horizontal and 1 in. long. The vectors **u** and **w** are $\frac{11}{16}$ in. long. **w** is vertical and **u** makes a 45° angle with the horizontal. All vectors must be drawn to scale.



25. $3\left(\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right)$ **27.** $5(\mathbf{k})$

29. $\sqrt{\frac{1}{2}}\left(\frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k}\right)$

31. **(a)** $2\mathbf{i}$ **(b)** $-\sqrt{3}\mathbf{k}$ **(c)** $\frac{3}{10}\mathbf{j} + \frac{2}{5}\mathbf{k}$ **(d)** $6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$

33. $\frac{7}{13}(12\mathbf{i} - 5\mathbf{k})$

35. **(a)** $\frac{3}{5\sqrt{2}}\mathbf{i} + \frac{4}{5\sqrt{2}}\mathbf{j} - \frac{1}{\sqrt{2}}\mathbf{k}$ **(b)** $(1/2, 3, 5/2)$

37. **(a)** $-\frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k}$ **(b)** $\left(\frac{5}{2}, \frac{7}{2}, \frac{9}{2}\right)$

39. $A(4, -3, 5)$ **41.** $a = \frac{3}{2}, b = \frac{1}{2}$

43. $\approx \langle -338.095, 725.046 \rangle$

45. $|\mathbf{F}_1| = \dfrac{100\cos 45°}{\sin 75°} \approx 73.205$ N,

$|\mathbf{F}_2| = \dfrac{100\cos 30°}{\sin 75°} \approx 89.658$ N,

$\mathbf{F}_1 = \langle -|\mathbf{F}_1|\cos 30°, |\mathbf{F}_1|\sin 30° \rangle \approx \langle -63.397, 36.603 \rangle$,

$\mathbf{F}_2 = \langle |\mathbf{F}_2|\cos 45°, |\mathbf{F}_2|\sin 45° \rangle \approx \langle 63.397, 63.397 \rangle$

47. $w = \dfrac{100\sin 75°}{\cos 40°} \approx 126.093$ N,

$|\mathbf{F}_1| = \dfrac{w\cos 35°}{\sin 75°} \approx 106.933$ N

49. **(a)** $(5\cos 60°, 5\sin 60°) = \left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

(b) $(5\cos 60° + 10\cos 315°, 5\sin 60° + 10\sin 315°) = \left(\frac{5 + 10\sqrt{2}}{2}, \frac{5\sqrt{3} - 10\sqrt{2}}{2}\right)$

51. **(a)** $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} - 3\mathbf{k}$ **(b)** $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ **(c)** $(2, 2, 1)$

Section 12.3, pp. 724–726

1. **(a)** $-25, 5, 5$ **(b)** -1 **(c)** -5 **(d)** $-2\mathbf{i} + 4\mathbf{j} - \sqrt{5}\mathbf{k}$

3. **(a)** $25, 15, 5$ **(b)** $\frac{1}{3}$ **(c)** $\frac{5}{3}$ **(d)** $\frac{1}{9}(10\mathbf{i} + 11\mathbf{j} - 2\mathbf{k})$

5. **(a)** $2, \sqrt{34}, \sqrt{3}$ **(b)** $\frac{2}{\sqrt{3}\sqrt{34}}$ **(c)** $\frac{2}{\sqrt{34}}$

(d) $\frac{1}{17}(5\mathbf{j} - 3\mathbf{k})$

7. **(a)** $10 + \sqrt{17}, \sqrt{26}, \sqrt{21}$ **(b)** $\frac{10 + \sqrt{17}}{\sqrt{546}}$

(c) $\frac{10 + \sqrt{17}}{\sqrt{26}}$ **(d)** $\frac{10 + \sqrt{17}}{26}(5\mathbf{i} + \mathbf{j})$

9. 0.75 rad **11.** 1.77 rad

13. Angle at $A = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 63.435$ degrees, angle at

$B = \cos^{-1}\left(\frac{3}{5}\right) \approx 53.130$ degrees, angle at

$C = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 63.435$ degrees.

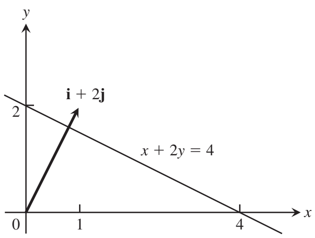
23. Horizontal component: ≈ 1188 ft/sec, vertical component: ≈ 167 ft/sec

25. **(a)** Since $|\cos\theta| \le 1$, we have $|\mathbf{u}\cdot\mathbf{v}| = |\mathbf{u}||\mathbf{v}||\cos\theta| \le |\mathbf{u}||\mathbf{v}|(1) = |\mathbf{u}||\mathbf{v}|$.

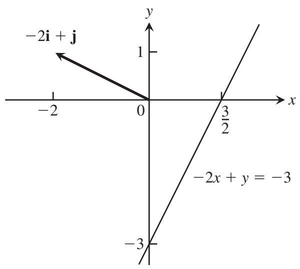
(b) We have equality precisely when $|\cos\theta| = 1$ or when one or both of **u** and **v** are **0**. In the case of nonzero vectors, we have equality when $\theta = 0$ or π, that is, when the vectors are parallel.

27. a

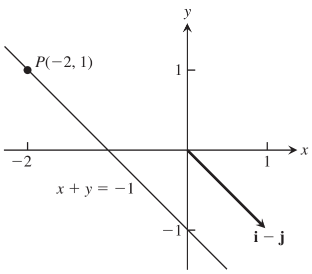
33. $x + 2y = 4$



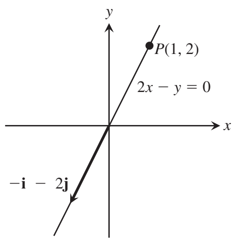
35. $-2x + y = -3$



37. $x + y = -1$



39. $2x - y = 0$



41. 5 J **43.** 3464 J **45.** $\frac{\pi}{4}$ **47.** $\frac{\pi}{6}$ **49.** 0.14

Section 12.4, pp. 730–732

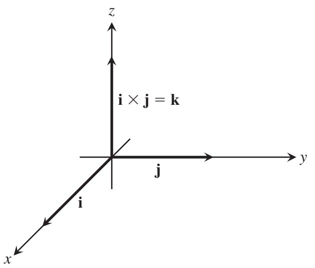
1. $|\mathbf{u}\times\mathbf{v}| = 3$, direction is $\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$; $|\mathbf{v}\times\mathbf{u}| = 3$, direction is $-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$

3. $|\mathbf{u}\times\mathbf{v}| = 0$, no direction; $|\mathbf{v}\times\mathbf{u}| = 0$, no direction

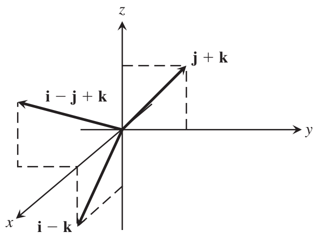
5. $|\mathbf{u}\times\mathbf{v}| = 6$, direction is $-\mathbf{k}$; $|\mathbf{v}\times\mathbf{u}| = 6$, direction is $\mathbf{k}$

7. $|\mathbf{u}\times\mathbf{v}| = 6\sqrt{5}$, direction is $\frac{1}{\sqrt{5}}\mathbf{i} - \frac{2}{\sqrt{5}}\mathbf{k}$; $|\mathbf{v}\times\mathbf{u}| = 6\sqrt{5}$, direction is $-\frac{1}{\sqrt{5}}\mathbf{i} + \frac{2}{\sqrt{5}}\mathbf{k}$

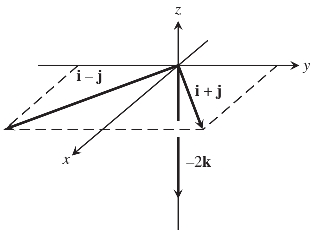
9.



11.



13.



15. **(a)** $2\sqrt{6}$ **(b)** $\pm\frac{1}{\sqrt{6}}(2\mathbf{i}+\mathbf{j}+\mathbf{k})$

17. **(a)** $\frac{\sqrt{2}}{2}$ **(b)** $\pm\frac{1}{\sqrt{2}}(\mathbf{i}-\mathbf{j})$

19. 8 **21.** 7 **23.** **(a)** None **(b)** **u** and **w**

25. $10\sqrt{3}$ ft-lb

27. **(a)** True **(b)** Not always true **(c)** True **(d)** True **(e)** Not always true **(f)** True **(g)** True **(h)** True

29. **(a)** $\text{proj}_{\mathbf{v}}\,\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\mathbf{v}$ **(b)** $\pm\,\mathbf{u}\times\mathbf{v}$ **(c)** $\pm\,(\mathbf{u}\times\mathbf{v})\times\mathbf{w}$ **(d)** $|(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}|$ **(e)** $(\mathbf{u}\times\mathbf{v})\times(\mathbf{u}\times\mathbf{w})$ **(f)** $|\mathbf{u}|\frac{\mathbf{v}}{|\mathbf{v}|}$

31. **(a)** Yes **(b)** No **(c)** Yes **(d)** No

33. No, **v** need not equal **w**. For example, $\mathbf{i}+\mathbf{j}\neq-\mathbf{i}+\mathbf{j}$, but $\mathbf{i}\times(\mathbf{i}+\mathbf{j}) = \mathbf{i}\times\mathbf{i}+\mathbf{i}\times\mathbf{j} = \mathbf{0}+\mathbf{k} = \mathbf{k}$ and $\mathbf{i}\times(-\mathbf{i}+\mathbf{j}) = -\mathbf{i}\times\mathbf{i}+\mathbf{i}\times\mathbf{j} = \mathbf{0}+\mathbf{k} = \mathbf{k}$.

35. 2 **37.** 13 **39.** $\sqrt{129}$ **41.** $\frac{11}{2}$ **43.** $\frac{25}{2}$

45. $\frac{3}{2}$ **47.** $\frac{\sqrt{21}}{2}$

49. If $\mathbf{A} = a_1\mathbf{i}+a_2\mathbf{j}$ and $\mathbf{B} = b_1\mathbf{i}+b_2\mathbf{j}$, then

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & 0\\ b_1 & b_2 & 0\end{vmatrix} = \begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}\mathbf{k}$$

and the triangle's area is

$$\frac{1}{2}\left|\mathbf{A}\times\mathbf{B}\right| = \pm\frac{1}{2}\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}.$$

The applicable sign is (+) if the acute angle from **A** to **B** runs counterclockwise in the *xy*-plane, and (−) if it runs clockwise.

Section 12.5, pp. 738–740

1. $x = 3+t,\quad y = -4+t,\quad z = -1+t$

3. $x = -2+5t,\quad y = 5t,\quad z = 3-5t$

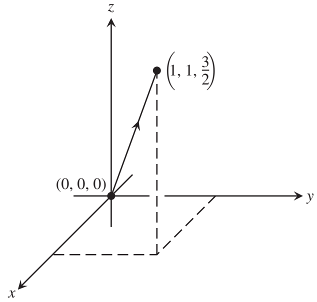
5. $x = 0,\quad y = 2t,\quad z = t$

7. $x = 1,\quad y = 1,\quad z = 1+t$

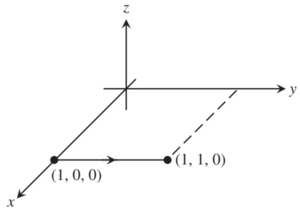
9. $x = t,\quad y = -7+2t,\quad z = 2t$

11. $x = t,\quad y = 0,\quad z = 0$

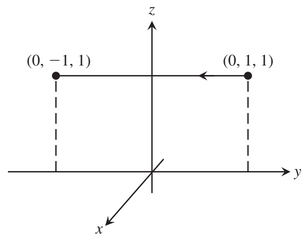
13. $x = t,\quad y = t,\quad z = \frac{3}{2}t,\quad 0\le t\le 1$



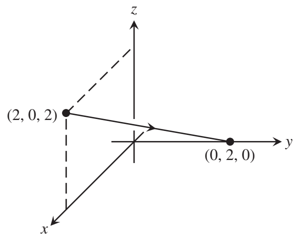
15. $x = 1,\quad y = 1+t,\quad z = 0,\quad -1\le t\le 0$



17. $x = 0,\quad y = 1-2t,\quad z = 1,\quad 0\le t\le 1$



19. $x = 2-2t,\quad y = 2t,\quad z = 2-2t,\quad 0\le t\le 1$



21. $3x-2y-z = -3$ **23.** $7x-5y-4z = 6$

25. $x+3y+4z = 34$ **27.** $(1, 2, 3), -20x+12y+z = 7$

29. $y+z = 3$ **31.** $x-y+z = 0$ **33.** $2\sqrt{30}$ **35.** 0

37. $\frac{9\sqrt{42}}{7}$ **39.** 3 **41.** $19/5$ **43.** $5/3$ **45.** $9/\sqrt{41}$

47. $\pi/4$ **49.** 1.38 rad **51.** 0.82 rad **53.** $\left(\frac{3}{2}, -\frac{3}{2}, \frac{1}{2}\right)$

55. (1, 1, 0) **57.** $x = 1-t,\quad y = 1+t,\quad z = -1$

59. $x = 4,\quad y = 3+6t,\quad z = 1+3t$

61. $L1$ intersects $L2$; $L2$ is parallel to $L3$, $\sqrt{5}/3$; $L1$ and $L3$ are skew, $10\sqrt{2}/3$

63. $x = 2+2t,\quad y = -4-t,\quad z = 7+3t;\quad x = -2-t,\quad y = -2+(1/2)t,\quad z = 1-(3/2)t$

65. $\left(0, -\frac{1}{2}, -\frac{3}{2}\right), (-1, 0, -3), (1, -1, 0)$

69. Many possible answers. One possibility: $x+y = 3$ and $2y+z = 7$.

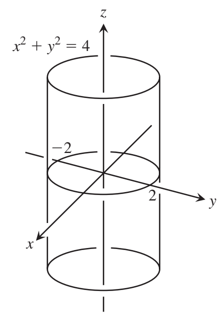
71. $(x/a)+(y/b)+(z/c) = 1$ describes all planes *except* those through the origin or parallel to a coordinate axis.

Section 12.6, pp. 744–745

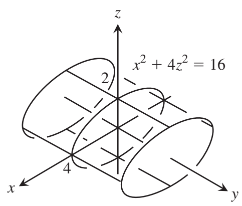
1. (d), ellipsoid **3.** (a), cylinder **5.** (l), hyperbolic paraboloid

7. (b), cylinder **9.** (k), hyperbolic paraboloid **11.** (h), cone

13.



15.



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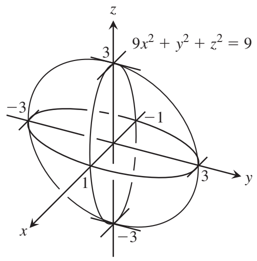

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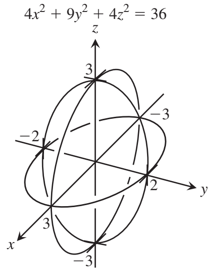

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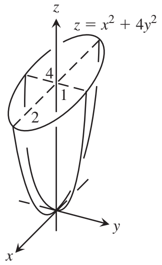

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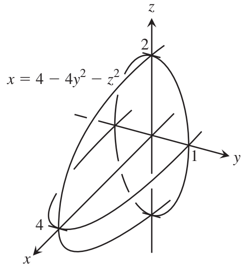

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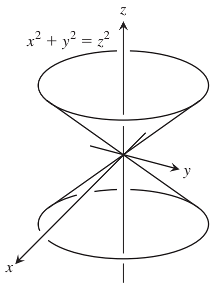

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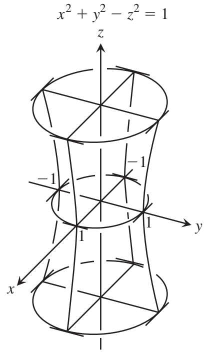

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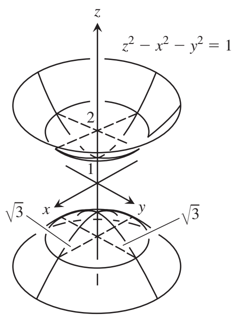

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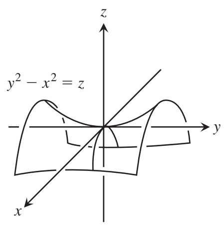

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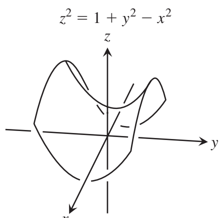

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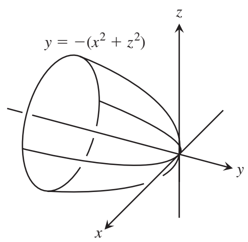

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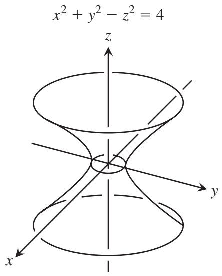

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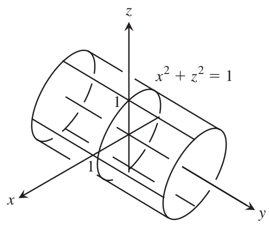

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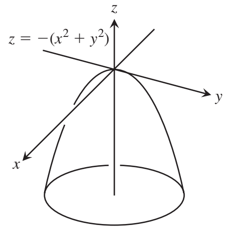
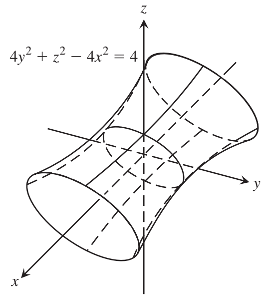


43.



45. **(a)** $\dfrac{2\pi(9 - c^2)}{9}$ **(b)** 8π **(c)** $\dfrac{4\pi abc}{3}$

Practice Exercises, pp. 746–747

1. **(a)** $\langle -17, 32 \rangle$ **(b)** $\sqrt{1313}$

3. **(a)** $\langle 6, -8 \rangle$ **(b)** 10

5. $\left\langle -\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2} \right\rangle$ [assuming counterclockwise]

7. $\left\langle \dfrac{8}{\sqrt{17}}, -\dfrac{2}{\sqrt{17}} \right\rangle$

9. Length $= 2$, direction is $\dfrac{1}{\sqrt{2}}\mathbf{i} + \dfrac{1}{\sqrt{2}}\mathbf{j}$.

11. $\mathbf{v}(\pi/2) = 2(-\mathbf{i})$

13. Length $= 7$, direction is $\dfrac{2}{7}\mathbf{i} - \dfrac{3}{7}\mathbf{j} + \dfrac{6}{7}\mathbf{k}$.

15. $\dfrac{8}{\sqrt{33}}\mathbf{i} - \dfrac{2}{\sqrt{33}}\mathbf{j} + \dfrac{8}{\sqrt{33}}\mathbf{k}$

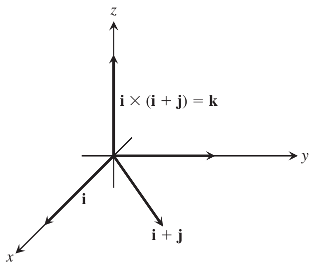
17. $|\mathbf{v}| = \sqrt{2}$, $|\mathbf{u}| = 3$, $\mathbf{v}\cdot\mathbf{u} = \mathbf{u}\cdot\mathbf{v} = 3$, $\mathbf{v}\times\mathbf{u} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$,

$\mathbf{u}\times\mathbf{v} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $|\mathbf{v}\times\mathbf{u}| = 3$, $\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{\pi}{4}$,

$|\mathbf{u}|\cos\theta = \dfrac{3}{\sqrt{2}}$, $\text{proj}_{\mathbf{v}}\,\mathbf{u} = \dfrac{3}{2}(\mathbf{i} + \mathbf{j})$

19. $\dfrac{4}{3}(2\mathbf{i} + \mathbf{j} - \mathbf{k})$

21. $\mathbf{u}\times\mathbf{v} = \mathbf{k}$



23. $2\sqrt{7}$ **25.** **(a)** $\sqrt{14}$ **(b)** 1 **29.** $\sqrt{78}/3$

31. $x = 1 - 3t, \quad y = 2, \quad z = 3 + 7t$ **33.** $\sqrt{2}$

35. $2x + y + z = 5$ **37.** $-9x + y + 7z = 4$

39. $\left(0, -\frac{1}{2}, -\frac{3}{2}\right), (-1, 0, -3), (1, -1, 0)$ **41.** $\pi/3$

43. $x = -5 + 5t, \quad y = 3 - t, \quad z = -3t$

45. **(b)** $x = -12t, \quad y = 19/12 + 15t, \quad z = 1/6 + 6t$

47. Yes; **v** is parallel to the plane.

49. 3 **51.** $-3\mathbf{j} + 3\mathbf{k}$

53. $\frac{2}{\sqrt{35}}(5\mathbf{i} - \mathbf{j} - 3\mathbf{k})$ **55.** $\left(\frac{11}{9}, \frac{26}{9}, -\frac{7}{9}\right)$

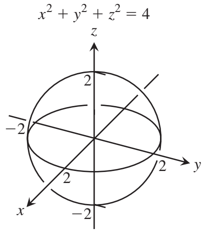
57. $(1, -2, -1); x = 1 - 5t, \quad y = -2 + 3t, \quad z = -1 + 4t$

59. $2x + 7y + 2z + 10 = 0$

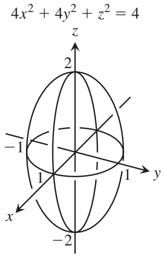
61. **(a)** No **(b)** No **(c)** No **(d)** No **(e)** Yes

63. $11/\sqrt{107}$

65.



67.



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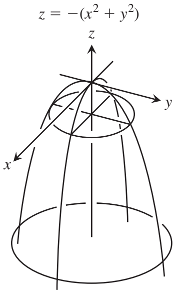

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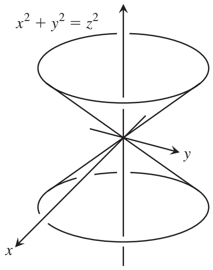

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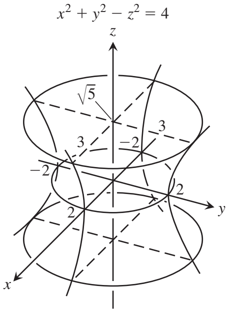

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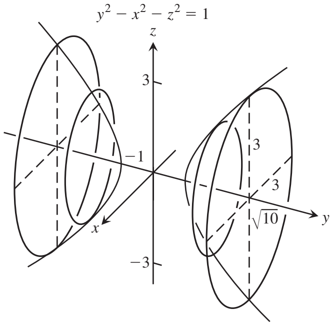


Additional and Advanced Exercises, pp. 748–750

1. $(26, 23, -1/3)$ **3.** $|\mathbf{F}| = 20$ lb

5. **(a)** $|\mathbf{F}_1| = 80$ lb, $|\mathbf{F}_2| = 60$ lb, $\mathbf{F}_1 = \langle -48, 64 \rangle$,
$\mathbf{F}_2 = \langle 48, 36 \rangle, \quad \alpha = \tan^{-1}\frac{4}{3}, \quad \beta = \tan^{-1}\frac{3}{4}$

(b) $|\mathbf{F}_1| = \frac{2400}{13} \approx 184.615$ lb, $|\mathbf{F}_2| = \frac{1000}{13} \approx 76.923$ lb,
$\mathbf{F}_1 = \left\langle \frac{-12{,}000}{169}, \frac{28{,}800}{169} \right\rangle \approx \langle -71.006, 170.414 \rangle$,
$\mathbf{F}_2 = \left\langle \frac{12{,}000}{169}, \frac{5000}{169} \right\rangle \approx \langle 71.006, 29.586 \rangle$,
$\alpha = \tan^{-1}\frac{12}{5}, \beta = \tan^{-1}\frac{5}{12}$

9. **(a)** $\theta = \tan^{-1}\sqrt{2} \approx 54.74°$ **(b)** $\theta = \tan^{-1} 2\sqrt{2} \approx 70.53°$

13. **(b)** $\frac{6}{\sqrt{14}}$ **(c)** $2x - y + 2z = 8$
(d) $x - 2y + z = 3 + 5\sqrt{6}$ and $x - 2y + z = 3 - 5\sqrt{6}$

15. $\frac{32}{41}\mathbf{i} + \frac{23}{41}\mathbf{j} - \frac{13}{41}\mathbf{k}$

17. **(a)** **0, 0** **(b)** $-10\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}, -9\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$
(c) $-4\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}, \mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$
(d) $-10\mathbf{i} - 10\mathbf{k}, -12\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$

19. The formula is always true.

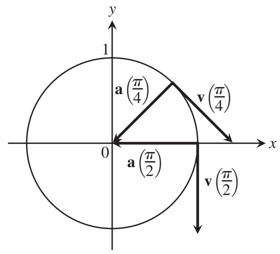
Chapter 13

Section 13.1, pp. 757–759

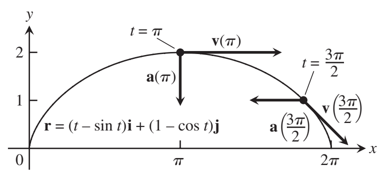
1. $y = x^2 - 2x, \quad \mathbf{v} = \mathbf{i} + 2\mathbf{j}, \quad \mathbf{a} = 2\mathbf{j}$

3. $y = \frac{2}{9}x^2, \quad \mathbf{v} = 3\mathbf{i} + 4\mathbf{j}, \quad \mathbf{a} = 3\mathbf{i} + 8\mathbf{j}$

5. $t = \frac{\pi}{4}$: $\mathbf{v} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}, \quad \mathbf{a} = \frac{-\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$;
$t = \pi/2$: $\mathbf{v} = -\mathbf{j}, \quad \mathbf{a} = -\mathbf{i}$



7. $t = \pi$: $\mathbf{v} = 2\mathbf{i}, \quad \mathbf{a} = -\mathbf{j}; \quad t = \frac{3\pi}{2}$: $\mathbf{v} = \mathbf{i} - \mathbf{j}, \quad \mathbf{a} = -\mathbf{i}$



9. $\mathbf{v} = \mathbf{i} + 2t\mathbf{j} + 2\mathbf{k}$; $\mathbf{a} = 2\mathbf{j}$; speed: 3; direction: $\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$;
$\mathbf{v}(1) = 3\left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$

11. $\mathbf{v} = (-2\sin t)\mathbf{i} + (3\cos t)\mathbf{j} + 4\mathbf{k}$;
$\mathbf{a} = (-2\cos t)\mathbf{i} - (3\sin t)\mathbf{j}$; speed: $2\sqrt{5}$;
direction: $(-1/\sqrt{5})\mathbf{i} + (2/\sqrt{5})\mathbf{k}$;
$\mathbf{v}(\pi/2) = 2\sqrt{5}\left[(-1/\sqrt{5})\mathbf{i} + (2/\sqrt{5})\mathbf{k}\right]$

13. $\mathbf{v} = \left(\frac{2}{t+1}\right)\mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$; $\mathbf{a} = \left(\frac{-2}{(t+1)^2}\right)\mathbf{i} + 2\mathbf{j} + \mathbf{k}$;
speed: $\sqrt{6}$; direction: $\frac{1}{\sqrt{6}}\mathbf{i} + \frac{2}{\sqrt{6}}\mathbf{j} + \frac{1}{\sqrt{6}}\mathbf{k}$;
$\mathbf{v}(1) = \sqrt{6}\left(\frac{1}{\sqrt{6}}\mathbf{i} + \frac{2}{\sqrt{6}}\mathbf{j} + \frac{1}{\sqrt{6}}\mathbf{k}\right)$

15. $\pi/2$ **17.** $\pi/2$

19. $x = t, \quad y = -1, \quad z = 1 + t$ **21.** $x = t, \quad y = \frac{1}{3}t, \quad z = t$

23. (a) (i): It has constant speed 1. **(ii):** Yes
(iii): Counterclockwise **(iv):** Yes
(b) (i): It has constant speed 2. **(ii):** Yes
(iii): Counterclockwise **(iv):** Yes
(c) (i): It has constant speed 1. **(ii):** Yes
(iii): Counterclockwise
(iv): It starts at $(0, -1)$ instead of $(1, 0)$.
(d) (i): It has constant speed 1. **(ii):** Yes
(iii): Clockwise **(iv):** Yes
(e) (i): It has variable speed. **(ii):** No
(iii): Counterclockwise **(iv):** Yes

25. $\mathbf{v} = 2\sqrt{5}\mathbf{i} + \sqrt{5}\mathbf{j}$

Section 13.2, pp. 765–768

1. $(1/4)\mathbf{i} + 7\mathbf{j} + (3/2)\mathbf{k}$ **3.** $\left(\frac{\pi + 2\sqrt{2}}{2}\right)\mathbf{j} + 2\mathbf{k}$

5. $(\ln 4)\mathbf{i} + (\ln 4)\mathbf{j} + (\ln 2)\mathbf{k}$

7. $\frac{e-1}{2}\mathbf{i} + \frac{e-1}{e}\mathbf{j} + \mathbf{k}$ **9.** $\mathbf{i} - \mathbf{j} + \frac{\pi}{4}\mathbf{k}$

11. $\mathbf{r}(t) = \left(\frac{-t^2}{2} + 1\right)\mathbf{i} + \left(\frac{-t^2}{2} + 2\right)\mathbf{j} + \left(\frac{-t^2}{2} + 3\right)\mathbf{k}$

13. $\mathbf{r}(t) = ((t+1)^{3/2} - 1)\mathbf{i} + (-e^{-t} + 1)\mathbf{j} + (\ln(t+1) + 1)\mathbf{k}$

15. $\mathbf{r}(t) = 8t\mathbf{i} + 8t\mathbf{j} + (-16t^2 + 100)\mathbf{k}$

17. $\mathbf{r}(t) = \left(\frac{3}{2}t^2 + \frac{6}{\sqrt{11}}t + 1\right)\mathbf{i} - \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{11}}t - 2\right)\mathbf{j}$
$+\left(\frac{1}{2}t^2 + \frac{2}{\sqrt{11}}t + 3\right)\mathbf{k} = \left(\frac{1}{2}t^2 + \frac{2t}{\sqrt{11}}\right)(3\mathbf{i} - \mathbf{j} + \mathbf{k})$
$+(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$

19. 50 sec

21. (a) 72.2 sec; 25,510 m **(b)** 4020 m **(c)** 6378 m

23. (a) $v_0 \approx 9.9$ m/sec **(b)** $\alpha \approx 18.4°$ or $71.6°$

25. 39.3° or 50.7° **31. (b)** $\mathbf{v}_0$ would bisect $\angle AOR$.

33. (a) (Assuming that "x" is zero at the point of impact)
$\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}$, where $x(t) = (35 \cos 27°)t$ and $y(t) = 4 + (35 \sin 27°)t - 16t^2$.
(b) At $t \approx 0.497$ sec, it reaches its maximum height of about 7.945 ft.
(c) Range ≈ 37.45 ft; flight time ≈ 1.201 sec
(d) At $t \approx 0.254$ and $t \approx 0.740$ sec, when it is ≈ 29.532 and ≈ 14.376 ft from where it will land
(e) Yes. It changes things because the ball won't clear the net.

35. 4.00 ft, 7.80 ft/sec

43. (a) $\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}$; where
$x(t) = \left(\frac{1}{0.08}\right)(1 - e^{-0.08t})(152 \cos 20° - 17.6)$ and
$y(t) = 3 + \left(\frac{152}{0.08}\right)(1 - e^{-0.08t})(\sin 20°)$
$+\left(\frac{32}{0.08^2}\right)(1 - 0.08t - e^{-0.08t})$
(b) At $t \approx 1.527$ sec it reaches a maximum height of about 41.893 feet.
(c) Range ≈ 351.734 ft; flight time ≈ 3.181 sec
(d) At $t \approx 0.877$ and 2.190 sec, when it is about 106.028 and 251.530 ft from home plate
(e) No

Section 13.3, pp. 771–772

1. $\mathbf{T} = \left(-\frac{2}{3}\sin t\right)\mathbf{i} + \left(\frac{2}{3}\cos t\right)\mathbf{j} + \frac{\sqrt{5}}{3}\mathbf{k}$, 3π

3. $\mathbf{T} = \frac{1}{\sqrt{1+t}}\mathbf{i} + \frac{\sqrt{t}}{\sqrt{1+t}}\mathbf{k}$, $\frac{52}{3}$ **5.** $\mathbf{T} = -\cos t\mathbf{j} + \sin t\mathbf{k}$, $\frac{3}{2}$

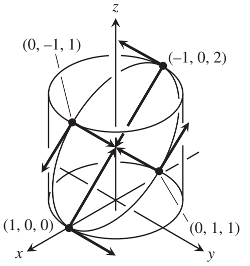
7. $\mathbf{T} = \left(\frac{\cos t - t \sin t}{t+1}\right)\mathbf{i} + \left(\frac{\sin t + t \cos t}{t+1}\right)\mathbf{j}$
$+\left(\frac{\sqrt{2}t^{1/2}}{t+1}\right)\mathbf{k}$, $\frac{\pi^2}{2} + \pi$

9. $(0, 5, 24\pi)$

11. $s(t) = 5t, \quad L = \frac{5\pi}{2}$

13. $s(t) = \sqrt{3}e^t - \sqrt{3}, \quad L = \frac{3\sqrt{3}}{4}$ **15.** $\sqrt{2} + \ln(1 + \sqrt{2})$

17. (a) Cylinder is $x^2 + y^2 = 1$; plane is $x + z = 1$.
(b) and **(c)**



(d) $L = \int_0^{2\pi} \sqrt{1 + \sin^2 t}\, dt$ **(e)** $L \approx 7.64$

Section 13.4, pp. 777–778

1. $\mathbf{T} = (\cos t)\mathbf{i} - (\sin t)\mathbf{j}, \quad \mathbf{N} = (-\sin t)\mathbf{i} - (\cos t)\mathbf{j}, \quad \kappa = \cos t$

3. $\mathbf{T} = \frac{1}{\sqrt{1+t^2}}\mathbf{i} - \frac{t}{\sqrt{1+t^2}}\mathbf{j}, \quad \mathbf{N} = \frac{-t}{\sqrt{1+t^2}}\mathbf{i} - \frac{1}{\sqrt{1+t^2}}\mathbf{j},$
$\kappa = \frac{1}{2(\sqrt{1+t^2})^3}$

5. (b) $\cos x$

7. (b) $\mathbf{N} = \dfrac{-2e^{2t}}{\sqrt{1 + 4e^{4t}}}\mathbf{i} + \dfrac{1}{\sqrt{1 + 4e^{4t}}}\mathbf{j}$

(c) $\mathbf{N} = -\frac{1}{2}\left(\sqrt{4 - t^2}\,\mathbf{i} + t\mathbf{j}\right)$

9. $\mathbf{T} = \dfrac{3\cos t}{5}\mathbf{i} - \dfrac{3\sin t}{5}\mathbf{j} + \dfrac{4}{5}\mathbf{k}$, $\mathbf{N} = (-\sin t)\mathbf{i} - (\cos t)\mathbf{j}$, $\kappa = \dfrac{3}{25}$

11. $\mathbf{T} = \left(\dfrac{\cos t - \sin t}{\sqrt{2}}\right)\mathbf{i} + \left(\dfrac{\cos t + \sin t}{\sqrt{2}}\right)\mathbf{j}$,
$\mathbf{N} = \left(\dfrac{-\cos t - \sin t}{\sqrt{2}}\right)\mathbf{i} + \left(\dfrac{-\sin t + \cos t}{\sqrt{2}}\right)\mathbf{j}$, $\kappa = \dfrac{1}{e^t\sqrt{2}}$

13. $\mathbf{T} = \dfrac{t}{\sqrt{t^2+1}}\mathbf{i} + \dfrac{1}{\sqrt{t^2+1}}\mathbf{j}$, $\mathbf{N} = \dfrac{\mathbf{i}}{\sqrt{t^2+1}} - \dfrac{t\mathbf{j}}{\sqrt{t^2+1}}$,
$\kappa = \dfrac{1}{t(t^2+1)^{3/2}}$

15. $\mathbf{T} = \left(\operatorname{sech}\dfrac{t}{a}\right)\mathbf{i} + \left(\tanh\dfrac{t}{a}\right)\mathbf{j}$,
$\mathbf{N} = \left(-\tanh\dfrac{t}{a}\right)\mathbf{i} + \left(\operatorname{sech}\dfrac{t}{a}\right)\mathbf{j}$,
$\kappa = \dfrac{1}{a}\operatorname{sech}^2\dfrac{t}{a}$

19. $1/(2b)$

21. $\left(x - \dfrac{\pi}{2}\right)^2 + y^2 = 1$

23. $\kappa(x) = 2/(1 + 4x^2)^{3/2}$

25. $\kappa(x) = |\sin x|/(1 + \cos^2 x)^{3/2}$

Section 13.5, pp. 783–784

1. $\mathbf{a} = |a|\mathbf{N}$ **3.** $\mathbf{a}(1) = \dfrac{4}{3}\mathbf{T} + \dfrac{2\sqrt{5}}{3}\mathbf{N}$ **5.** $\mathbf{a}(0) = 2\mathbf{N}$

7. $\mathbf{r}\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j} - \mathbf{k}$, $\mathbf{T}\left(\dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j}$,
$\mathbf{N}\left(\dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}\mathbf{i} - \dfrac{\sqrt{2}}{2}\mathbf{j}$, $\mathbf{B}\left(\dfrac{\pi}{4}\right) = \mathbf{k}$; osculating plane: $z = -1$; normal plane: $-x + y = 0$; rectifying plane: $x + y = \sqrt{2}$

9. $\mathbf{B} = \left(\dfrac{4}{5}\cos t\right)\mathbf{i} - \left(\dfrac{4}{5}\sin t\right)\mathbf{j} - \dfrac{3}{5}\mathbf{k}$, $\tau = -\dfrac{4}{25}$

11. $\mathbf{B} = \mathbf{k}$, $\tau = 0$ **13.** $\mathbf{B} = -\mathbf{k}$, $\tau = 0$ **15.** $\mathbf{B} = \mathbf{k}$, $\tau = 0$

17. Yes. If the car is moving on a curved path ($\kappa \neq 0$), then $a_{\mathrm{N}} = \kappa|\mathbf{v}|^2 \neq 0$ and $\mathbf{a} \neq \mathbf{0}$.

23. $\kappa = \dfrac{1}{t}$, $\rho = t$

27. Components of **v**: −1.8701, 0.7089, 1.0000
Components of **a**: −1.6960, −2.0307, 0
Speed: 2.2361; Components of **T**: −0.8364, 0.3170, 0.4472
Components of **N**: −0.4143, −0.8998, −0.1369
Components of **B**: 0.3590, −0.2998, 0.8839; Curvature: 0.5060
Torsion: 0.2813; Tangential component of acceleration: 0.7746
Normal component of acceleration: 2.5298

29. Components of **v**: 2.0000, 0, −0.1629
Components of **a**: 0, −1.0000, −0.0086; Speed: 2.0066
Components of **T**: 0.9967, 0, −0.0812
Components of **N**: −0.0007, −1.0000, −0.0086
Components of **B**: −0.0812, 0.0086, 0.9967;
Curvature: 0.2484
Torsion: 0.0411; Tangential component of acceleration: 0.0007
Normal component of acceleration: 1.0000

Section 13.6, pp. 787–788

1. $\mathbf{v} = (3a\sin\theta)\mathbf{u}_r + 3a(1 - \cos\theta)\mathbf{u}_\theta$
$\mathbf{a} = 9a(2\cos\theta - 1)\mathbf{u}_r + (18a\sin\theta)\mathbf{u}_\theta$

3. $\mathbf{v} = 2ae^{a\theta}\mathbf{u}_r + 2e^{a\theta}\mathbf{u}_\theta$
$\mathbf{a} = 4e^{a\theta}(a^2 - 1)\mathbf{u}_r + 8ae^{a\theta}\mathbf{u}_\theta$

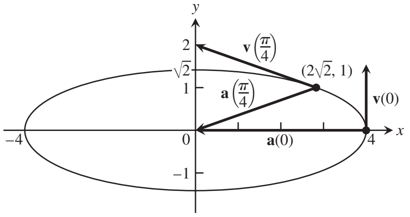
5. $\mathbf{v} = (-8\sin 4t)\mathbf{u}_r + (4\cos 4t)\mathbf{u}_\theta$
$\mathbf{a} = (-40\cos 4t)\mathbf{u}_r - (32\sin 4t)\mathbf{u}_\theta$

11. $\approx 29.93 \times 10^{10}$ m **13.** $\approx 2.25 \times 10^9$ km²/sec

15. $\approx 1.876 \times 10^{27}$ kg

Practice Exercises, pp. 788–790

1. $\dfrac{x^2}{16} + \dfrac{y^2}{2} = 1$



At $t = 0$: $a_{\mathrm{T}} = 0$, $a_{\mathrm{N}} = 4$, $\kappa = 2$;
At $t = \dfrac{\pi}{4}$: $a_{\mathrm{T}} = \dfrac{7}{3}$, $a_{\mathrm{N}} = \dfrac{4\sqrt{2}}{3}$, $\kappa = \dfrac{4\sqrt{2}}{27}$

3. $|\mathbf{v}|_{\max} = 1$ **5.** $\kappa = 1/5$ **7.** $dy/dt = -x$; clockwise

11. Shot put is on the ground, about 66 ft 3 in. from the stopboard.

15. Length $= \dfrac{\pi}{4}\sqrt{1 + \dfrac{\pi^2}{16}} + \ln\left(\dfrac{\pi}{4} + \sqrt{1 + \dfrac{\pi^2}{16}}\right)$

17. $\mathbf{T}(0) = \dfrac{2}{3}\mathbf{i} - \dfrac{2}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$; $\mathbf{N}(0) = \dfrac{1}{\sqrt{2}}\mathbf{i} + \dfrac{1}{\sqrt{2}}\mathbf{j}$;
$\mathbf{B}(0) = -\dfrac{1}{3\sqrt{2}}\mathbf{i} + \dfrac{1}{3\sqrt{2}}\mathbf{j} + \dfrac{4}{3\sqrt{2}}\mathbf{k}$; $\kappa = \dfrac{\sqrt{2}}{3}$; $\tau = \dfrac{1}{6}$

19. $\mathbf{T}(\ln 2) = \dfrac{1}{\sqrt{17}}\mathbf{i} + \dfrac{4}{\sqrt{17}}\mathbf{j}$; $\mathbf{N}(\ln 2) = -\dfrac{4}{\sqrt{17}}\mathbf{i} + \dfrac{1}{\sqrt{17}}\mathbf{j}$;
$\mathbf{B}(\ln 2) = \mathbf{k}$; $\kappa = \dfrac{8}{17\sqrt{17}}$; $\tau = 0$

21. $\mathbf{a}(0) = 10\mathbf{T} + 6\mathbf{N}$

23. $\mathbf{T} = \left(\dfrac{1}{\sqrt{2}}\cos t\right)\mathbf{i} - (\sin t)\mathbf{j} + \left(\dfrac{1}{\sqrt{2}}\cos t\right)\mathbf{k}$;
$\mathbf{N} = \left(-\dfrac{1}{\sqrt{2}}\sin t\right)\mathbf{i} - (\cos t)\mathbf{j} - \left(\dfrac{1}{\sqrt{2}}\sin t\right)\mathbf{k}$;
$\mathbf{B} = \dfrac{1}{\sqrt{2}}\mathbf{i} - \dfrac{1}{\sqrt{2}}\mathbf{k}$; $\kappa = \dfrac{1}{\sqrt{2}}$; $\tau = 0$

25. $\dfrac{\pi}{3}$ **27.** $x = 1 + t$, $y = t$, $z = -t$ **31.** $\kappa = \dfrac{1}{a}$

Additional and Advanced Exercises, pp. 790–791

1. (a) $\left.\dfrac{d\theta}{dt}\right|_{\theta=2\pi} = 2\sqrt{\dfrac{\pi g b}{a^2 + b^2}}$

(b) $\theta = \dfrac{gbt^2}{2(a^2 + b^2)}, \quad z = \dfrac{gb^2t^2}{2(a^2 + b^2)}$

(c) $\mathbf{v}(t) = \dfrac{gbt}{\sqrt{a^2 + b^2}}\mathbf{T};$

$\dfrac{d^2\mathbf{r}}{dt^2} = \dfrac{bg}{\sqrt{a^2 + b^2}}\mathbf{T} + a\left(\dfrac{bgt}{a^2 + b^2}\right)^2\mathbf{N}$

There is no component in the direction of **B**.

5. (a) $\dfrac{dx}{dt} = \dot{r}\cos\theta - r\dot{\theta}\sin\theta, \dfrac{dy}{dt} = \dot{r}\sin\theta + r\dot{\theta}\cos\theta$

(b) $\dfrac{dr}{dt} = \dot{x}\cos\theta + \dot{y}\sin\theta, r\dfrac{d\theta}{dt} = -\dot{x}\sin\theta + \dot{y}\cos\theta$

7. (a) $\mathbf{a}(1) = -9\mathbf{u}_r - 6\mathbf{u}_\theta, \mathbf{v}(1) = -\mathbf{u}_r + 3\mathbf{u}_\theta$ **(b)** 6.5 in.

9. (c) $\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta + \dot{z}\mathbf{k}, \mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta + \ddot{z}\mathbf{k}$

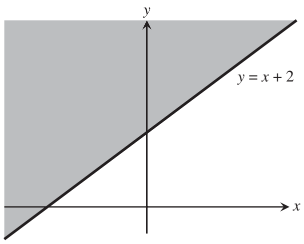
Chapter 14

Section 14.1, pp. 799–801

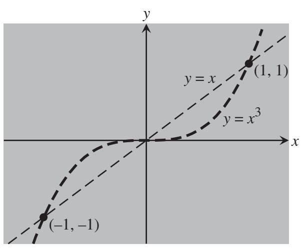
1. (a) 0 **(b)** 0 **(c)** 58 **(d)** 33

3. (a) 4/5 **(b)** 8/5 **(c)** 3 **(d)** 0

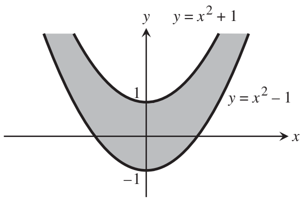
5. Domain: all points (x, y) on or above line $y = x + 2$



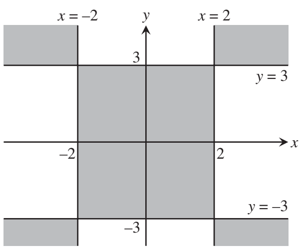
7. Domain: all points (x, y) not lying on the graph of $y = x$ or $y = x^3$



9. Domain: all points (x, y) satisfying $x^2 - 1 \le y \le x^2 + 1$



11. Domain: all points (x, y) for which $(x - 2)(x + 2)(y - 3)(y + 3) \ge 0$



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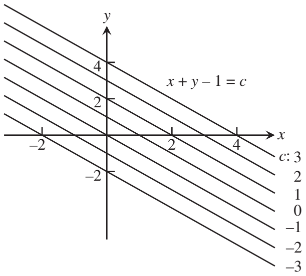
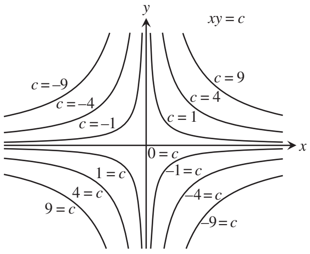


15.



17. (a) All points in the xy-plane **(b)** All reals
(c) The lines $y - x = c$ **(d)** No boundary points
(e) Both open and closed **(f)** Unbounded

19. (a) All points in the xy-plane **(b)** $z \ge 0$
(c) For $f(x, y) = 0$, the origin; for $f(x, y) \ne 0$, ellipses with the center $(0, 0)$, and major and minor axes along the x- and y-axes, respectively
(d) No boundary points **(e)** Both open and closed
(f) Unbounded

21. (a) All points in the xy-plane **(b)** All reals
(c) For $f(x, y) = 0$, the x- and y-axes; for $f(x, y) \ne 0$, hyperbolas with the x- and y-axes as asymptotes
(d) No boundary points **(e)** Both open and closed
(f) Unbounded

23. (a) All (x, y) satisfying $x^2 + y^2 < 16$ **(b)** $z \ge 1/4$
(c) Circles centered at the origin with radii $r < 4$
(d) Boundary is the circle $x^2 + y^2 = 16$
(e) Open **(f)** Bounded

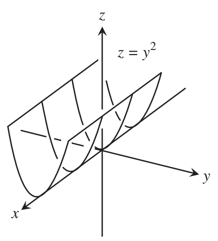
25. (a) $(x, y) \ne (0, 0)$ **(b)** All reals
(c) The circles with center $(0, 0)$ and radii $r > 0$
(d) Boundary is the single point $(0, 0)$
(e) Open **(f)** Unbounded

27. (a) All (x, y) satisfying $-1 \le y - x \le 1$
(b) $-\pi/2 \le z \le \pi/2$
(c) Straight lines of the form $y - x = c$ where $-1 \le c \le 1$
(d) Boundary is two straight lines $y = 1 + x$ and $y = -1 + x$
(e) Closed **(f)** Unbounded

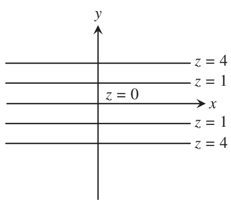
29. (a) Domain: all points (x, y) outside the circle $x^2 + y^2 = 1$
(b) Range: all reals
(c) Circles centered at the origin with radii $r > 1$
(d) Boundary: $x^2 + y^2 = 1$
(e) Open **(f)** Unbounded

31. (f) **33.** (a) **35.** (d)

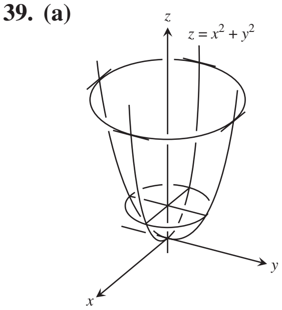
37. (a)



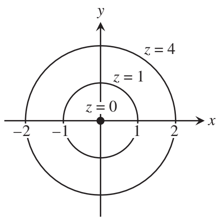
(b)



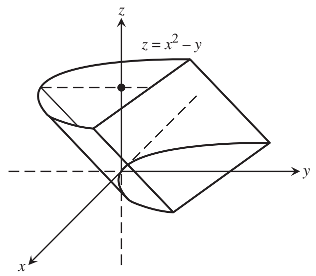
39. (a)



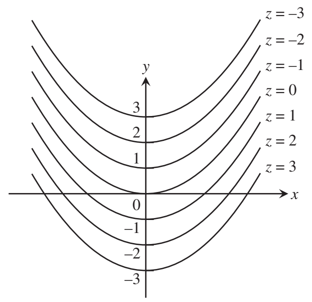
(b)



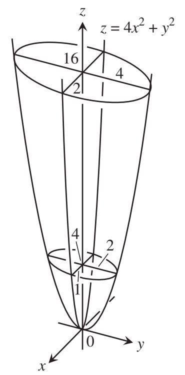
41. (a)



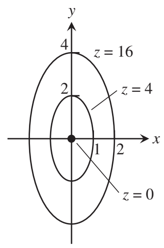
(b)



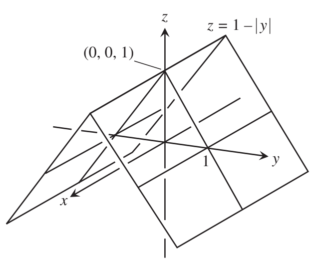
43. (a)



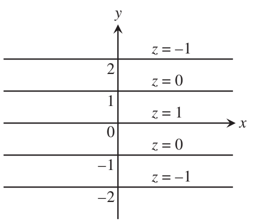
(b)



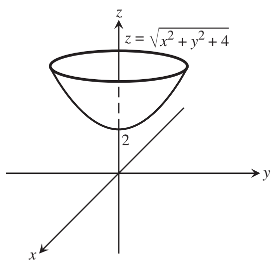
45. (a)



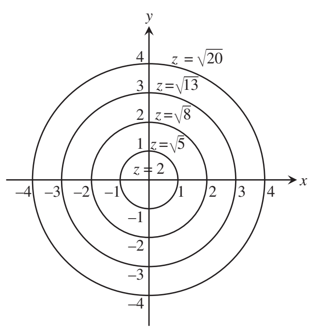
(b)



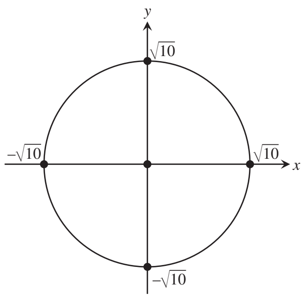
47. (a)



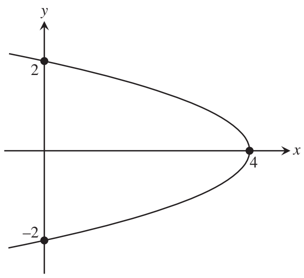
(b)



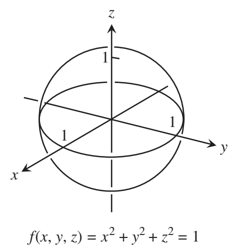
49. $x^2 + y^2 = 10$



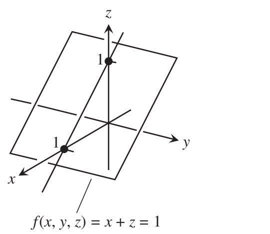
51. $x + y^2 = 4$



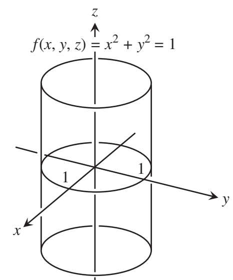
53.



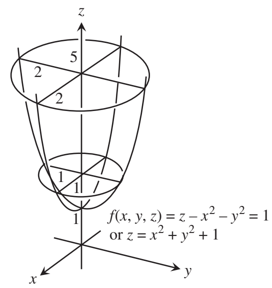
55.



57.

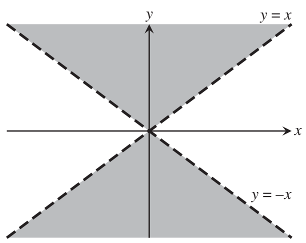


59.



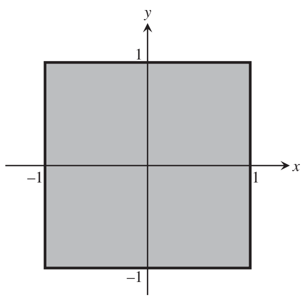
61. $\sqrt{x - y} - \ln z = 2$ **63.** $x^2 + y^2 + z^2 = 4$

65. Domain: all points (x, y) satisfying $|x| < |y|$



level curve: $y = 2x$

67. Domain: all points (x, y) satisfying $-1 \le x \le 1$ and $-1 \le y \le 1$



level curve: $\sin^{-1} y - \sin^{-1} x = \dfrac{\pi}{2}$

Section 14.2, pp. 807–810

1. $5/2$ **3.** $2\sqrt{6}$ **5.** 1 **7.** $1/2$ **9.** 1
11. $1/4$ **13.** 0 **15.** -1 **17.** 2 **19.** $1/4$
21. 1 **23.** 3 **25.** $19/12$ **27.** 2 **29.** 3
31. (a) All (x, y) **(b)** All (x, y) except $(0, 0)$
33. (a) All (x, y) except where $x = 0$ or $y = 0$ **(b)** All (x, y)
35. (a) All (x, y, z)
(b) All (x, y, z) except the interior of the cylinder $x^2 + y^2 = 1$

37. **(a)** All (x, y, z) with $z \neq 0$ **(b)** All (x, y, z) with $x^2 + z^2 \neq 1$

39. **(a)** All points (x, y, z) satisfying $z > x^2 + y^2 + 1$
(b) All points (x, y, z) satisfying $z \neq \sqrt{x^2 + y^2}$

41. Consider paths along $y = x, x > 0$, and along $y = x, x < 0$.

43. Consider the paths $y = kx^2$, k a constant.

45. Consider the paths $y = mx$, m a constant, $m \neq -1$.

47. Consider the paths $y = kx^2$, k a constant, $k \neq 0$.

49. Consider the paths $x = 1$ and $y = x$.

51. **(a)** 1 **(b)** 0 **(c)** Does not exist

55. The limit is 1. **57.** The limit is 0.

59. **(a)** $f(x, y)\big|_{y=mx} = \sin 2\theta$ where $\tan\theta = m$ **61.** 0

63. Does not exist **65.** $\pi/2$ **67.** $f(0, 0) = \ln 3$

69. $\delta = 0.1$ **71.** $\delta = 0.005$ **73.** $\delta = 0.04$

75. $\delta = \sqrt{0.015}$ **77.** $\delta = 0.005$

Section 14.3, pp. 819–821

1. $\frac{\partial f}{\partial x} = 4x, \frac{\partial f}{\partial y} = -3$ **3.** $\frac{\partial f}{\partial x} = 2x(y + 2), \frac{\partial f}{\partial y} = x^2 - 1$

5. $\frac{\partial f}{\partial x} = 2y(xy - 1), \frac{\partial f}{\partial y} = 2x(xy - 1)$

7. $\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}}, \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2 + y^2}}$

9. $\frac{\partial f}{\partial x} = \frac{-1}{(x + y)^2}, \frac{\partial f}{\partial y} = \frac{-1}{(x + y)^2}$

11. $\frac{\partial f}{\partial x} = \frac{-y^2 - 1}{(xy - 1)^2}, \frac{\partial f}{\partial y} = \frac{-x^2 - 1}{(xy - 1)^2}$

13. $\frac{\partial f}{\partial x} = e^{x+y+1}, \frac{\partial f}{\partial y} = e^{x+y+1}$ **15.** $\frac{\partial f}{\partial x} = \frac{1}{x + y}, \frac{\partial f}{\partial y} = \frac{1}{x + y}$

17. $\frac{\partial f}{\partial x} = 2\sin(x - 3y)\cos(x - 3y)$,
$\frac{\partial f}{\partial y} = -6\sin(x - 3y)\cos(x - 3y)$

19. $\frac{\partial f}{\partial x} = yx^{y-1}, \frac{\partial f}{\partial y} = x^y \ln x$ **21.** $\frac{\partial f}{\partial x} = -g(x), \frac{\partial f}{\partial y} = g(y)$

23. $f_x = y^2, f_y = 2xy, f_z = -4z$

25. $f_x = 1, f_y = -y(y^2 + z^2)^{-1/2}, f_z = -z(y^2 + z^2)^{-1/2}$

27. $f_x = \frac{yz}{\sqrt{1 - x^2y^2z^2}}, f_y = \frac{xz}{\sqrt{1 - x^2y^2z^2}}, f_z = \frac{xy}{\sqrt{1 - x^2y^2z^2}}$

29. $f_x = \frac{1}{x + 2y + 3z}, f_y = \frac{2}{x + 2y + 3z}, f_z = \frac{3}{x + 2y + 3z}$

31. $f_x = -2xe^{-(x^2+y^2+z^2)}, f_y = -2ye^{-(x^2+y^2+z^2)}, f_z = -2ze^{-(x^2+y^2+z^2)}$

33. $f_x = \operatorname{sech}^2(x + 2y + 3z), f_y = 2\operatorname{sech}^2(x + 2y + 3z)$,
$f_z = 3\operatorname{sech}^2(x + 2y + 3z)$

35. $\frac{\partial f}{\partial t} = -2\pi\sin(2\pi t - \alpha), \frac{\partial f}{\partial \alpha} = \sin(2\pi t - \alpha)$

37. $\frac{\partial h}{\partial \rho} = \sin\phi\cos\theta, \frac{\partial h}{\partial \phi} = \rho\cos\phi\cos\theta, \frac{\partial h}{\partial \theta} = -\rho\sin\phi\sin\theta$

39. $W_P(P, V, \delta, v, g) = V, W_V(P, V, \delta, v, g) = P + \frac{\delta v^2}{2g}$,
$W_\delta(P, V, \delta, v, g) = \frac{Vv^2}{2g}, W_v(P, V, \delta, v, g) = \frac{V\delta v}{g}$,
$W_g(P, V, \delta, v, g) = -\frac{V\delta v^2}{2g^2}$

41. $\frac{\partial f}{\partial x} = 1 + y, \frac{\partial f}{\partial y} = 1 + x, \frac{\partial^2 f}{\partial x^2} = 0, \frac{\partial^2 f}{\partial y^2} = 0, \frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial^2 f}{\partial x\,\partial y} = 1$

43. $\frac{\partial g}{\partial x} = 2xy + y\cos x, \frac{\partial g}{\partial y} = x^2 - \sin y + \sin x$,
$\frac{\partial^2 g}{\partial x^2} = 2y - y\sin x, \frac{\partial^2 g}{\partial y^2} = -\cos y$,
$\frac{\partial^2 g}{\partial y\,\partial x} = \frac{\partial^2 g}{\partial x\,\partial y} = 2x + \cos x$

45. $\frac{\partial r}{\partial x} = \frac{1}{x + y}, \frac{\partial r}{\partial y} = \frac{1}{x + y}, \frac{\partial^2 r}{\partial x^2} = \frac{-1}{(x + y)^2}, \frac{\partial^2 r}{\partial y^2} = \frac{-1}{(x + y)^2}$,
$\frac{\partial^2 r}{\partial y\,\partial x} = \frac{\partial^2 r}{\partial x\,\partial y} = \frac{-1}{(x + y)^2}$

47. $\frac{\partial w}{\partial x} = x^2y\sec^2(xy) + 2x\tan(xy), \frac{\partial w}{\partial y} = x^3\sec^2(xy)$,
$\frac{\partial^2 w}{\partial y\,\partial x} = \frac{\partial^2 w}{\partial x\,\partial y} = 2x^3y\sec^2(xy)\tan(xy) + 3x^2\sec^2(xy)$
$\frac{\partial^2 w}{\partial x^2} = 4xy\sec^2(xy) + 2x^2y^2\sec^2(xy)\tan(xy) + 2\tan(xy)$
$\frac{\partial^2 w}{\partial y^2} = 2x^4\sec^2(xy)\tan(xy)$

49. $\frac{\partial w}{\partial x} = \sin(x^2y) + 2x^2y\cos(x^2y), \frac{\partial w}{\partial y} = x^3\cos(x^2y)$,
$\frac{\partial^2 w}{\partial y\,\partial x} = \frac{\partial^2 w}{\partial x\,\partial y} = 3x^2\cos(x^2y) - 2x^4y\sin(x^2y)$
$\frac{\partial^2 w}{\partial x^2} = 6xy\cos(x^2y) - 4x^3y^2\sin(x^2y)$
$\frac{\partial^2 w}{\partial y^2} = -x^5\sin(x^2y)$

51. $\frac{\partial w}{\partial x} = \frac{2}{2x + 3y}, \frac{\partial w}{\partial y} = \frac{3}{2x + 3y}, \frac{\partial^2 w}{\partial y\,\partial x} = \frac{\partial^2 w}{\partial x\,\partial y} = \frac{-6}{(2x + 3y)^2}$

53. $\frac{\partial w}{\partial x} = y^2 + 2xy^3 + 3x^2y^4, \frac{\partial w}{\partial y} = 2xy + 3x^2y^2 + 4x^3y^3$,
$\frac{\partial^2 w}{\partial y\,\partial x} = \frac{\partial^2 w}{\partial x\,\partial y} = 2y + 6xy^2 + 12x^2y^3$

55. **(a)** x first **(b)** y first **(c)** x first
(d) x first **(e)** y first **(f)** y first

57. $f_x(1, 2) = -13, f_y(1, 2) = -2$

59. $f_x(-2, 3) = 1/2, f_y(-2, 3) = 3/4$ **61.** **(a)** 3 **(b)** 2

63. 12 **65.** -2 **67.** $\frac{\partial A}{\partial a} = \frac{a}{bc\sin A}, \frac{\partial A}{\partial b} = \frac{c\cos A - b}{bc\sin A}$

69. $v_x = \frac{\ln v}{(\ln u)(\ln v) - 1}$

71. $f_x(x, y) = 0$ for all points (x, y),
$f_y(x, y) = \begin{cases} 3y^2, & y \geq 0 \\ -2y, & y < 0 \end{cases}$,
$f_{xy}(x, y) = f_{yx}(x, y) = 0$ for all points (x, y)

89. Yes

Section 14.4, pp. 828–830

1. **(a)** $\frac{dw}{dt} = 0$, **(b)** $\frac{dw}{dt}(\pi) = 0$

3. **(a)** $\frac{dw}{dt} = 1$, **(b)** $\frac{dw}{dt}(3) = 1$

5. (a) $\dfrac{dw}{dt} = 4t\tan^{-1}t + 1$, **(b)** $\dfrac{dw}{dt}(1) = \pi + 1$

7. (a) $\dfrac{\partial z}{\partial u} = 4\cos v \ln(u \sin v) + 4\cos v$,

$\dfrac{\partial z}{\partial v} = -4u \sin v \ln(u \sin v) + \dfrac{4u\cos^2 v}{\sin v}$

(b) $\dfrac{\partial z}{\partial u} = \sqrt{2}(\ln 2 + 2)$, $\dfrac{\partial z}{\partial v} = -2\sqrt{2}(\ln 2 - 2)$

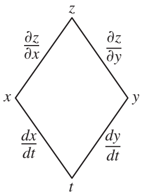
9. (a) $\dfrac{\partial w}{\partial u} = 2u + 4uv$, $\dfrac{\partial w}{\partial v} = -2v + 2u^2$

(b) $\dfrac{\partial w}{\partial u} = 3$, $\dfrac{\partial w}{\partial v} = -\dfrac{3}{2}$

11. (a) $\dfrac{\partial u}{\partial x} = 0$, $\dfrac{\partial u}{\partial y} = \dfrac{z}{(z-y)^2}$, $\dfrac{\partial u}{\partial z} = \dfrac{-y}{(z-y)^2}$

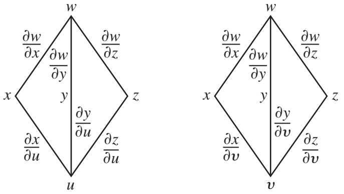
(b) $\dfrac{\partial u}{\partial x} = 0$, $\dfrac{\partial u}{\partial y} = 1$, $\dfrac{\partial u}{\partial z} = -2$

13. $\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\dfrac{dy}{dt}$

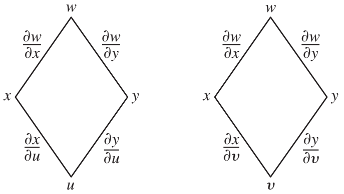


15. $\dfrac{\partial w}{\partial u} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial u}$,

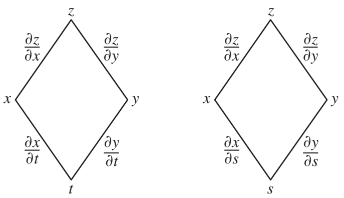
$\dfrac{\partial w}{\partial v} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial v} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial v}$



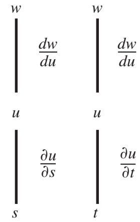
17. $\dfrac{\partial w}{\partial u} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u}$, $\dfrac{\partial w}{\partial v} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial v}$.



19. $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial t}$, $\dfrac{\partial z}{\partial s} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial s}$

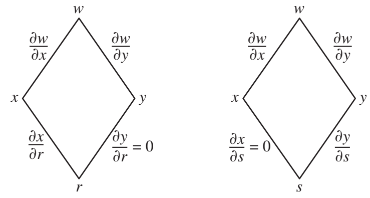


21. $\dfrac{\partial w}{\partial s} = \dfrac{dw}{du}\dfrac{\partial u}{\partial s}$, $\dfrac{\partial w}{\partial t} = \dfrac{dw}{du}\dfrac{\partial u}{\partial t}$



23. $\dfrac{\partial w}{\partial r} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial r} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial r} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial r}$ since $\dfrac{\partial y}{\partial r} = 0$,

$\dfrac{\partial w}{\partial s} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial s} = \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial s}$ since $\dfrac{\partial x}{\partial s} = 0$



25. $4/3$ **27.** $-4/5$ **29.** $\dfrac{\partial z}{\partial x} = \dfrac{1}{4}$, $\dfrac{\partial z}{\partial y} = -\dfrac{3}{4}$

31. $\dfrac{\partial z}{\partial x} = -1$, $\dfrac{\partial z}{\partial y} = -1$ **33.** 12 **35.** -7

37. $\dfrac{\partial z}{\partial u} = 2$, $\dfrac{\partial z}{\partial v} = 1$ **39.** $\dfrac{\partial w}{\partial t} = 2t\, e^{s^3+t^2}$, $\dfrac{\partial w}{\partial s} = 3s^2\, e^{s^3+t^2}$

41. -0.00005 amp/sec

47. $(\cos 1, \sin 1, 1)$ and $(\cos(-2), \sin(-2), -2)$

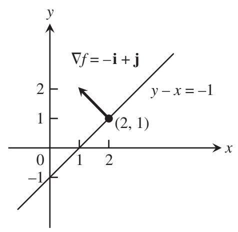
49. (a) Maximum at $\left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$ and $\left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$; minimum at $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$ and $\left(-\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$

(b) Max $= 6$, min $= 2$

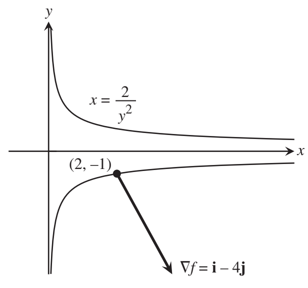
51. $2x\sqrt{x^8 + x^3} + \displaystyle\int_0^{x^2} \dfrac{3x^2}{2\sqrt{t^4 + x^3}}\, dt$

Section 14.5, p. 888

1.



3.



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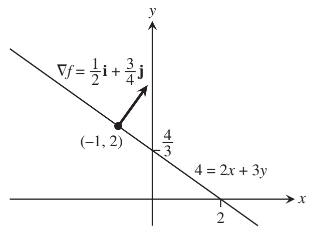


7. $\nabla f = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ **9.** $\nabla f = -\frac{26}{27}\mathbf{i} + \frac{23}{54}\mathbf{j} - \frac{23}{54}\mathbf{k}$

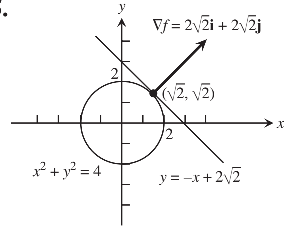
11. -4 **13.** $21/13$ **15.** 3 **17.** 2

19. $\mathbf{u} = -\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$, $(D_{\mathbf{u}}f)_{P_0} = \sqrt{2}$; $-\mathbf{u} = \frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$, $(D_{-\mathbf{u}}f)_{P_0} = -\sqrt{2}$

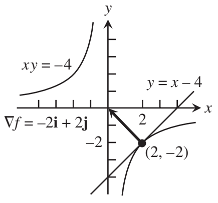
21. $\mathbf{u} = \frac{1}{3\sqrt{3}}\mathbf{i} - \frac{5}{3\sqrt{3}}\mathbf{j} - \frac{1}{3\sqrt{3}}\mathbf{k}$, $(D_{\mathbf{u}}f)_{P_0} = 3\sqrt{3}$; $-\mathbf{u} = -\frac{1}{3\sqrt{3}}\mathbf{i} + \frac{5}{3\sqrt{3}}\mathbf{j} + \frac{1}{3\sqrt{3}}\mathbf{k}$, $(D_{-\mathbf{u}}f)_{P_0} = -3\sqrt{3}$

23. $\mathbf{u} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$, $(D_{\mathbf{u}}f)_{P_0} = 2\sqrt{3}$; $-\mathbf{u} = -\frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$, $(D_{-\mathbf{u}}f)_{P_0} = -2\sqrt{3}$

25.



27.



29. **(a)** $\mathbf{u} = \frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$, $D_{\mathbf{u}}f(1, -1) = 5$
(b) $\mathbf{u} = -\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$, $D_{\mathbf{u}}f(1, -1) = -5$
(c) $\mathbf{u} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$, $\mathbf{u} = -\frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$
(d) $\mathbf{u} = -\mathbf{j}$, $\mathbf{u} = \frac{24}{25}\mathbf{i} - \frac{7}{25}\mathbf{j}$
(e) $\mathbf{u} = -\mathbf{i}$, $\mathbf{u} = \frac{7}{25}\mathbf{i} + \frac{24}{25}\mathbf{j}$

31. $\mathbf{u} = \frac{7}{\sqrt{53}}\mathbf{i} - \frac{2}{\sqrt{53}}\mathbf{j}$, $-\mathbf{u} = -\frac{7}{\sqrt{53}}\mathbf{i} + \frac{2}{\sqrt{53}}\mathbf{j}$

33. No, the maximum rate of change is $\sqrt{185} < 14$.

35. $-7/\sqrt{5}$

Section 14.6, pp. 845–848

1. **(a)** $x + y + z = 3$
(b) $x = 1 + 2t$, $y = 1 + 2t$, $z = 1 + 2t$

3. **(a)** $2x - z - 2 = 0$
(b) $x = 2 - 4t$, $y = 0$, $z = 2 + 2t$

5. **(a)** $2x + 2y + z - 4 = 0$
(b) $x = 2t$, $y = 1 + 2t$, $z = 2 + t$

7. **(a)** $x + y + z - 1 = 0$ **(b)** $x = t$, $y = 1 + t$, $z = t$

9. $2x - z - 2 = 0$ **11.** $x - y + 2z - 1 = 0$

13. $x = 1$, $y = 1 + 2t$, $z = 1 - 2t$

15. $x = 1 - 2t$, $y = 1$, $z = \frac{1}{2} + 2t$

17. $x = 1 + 90t$, $y = 1 - 90t$, $z = 3$

19. $df = \frac{9}{11{,}830} \approx 0.0008$ **21.** $dg = 0$

23. **(a)** $\frac{\sqrt{3}}{2}\sin\sqrt{3} - \frac{1}{2}\cos\sqrt{3} \approx 0.935°\text{C/ft}$
(b) $\sqrt{3}\sin\sqrt{3} - \cos\sqrt{3} \approx 1.87°\text{C/sec}$

25. **(a)** $L(x, y) = 1$ **(b)** $L(x, y) = 2x + 2y - 1$

27. **(a)** $L(x, y) = 3x - 4y + 5$
(b) $L(x, y) = 3x - 4y + 5$

29. **(a)** $L(x, y) = 1 + x$ **(b)** $L(x, y) = -y + \frac{\pi}{2}$

31. **(a)** $W(20, 25) = 11°\text{F}$, $W(30, -10) = -39°\text{F}$, $W(15, 15) = 0°\text{F}$
(b) $W(10, -40) \approx -65.5°\text{F}$, $W(50, -40) \approx -88°\text{F}$, $W(60, 30) \approx 10.2°\text{F}$
(c) $L(v, T) \approx -0.36(v - 25) + 1.337(T - 5) - 17.4088$
(d) **i)** $L(24, 6) \approx -15.7°\text{F}$
ii) $L(27, 2) \approx -22.1°\text{F}$
iii) $L(5, -10) \approx -30.2°\text{F}$

33. $L(x, y) = 7 + x - 6y$; 0.06 **35.** $L(x, y) = x + y + 1$; 0.08

37. $L(x, y) = 1 + x$; 0.0222

39. **(a)** $L(x, y, z) = 2x + 2y + 2z - 3$ **(b)** $L(x, y, z) = y + z$
(c) $L(x, y, z) = 0$

41. **(a)** $L(x, y, z) = x$ **(b)** $L(x, y, z) = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y$
(c) $L(x, y, z) = \frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z$

43. **(a)** $L(x, y, z) = 2 + x$
(b) $L(x, y, z) = x - y - z + \frac{\pi}{2} + 1$
(c) $L(x, y, z) = x - y - z + \frac{\pi}{2} + 1$

45. $L(x, y, z) = 2x - 6y - 2z + 6$, 0.0024

47. $L(x, y, z) = x + y - z - 1$, 0.00135

49. Maximum error (estimate) ≤ 0.31 in magnitude

51. Pay more attention to the smaller of the two dimensions. It will generate the larger partial derivative.

53. f is most sensitive to a change in d.

Section 14.7, pp. 855–857

1. $f(-3, 3) = -5$, local minimum **3.** $f(-2, 1)$, saddle point

5. $f\left(3, \frac{3}{2}\right) = \frac{17}{2}$, local maximum

7. $f(2, -1) = -6$, local minimum **9.** $f(1, 2)$, saddle point

11. $f\left(\frac{16}{7}, 0\right) = -\frac{16}{7}$, local maximum

13. $f(0, 0)$, saddle point; $f\left(-\frac{2}{3}, \frac{2}{3}\right) = \frac{170}{27}$, local maximum

15. $f(0, 0) = 0$, local minimum; $f(1, -1)$, saddle point

17. $f(0, \pm\sqrt{5})$, saddle points; $f(-2, -1) = 30$, local maximum; $f(2, 1) = -30$, local minimum

19. $f(0, 0)$, saddle point; $f(1, 1) = 2$, $f(-1, -1) = 2$, local maxima

21. $f(0, 0) = -1$, local maximum

23. $f(n\pi, 0)$, saddle points, for every integer n

25. $f(2, 0) = e^{-4}$, local minimum

27. $f(0, 0) = 0$, local minimum; $f(0, 2)$, saddle point

29. $f\left(\frac{1}{2}, 1\right) = \ln\left(\frac{1}{4}\right) - 3$, local maximum

31. Absolute maximum: 1 at $(0, 0)$; absolute minimum: -5 at $(1, 2)$

33. Absolute maximum: 4 at $(0, 2)$; absolute minimum: 0 at $(0, 0)$

35. Absolute maximum: 11 at $(0, -3)$; absolute minimum: -10 at $(4, -2)$

37. Absolute maximum: 4 at (2, 0); absolute minimum: $\frac{3\sqrt{2}}{2}$ at $\left(3, -\frac{\pi}{4}\right), \left(3, \frac{\pi}{4}\right), \left(1, -\frac{\pi}{4}\right)$, and $\left(1, \frac{\pi}{4}\right)$

39. $a = -3, b = 2$

41. Hottest is $2\frac{1}{4}^\circ$ at $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$; coldest is $-\frac{1}{4}^\circ$ at $\left(\frac{1}{2}, 0\right)$.

43. **(a)** $f(0, 0)$, saddle point **(b)** $f(1, 2)$, local minimum **(c)** $f(1, -2)$, local minimum; $f(-1, -2)$, saddle point

49. $\left(\frac{1}{6}, \frac{1}{3}, \frac{355}{36}\right)$ **51.** $\left(\frac{9}{7}, \frac{6}{7}, \frac{3}{7}\right)$ **53.** 3, 3, 3 **55.** 12

57. $\frac{4}{\sqrt{3}} \times \frac{4}{\sqrt{3}} \times \frac{4}{\sqrt{3}}$ **59.** 2 ft × 2 ft × 1 ft

61. **(a)** On the semicircle, max $f = 2\sqrt{2}$ at $t = \pi/4$, min $f = -2$ at $t = \pi$. On the quarter circle, max $f = 2\sqrt{2}$ at $t = \pi/4$, min $f = 2$ at $t = 0, \pi/2$.
(b) On the semicircle, max $g = 2$ at $t = \pi/4$, min $g = -2$ at $t = 3\pi/4$. On the quarter circle, max $g = 2$ at $t = \pi/4$, min $g = 0$ at $t = 0, \pi/2$.
(c) On the semicircle, max $h = 8$ at $t = 0, \pi$; min $h = 4$ at $t = \pi/2$. On the quarter circle, max $h = 8$ at $t = 0$, min $h = 4$ at $t = \pi/2$.

63. **i)** min $f = -1/2$ at $t = -1/2$; no max
ii) max $f = 0$ at $t = -1, 0$; min $f = -1/2$ at $t = -1/2$
iii) max $f = 4$ at $t = 1$; min $f = 0$ at $t = 0$

67. $y = -\frac{20}{13}x + \frac{9}{13}$, $y|_{x=4} = -\frac{71}{13}$

Section 14.8, pp. 864–866

1. $\left(\pm\frac{1}{\sqrt{2}}, \frac{1}{2}\right), \left(\pm\frac{1}{\sqrt{2}}, -\frac{1}{2}\right)$ **3.** 39 **5.** $\left(3, \pm 3\sqrt{2}\right)$

7. **(a)** 8 **(b)** 64

9. $r = 2$ cm, $h = 4$ cm **11.** Length $= 4\sqrt{2}$, width $= 3\sqrt{2}$

13. $f(0, 0) = 0$ is minimum; $f(2, 4) = 20$ is maximum.

15. Lowest $= 0°$, highest $= 125°$

17. $\left(\frac{3}{2}, 2, \frac{5}{2}\right)$ **19.** 1 **21.** (0, 0, 2), (0, 0, −2)

23. $f(1, -2, 5) = 30$ is maximum; $f(-1, 2, -5) = -30$ is minimum.

25. 3, 3, 3 **27.** $\frac{2}{\sqrt{3}}$ by $\frac{2}{\sqrt{3}}$ by $\frac{2}{\sqrt{3}}$ units

29. $(\pm 4/3, -4/3, -4/3)$ **31.** ≈24,322 units

33. $U(8, 14) = \$128$ **37.** $f(2/3, 4/3, -4/3) = \frac{4}{3}$

39. (2, 4, 4) **41.** Maximum is $1 + 6\sqrt{3}$ at $\left(\pm\sqrt{6}, \sqrt{3}, 1\right)$; minimum is $1 - 6\sqrt{3}$ at $\left(\pm\sqrt{6}, -\sqrt{3}, 1\right)$.

43. Maximum is 4 at $(0, 0, \pm 2)$; minimum is 2 at $\left(\pm\sqrt{2}, \pm\sqrt{2}, 0\right)$.

Section 14.9, p. 870

1. Quadratic: $x + xy$; cubic: $x + xy + \frac{1}{2}xy^2$

3. Quadratic: xy; cubic: xy

5. Quadratic: $y + \frac{1}{2}(2xy - y^2)$;
cubic: $y + \frac{1}{2}(2xy - y^2) + \frac{1}{6}(3x^2y - 3xy^2 + 2y^3)$

7. Quadratic: $\frac{1}{2}(2x^2 + 2y^2) = x^2 + y^2$; cubic: $x^2 + y^2$

9. Quadratic: $1 + (x + y) + (x + y)^2$;
cubic: $1 + (x + y) + (x + y)^2 + (x + y)^3$

11. Quadratic: $1 - \frac{1}{2}x^2 - \frac{1}{2}y^2$; $E(x, y) \le 0.00134$

Section 14.10, pp. 874–875

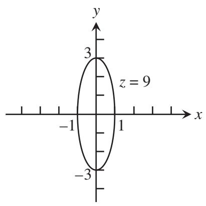
1. **(a)** 0 **(b)** $1 + 2z$ **(c)** $1 + 2z$

3. **(a)** $\frac{\partial U}{\partial P} + \frac{\partial U}{\partial T}\left(\frac{V}{nR}\right)$ **(b)** $\frac{\partial U}{\partial P}\left(\frac{nR}{V}\right) + \frac{\partial U}{\partial T}$

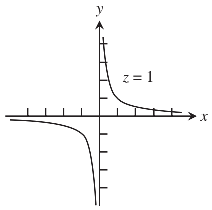
5. **(a)** 5 **(b)** 5 **7.** $\left(\frac{\partial x}{\partial r}\right)_\theta = \cos\theta \quad \left(\frac{\partial r}{\partial x}\right)_y = \frac{x}{\sqrt{x^2 + y^2}}$

Practice Exercises, pp. 876–879

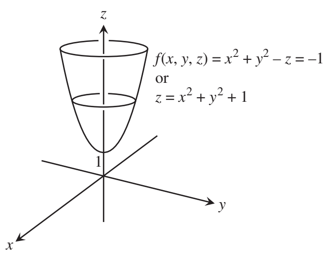
1. Domain: all points in the xy-plane; range: $z \ge 0$. Level curves are ellipses with major axis along the y-axis and minor axis along the x-axis.



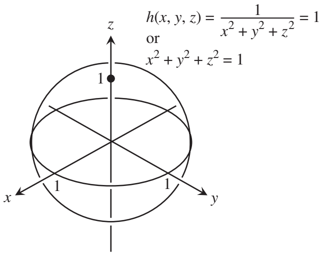
3. Domain: all (x, y) such that $x \ne 0$ and $y \ne 0$; range: $z \ne 0$. Level curves are hyperbolas with the x- and y-axes as asymptotes.



5. Domain: all points in xyz-space; range: all real numbers. Level surfaces are paraboloids of revolution with the z-axis as axis.



7. Domain: all (x, y, z) such that $(x, y, z) \neq (0, 0, 0)$; range: positive real numbers. Level surfaces are spheres with center $(0, 0, 0)$ and radius $r > 0$.



9. -2 **11.** $1/2$ **13.** 1 **15.** Let $y = kx^2, k \neq 1$

17. No; $\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist.

19. $\dfrac{\partial g}{\partial r} = \cos\theta + \sin\theta, \dfrac{\partial g}{\partial \theta} = -r\sin\theta + r\cos\theta$

21. $\dfrac{\partial f}{\partial R_1} = -\dfrac{1}{R_1{}^2}, \dfrac{\partial f}{\partial R_2} = -\dfrac{1}{R_2{}^2}, \dfrac{\partial f}{\partial R_3} = -\dfrac{1}{R_3{}^2}$

23. $\dfrac{\partial P}{\partial n} = \dfrac{RT}{V}, \dfrac{\partial P}{\partial R} = \dfrac{nT}{V}, \dfrac{\partial P}{\partial T} = \dfrac{nR}{V}, \dfrac{\partial P}{\partial V} = -\dfrac{nRT}{V^2}$

25. $\dfrac{\partial^2 g}{\partial x^2} = 0, \dfrac{\partial^2 g}{\partial y^2} = \dfrac{2x}{y^3}, \dfrac{\partial^2 g}{\partial y\,\partial x} = \dfrac{\partial^2 g}{\partial x\,\partial y} = -\dfrac{1}{y^2}$

27. $\dfrac{\partial^2 f}{\partial x^2} = -30x + \dfrac{2 - 2x^2}{(x^2+1)^2}, \dfrac{\partial^2 f}{\partial y^2} = 0, \dfrac{\partial^2 f}{\partial y\,\partial x} = \dfrac{\partial^2 f}{\partial x\,\partial y} = 1$

29. $\left.\dfrac{dw}{dt}\right|_{t=0} = -1$

31. $\left.\dfrac{\partial w}{\partial r}\right|_{(r,s)=(\pi,0)} = 2, \left.\dfrac{\partial w}{\partial s}\right|_{(r,s)=(\pi,0)} = 2 - \pi$

33. $\left.\dfrac{df}{dt}\right|_{t=1} = -(\sin 1 + \cos 2)(\sin 1) + (\cos 1 + \cos 2)(\cos 1) - 2(\sin 1 + \cos 1)(\sin 2)$

35. $\left.\dfrac{dy}{dx}\right|_{(x,y)=(0,1)} = -1$

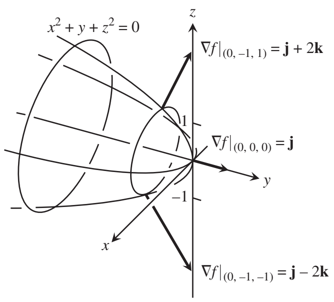
37. Increases most rapidly in the direction $\mathbf{u} = -\dfrac{\sqrt{2}}{2}\mathbf{i} - \dfrac{\sqrt{2}}{2}\mathbf{j}$; decreases most rapidly in the direction $-\mathbf{u} = \dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j}$; $D_{\mathbf{u}}f = \dfrac{\sqrt{2}}{2}; D_{-\mathbf{u}}f = -\dfrac{\sqrt{2}}{2}; D_{\mathbf{u}_1}f = -\dfrac{7}{10}$ where $\mathbf{u}_1 = \dfrac{\mathbf{v}}{|\mathbf{v}|}$

39. Increases most rapidly in the direction $\mathbf{u} = \dfrac{2}{7}\mathbf{i} + \dfrac{3}{7}\mathbf{j} + \dfrac{6}{7}\mathbf{k}$; decreases most rapidly in the direction $-\mathbf{u} = -\dfrac{2}{7}\mathbf{i} - \dfrac{3}{7}\mathbf{j} - \dfrac{6}{7}\mathbf{k}$; $D_{\mathbf{u}}f = 7; D_{-\mathbf{u}}f = -7; D_{\mathbf{u}_1}f = 7$ where $\mathbf{u}_1 = \dfrac{\mathbf{v}}{|\mathbf{v}|}$

41. $\pi/\sqrt{2}$

43. **(a)** $f_x(1, 2) = f_y(1, 2) = 2$ **(b)** $14/5$

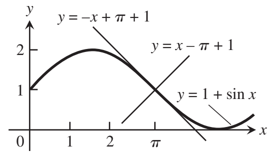
45.



47. Tangent: $4x - y - 5z = 4$; normal line: $x = 2 + 4t, y = -1 - t, z = 1 - 5t$

49. $2y - z - 2 = 0$

51. Tangent: $x + y = \pi + 1$; normal line: $y = x - \pi + 1$



53. $x = 1 - 2t, y = 1, z = 1/2 + 2t$

55. Answers will depend on the upper bound used for $|f_{xx}|, |f_{xy}|, |f_{yy}|$. With $M = \sqrt{2}/2$, $|E| \leq 0.0142$. With $M = 1$, $|E| \leq 0.02$.

57. $L(x, y, z) = y - 3z, L(x, y, z) = x + y - z - 1$

59. Be more careful with the diameter.

61. $dI = 0.038$, % change in $I = 15.83\%$, more sensitive to voltage change

63. **(a)** 5% **65.** Local minimum of -8 at $(-2, -2)$

67. Saddle point at $(0, 0)$, $f(0, 0) = 0$; local maximum of $1/4$ at $(-1/2, -1/2)$

69. Saddle point at $(0, 0)$, $f(0, 0) = 0$; local minimum of -4 at $(0, 2)$; local maximum of 4 at $(-2, 0)$; saddle point at $(-2, 2)$, $f(-2, 2) = 0$

71. Absolute maximum: 28 at $(0, 4)$; absolute minimum: $-9/4$ at $(3/2, 0)$

73. Absolute maximum: 18 at $(2, -2)$; absolute minimum: $-17/4$ at $(-2, 1/2)$

75. Absolute maximum: 8 at $(-2, 0)$; absolute minimum: -1 at $(1, 0)$

77. Absolute maximum: 4 at $(1, 0)$; absolute minimum: -4 at $(0, -1)$

79. Absolute maximum: 1 at $(0, \pm 1)$ and $(1, 0)$; absolute minimum: -1 at $(-1, 0)$

81. Maximum: 5 at $(0, 1)$; minimum: $-1/3$ at $(0, -1/3)$

83. Maximum: $\sqrt{3}$ at $\left(\dfrac{1}{\sqrt{3}}, -\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$; minimum: $-\sqrt{3}$ at $\left(-\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, -\dfrac{1}{\sqrt{3}}\right)$

85. Width $= \left(\dfrac{c^2V}{ab}\right)^{1/3}$, depth $= \left(\dfrac{b^2V}{ac}\right)^{1/3}$, height $= \left(\dfrac{a^2V}{bc}\right)^{1/3}$

87. Maximum: $\frac{3}{2}$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \sqrt{2}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\sqrt{2}\right)$; minimum: $\frac{1}{2}$ at $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\sqrt{2}\right)$ and $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \sqrt{2}\right)$

89. $\frac{\partial w}{\partial x} = \cos\theta \frac{\partial w}{\partial r} - \frac{\sin\theta}{r}\frac{\partial w}{\partial \theta}, \frac{\partial w}{\partial y} = \sin\theta \frac{\partial w}{\partial r} + \frac{\cos\theta}{r}\frac{\partial w}{\partial \theta}$

95. $(t, -t \pm 4, t)$, t a real number

101. **(a)** $(2y + x^2z)e^{yz}$ **(b)** $x^2e^{yz}\left(y - \frac{z}{2y}\right)$ **(c)** $(1 + x^2y)e^{yz}$

Additional and Advanced Exercises, pp. 879–881

1. $f_{xy}(0, 0) = -1, f_{yx}(0, 0) = 1$

7. (c) $\frac{r^2}{2} = \frac{1}{2}(x^2 + y^2 + z^2)$ **13.** $V = \frac{\sqrt{3}abc}{2}$

17. $f(x, y) = \frac{y}{2} + 4, g(x, y) = \frac{x}{2} + \frac{9}{2}$

19. $y = 2\ln|\sin x| + \ln 2$

21. **(a)** $\frac{1}{\sqrt{53}}(2\mathbf{i} + 7\mathbf{j})$ **(b)** $\frac{-1}{\sqrt{29{,}097}}(98\mathbf{i} - 127\mathbf{j} + 58\mathbf{k})$

23. $w = e^{-c^2\pi^2 t}\sin \pi x$

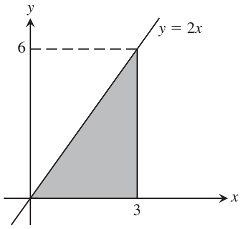
Chapter 15

Section 15.1, pp. 886–887

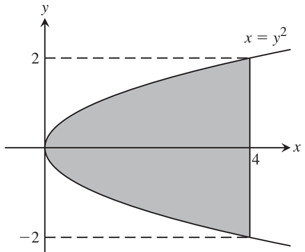
1. 24 **3.** 1 **5.** 16 **7.** $2\ln 2 - 1$ **9.** $(3/2)(5 - e)$
11. $3/2$ **13.** $\ln 2$ **15.** 14 **17.** 0 **19.** $1/2$
21. $2\ln 2$ **23.** $(\ln 2)^2$ **25.** $8/3$ **27.** 1 **29.** $\sqrt{2}$
31. $2/27$ **33.** $\frac{3}{2}\ln 3 - 1$ **35.** **(a)** $1/3$ **(b)** $2/3$

Section 15.2, pp. 894–896

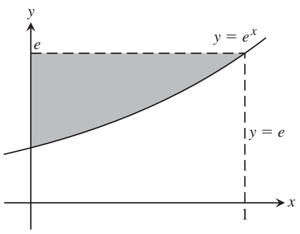
1.



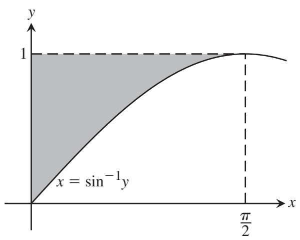
3.



5.



7.



9. **(a)** $0 \le x \le 2, x^3 \le y \le 8$
(b) $0 \le y \le 8, 0 \le x \le y^{1/3}$

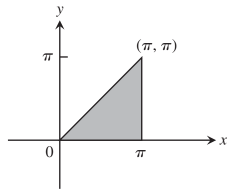
11. **(a)** $0 \le x \le 3, x^2 \le y \le 3x$
(b) $0 \le y \le 9, \frac{y}{3} \le x \le \sqrt{y}$

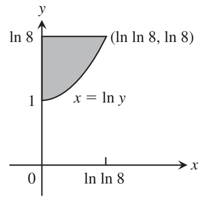
13. **(a)** $0 \le x \le 9, 0 \le y \le \sqrt{x}$
(b) $0 \le y \le 3, y^2 \le x \le 9$

15. **(a)** $0 \le x \le \ln 3, e^{-x} \le y \le 1$
(b) $\frac{1}{3} \le y \le 1, -\ln y \le x \le \ln 3$

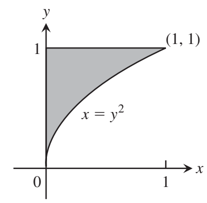
17. **(a)** $0 \le x \le 1, x \le y \le 3 - 2x$
(b) $0 \le y \le 1, 0 \le x \le y \cup 1 \le y \le 3, 0 \le x \le \frac{3 - y}{2}$

19. $\frac{\pi^2}{2} + 2$ **21.** $8\ln 8 - 16 + e$



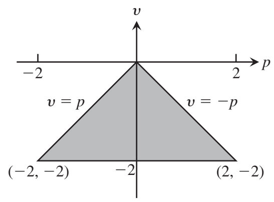


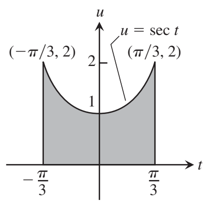
23. $e - 2$



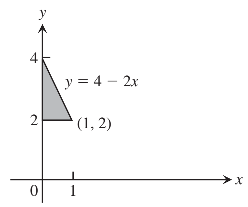
25. $\frac{3}{2}\ln 2$ **27.** $-1/10$

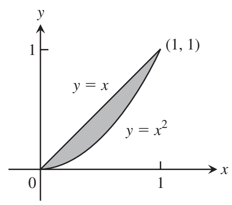
29. 8 **31.** 2π



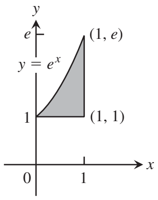


33. $\int_2^4 \int_0^{(4-y)/2} dx\, dy$ **35.** $\int_0^1 \int_{x^2}^{x} dy\, dx$

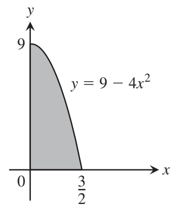




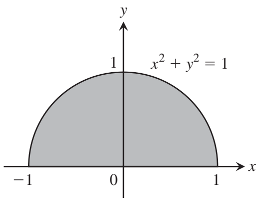
37. $\int_1^e \int_{\ln y}^1 dx\, dy$



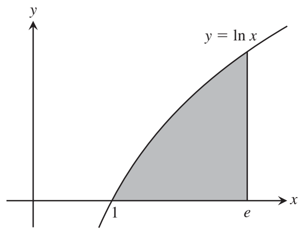
39. $\int_0^9 \int_0^{(\sqrt{9-y})/2} 16x\, dx\, dy$



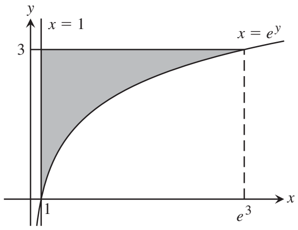
41. $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} 3y\, dy\, dx$



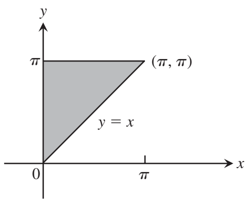
43. $\int_0^1 \int_{e^y}^e xy\, dx\, dy$



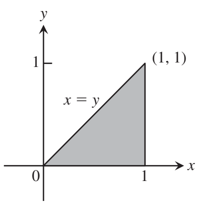
45. $\int_1^{e^3} \int_{\ln x}^3 (x + y)\, dy\, dx$



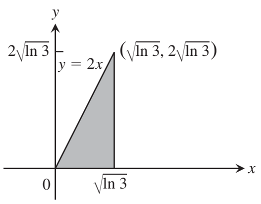
47. 2



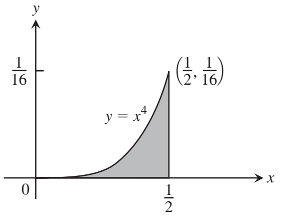
49. $\frac{e-2}{2}$



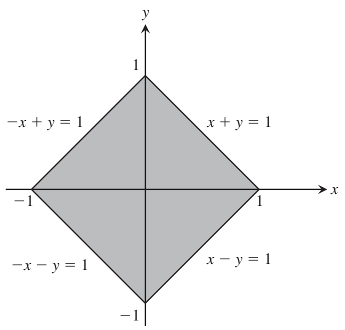
51. 2



53. $1/(80\pi)$

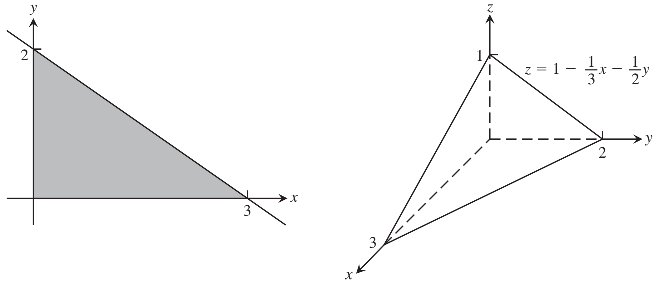


55. $-2/3$



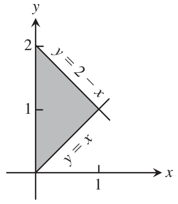
57. $4/3$ **59.** $625/12$ **61.** 16 **63.** 20 **65.** $2(1 + \ln 2)$

67.



69. 1 **71.** π^2 **73.** $-\frac{3}{32}$ **75.** $\frac{20\sqrt{3}}{9}$

77. $\int_0^1 \int_x^{2-x} (x^2 + y^2)\, dy\, dx = \frac{4}{3}$



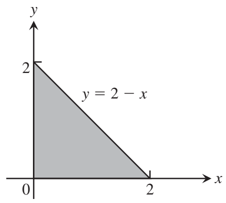
79. R is the set of points (x, y) such that $x^2 + 2y^2 < 4$.

81. No, by Fubini's Theorem, the two orders of integration must give the same result.

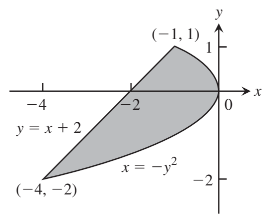
85. 0.603 **87.** 0.233

Section 15.3, p. 899

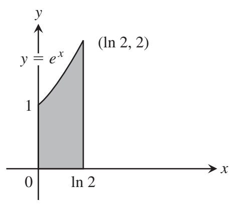
1. $\int_0^2 \int_0^{2-x} dy\, dx = 2$ or $\int_0^2 \int_0^{2-y} dx\, dy = 2$



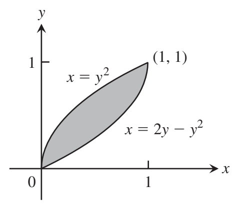
3. $\int_{-2}^1 \int_{y-2}^{-y^2} dx\, dy = \frac{9}{2}$



5. $\int_0^{\ln 2} \int_0^{e^x} dy\, dx = 1$

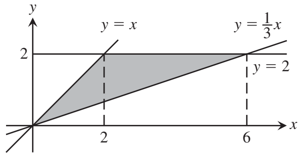


7. $\int_0^1 \int_{y^2}^{2y-y^2} dx\, dy = \frac{1}{3}$



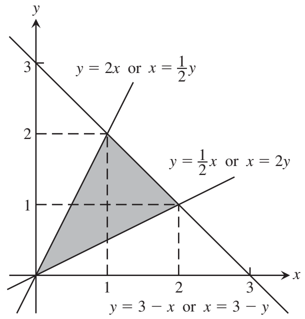
9. $\int_0^2 \int_y^{3y} 1\,dx\,dy = 4$ or

$\int_0^2 \int_{x/3}^{x} 1\,dy\,dx + \int_2^6 \int_{x/3}^{2} 1\,dy\,dx = 4$

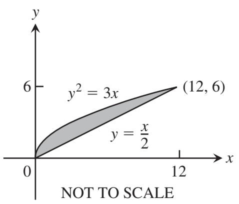


11. $\int_0^1 \int_{x/2}^{2x} 1\,dy\,dx + \int_1^2 \int_{x/2}^{3-x} 1\,dy\,dx = \frac{3}{2}$ or

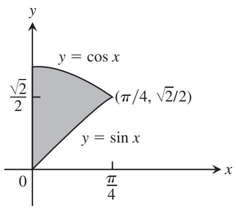
$\int_0^1 \int_{y/2}^{2y} 1\,dx\,dy + \int_1^2 \int_{y/2}^{3-y} 1\,dx\,dy = \frac{3}{2}$



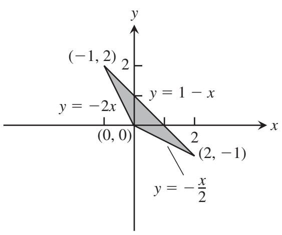
13. 12



15. $\sqrt{2} - 1$



17. $\frac{3}{2}$



19. **(a)** 0 **(b)** $4/\pi^2$ **21.** $8/3$ **23.** $\pi - 2$

25. $40{,}000(1 - e^{-2})\ln(7/2) \approx 43{,}329$

Section 15.4, pp. 904–906

1. $\frac{\pi}{2} \le \theta \le 2\pi, 0 \le r \le 9$ **3.** $\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}, 0 \le r \le \csc\theta$

5. $0 \le \theta \le \frac{\pi}{6}, 1 \le r \le 2\sqrt{3}\sec\theta$;

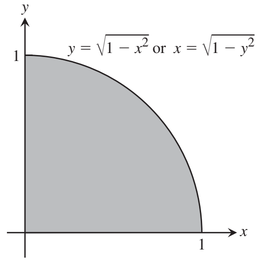
$\frac{\pi}{6} \le \theta \le \frac{\pi}{2}, 1 \le r \le 2\csc\theta$

7. $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, 0 \le r \le 2\cos\theta$ **9.** $\frac{\pi}{2}$

11. 2π **13.** 36 **15.** $2 - \sqrt{3}$ **17.** $(1 - \ln 2)\pi$

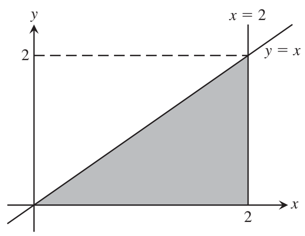
19. $(2\ln 2 - 1)(\pi/2)$ **21.** $\frac{2(1 + \sqrt{2})}{3}$

23.



$\int_0^1 \int_0^{\sqrt{1-x^2}} xy\,dy\,dx$ or $\int_0^1 \int_0^{\sqrt{1-y^2}} xy\,dx\,dy$

25.



$\int_0^2 \int_0^x y^2(x^2 + y^2)\,dy\,dx$ or $\int_0^2 \int_y^2 y^2(x^2 + y^2)\,dx\,dy$

27. $2(\pi - 2)$ **29.** 12π **31.** $(3\pi/8) + 1$ **33.** $\frac{2a}{3}$

35. $\frac{2a}{3}$ **37.** $2\pi(2 - \sqrt{e})$ **39.** $\frac{4}{3} + \frac{5\pi}{8}$

41. **(a)** $\frac{\sqrt{\pi}}{2}$ **(b)** 1 **43.** $\pi\ln 4$, no **45.** $\frac{1}{2}(a^2 + 2h^2)$

47. $\frac{8}{9}(3\pi - 4)$

Section 15.5, pp. 912–915

1. $1/6$

3. $\int_0^1 \int_0^{2-2x} \int_0^{3-3x-3y/2} dz\,dy\,dx, \int_0^2 \int_0^{1-y/2} \int_0^{3-3x-3y/2} dz\,dx\,dy,$

$\int_0^1 \int_0^{3-3x} \int_0^{2-2x-2z/3} dy\,dz\,dx, \int_0^3 \int_0^{1-z/3} \int_0^{2-2x-2z/3} dy\,dx\,dz,$

$\int_0^2 \int_0^{3-3y/2} \int_0^{1-y/2-z/3} dx\,dz\,dy, \int_0^3 \int_0^{2-2z/3} \int_0^{1-y/2-z/3} dx\,dy\,dz.$

The value of all six integrals is 1.

5. $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{8-x^2-y^2} 1\,dz\,dx\,dy$, $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{x^2+y^2}^{8-x^2-y^2} 1\,dz\,dx\,dy$,

$\int_{-2}^{2}\int_{4}^{8-y^2}\int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} 1\,dx\,dz\,dy + \int_{-2}^{2}\int_{y^2}^{4}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} 1\,dx\,dz\,dy$,

$\int_{4}^{8}\int_{-\sqrt{8-z}}^{\sqrt{8-z}}\int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} 1\,dx\,dy\,dz + \int_{0}^{4}\int_{-\sqrt{z}}^{\sqrt{z}}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} 1\,dx\,dy\,dz$,

$\int_{-2}^{2}\int_{4}^{8-x^2}\int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} 1\,dy\,dz\,dx + \int_{-2}^{2}\int_{x^2}^{4}\int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} 1\,dy\,dz\,dx$,

$\int_{4}^{8}\int_{-\sqrt{8-z}}^{\sqrt{8-z}}\int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} 1\,dy\,dx\,dz + \int_{0}^{4}\int_{-\sqrt{z}}^{\sqrt{z}}\int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} 1\,dy\,dx\,dz$.

The value of all six integrals is 16π.

7. 1 **9.** 6 **11.** $\frac{5(2-\sqrt{3})}{4}$ **13.** 18

15. $7/6$ **17.** 0 **19.** $\frac{1}{2} - \frac{\pi}{8}$

21. (a) $\int_{-1}^{1}\int_{0}^{1-x^2}\int_{x^2}^{1-z} dy\,dz\,dx$ **(b)** $\int_{0}^{1}\int_{-\sqrt{1-z}}^{\sqrt{1-z}}\int_{x^2}^{1-z} dy\,dx\,dz$

(c) $\int_{0}^{1}\int_{0}^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy\,dz$ **(d)** $\int_{0}^{1}\int_{0}^{1-y}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dz\,dy$

(e) $\int_{0}^{1}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{1-y} dz\,dx\,dy$

23. $2/3$ **25.** $20/3$ **27.** 1 **29.** $16/3$ **31.** $8\pi - \frac{32}{3}$

33. 2 **35.** 4π **37.** $31/3$ **39.** 1 **41.** $2 \sin 4$

43. 4 **45.** $a = 3$ or $a = 13/3$

47. The domain is the set of all points (x, y, z) such that $4x^2 + 4y^2 + z^2 \le 4$.

Section 15.6, pp. 920–922

1. $\bar{x} = 5/14, \bar{y} = 38/35$ **3.** $\bar{x} = 64/35, \bar{y} = 5/7$

5. $\bar{x} = \bar{y} = 4a/(3\pi)$ **7.** $I_x = I_y = 4\pi, I_0 = 8\pi$

9. $\bar{x} = -1, \bar{y} = 1/4$ **11.** $I_x = 64/105$

13. $\bar{x} = 3/8, \bar{y} = 17/16$ **15.** $\bar{x} = 11/3, \bar{y} = 14/27, I_y = 432$

17. $\bar{x} = 0, \bar{y} = 13/31, I_y = 7/5$

19. $\bar{x} = 0, \bar{y} = 7/10; I_x = 9/10, I_y = 3/10, I_0 = 6/5$

21. $I_x = \frac{M}{3}(b^2 + c^2), I_y = \frac{M}{3}(a^2 + c^2), I_z = \frac{M}{3}(a^2 + b^2)$

23. $\bar{x} = \bar{y} = 0, \bar{z} = 12/5, I_x = 7904/105 \approx 75.28$, $I_y = 4832/63 \approx 76.70, I_z = 256/45 \approx 5.69$

25. (a) $\bar{x} = \bar{y} = 0, \bar{z} = 8/3$ **(b)** $c = 2\sqrt{2}$

27. $I_L = 1386$

29. (a) $4/3$ **(b)** $\bar{x} = 4/5, \bar{y} = \bar{z} = 2/5$

31. (a) $5/2$ **(b)** $\bar{x} = \bar{y} = \bar{z} = 8/15$ **(c)** $I_x = I_y = I_z = 11/6$

33. 3

37. (a) $I_{c.m.} = \frac{abc(a^2 + b^2)}{12}, R_{c.m.} = \sqrt{\frac{a^2 + b^2}{12}}$

(b) $I_L = \frac{abc(a^2 + 7b^2)}{3}, R_L = \sqrt{\frac{a^2 + 7b^2}{3}}$

Section 15.7, pp. 930–934

1. $\frac{4\pi(\sqrt{2} - 1)}{3}$ **3.** $\frac{17\pi}{5}$ **5.** $\pi(6\sqrt{2} - 8)$ **7.** $\frac{3\pi}{10}$

9. $\pi/3$

11. (a) $\int_{0}^{2\pi}\int_{0}^{1}\int_{0}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

(b) $\int_{0}^{2\pi}\int_{0}^{\sqrt{3}}\int_{0}^{1} r\,dr\,dz\,d\theta + \int_{0}^{2\pi}\int_{\sqrt{3}}^{2}\int_{0}^{\sqrt{4-z^2}} r\,dr\,dz\,d\theta$

(c) $\int_{0}^{1}\int_{0}^{\sqrt{4-r^2}}\int_{0}^{2\pi} r\,d\theta\,dz\,dr$

13. $\int_{-\pi/2}^{\pi/2}\int_{0}^{\cos\theta}\int_{0}^{3r^2} f(r, \theta, z)\,dz\,r\,dr\,d\theta$

15. $\int_{0}^{\pi}\int_{0}^{2\sin\theta}\int_{0}^{4-r\sin\theta} f(r, \theta, z)\,dz\,r\,dr\,d\theta$

17. $\int_{-\pi/2}^{\pi/2}\int_{1}^{1+\cos\theta}\int_{0}^{4} f(r, \theta, z)\,dz\,r\,dr\,d\theta$

19. $\int_{0}^{\pi/4}\int_{0}^{\sec\theta}\int_{0}^{2-r\sin\theta} f(r, \theta, z)\,dz\,r\,dr\,d\theta$ **21.** π^2

23. $\pi/3$ **25.** 5π **27.** 2π **29.** $\left(\frac{8 - 5\sqrt{2}}{2}\right)\pi$

31. (a) $\int_{0}^{2\pi}\int_{0}^{\pi/6}\int_{0}^{2} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta +$

$\int_{0}^{2\pi}\int_{\pi/6}^{\pi/2}\int_{0}^{\csc\phi} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$

(b) $\int_{0}^{2\pi}\int_{1}^{2}\int_{\pi/6}^{\sin^{-1}(1/\rho)} \rho^2 \sin\phi\,d\phi\,d\rho\,d\theta +$

$\int_{0}^{2\pi}\int_{0}^{2}\int_{0}^{\pi/6} \rho^2 \sin\phi\,d\phi\,d\rho\,d\theta +$

$\int_{0}^{2\pi}\int_{0}^{1}\int_{\pi/6}^{\pi/2} \rho^2 \sin\phi\,d\phi\,d\rho\,d\theta$

33. $\int_{0}^{2\pi}\int_{0}^{\pi/2}\int_{\cos\phi}^{2} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta = \frac{31\pi}{6}$

35. $\int_{0}^{2\pi}\int_{0}^{\pi}\int_{0}^{1-\cos\phi} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta = \frac{8\pi}{3}$

37. $\int_{0}^{2\pi}\int_{\pi/4}^{\pi/2}\int_{0}^{2\cos\phi} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta = \frac{\pi}{3}$

39. (a) $8\int_{0}^{\pi/2}\int_{0}^{\pi/2}\int_{0}^{2} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$

(b) $8\int_{0}^{\pi/2}\int_{0}^{2}\int_{0}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

(c) $8\int_{0}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} dz\,dy\,dx$

41. (a) $\int_0^{2\pi}\int_0^{\pi/3}\int_{\sec\phi}^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

(b) $\int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta$

(c) $\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}}\int_1^{\sqrt{4-x^2-y^2}} dz \, dy \, dx$ **(d)** $5\pi/3$

43. $8\pi/3$ **45.** $9/4$ **47.** $\frac{3\pi - 4}{18}$ **49.** $\frac{2\pi a^3}{3}$

51. $5\pi/3$ **53.** $\pi/2$ **55.** $\frac{4(2\sqrt{2} - 1)\pi}{3}$ **57.** 16π

59. $5\pi/2$ **61.** $\frac{4\pi(8 - 3\sqrt{3})}{3}$ **63.** $2/3$ **65.** $3/4$

67. $\bar{x} = \bar{y} = 0, \bar{z} = 3/8$ **69.** $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 3/8)$

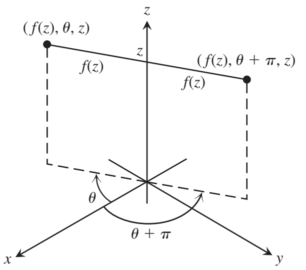
71. $\bar{x} = \bar{y} = 0, \bar{z} = 5/6$ **73.** $I_x = \pi/4$ **75.** $\frac{a^4 h\pi}{10}$

77. (a) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{4}{5}\right), I_z = \frac{\pi}{12}$

(b) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{5}{6}\right), I_z = \frac{\pi}{14}$

81. $\frac{3M}{\pi R^3}$

85. The surface's equation $r = f(z)$ tells us that the point $(r, \theta, z) = (f(z), \theta, z)$ will lie on the surface for all θ. In particular, $(f(z), \theta + \pi, z)$ lies on the surface whenever $(f(z), \theta, z)$ lies on the surface, so the surface is symmetric with respect to the z-axis.



Section 15.8, pp. 942–944

1. (a) $x = \frac{u + v}{3}, y = \frac{v - 2u}{3}; \frac{1}{3}$

(b) Triangular region with boundaries $u = 0, v = 0$, and $u + v = 3$

3. (a) $x = \frac{1}{5}(2u - v), y = \frac{1}{10}(3v - u); \frac{1}{10}$

(b) Triangular region with boundaries $3v = u, v = 2u$, and $3u + v = 10$

7. $64/5$ **9.** $\int_1^2\int_1^3 (u + v)\frac{2u}{v} \, du \, dv = 8 + \frac{52}{3}\ln 2$

11. $\frac{\pi ab(a^2 + b^2)}{4}$ **13.** $\frac{1}{3}\left(1 + \frac{3}{e^2}\right) \approx 0.4687$

15. $\frac{225}{16}$ **17.** 12 **19.** $\frac{a^2b^2c^2}{6}$

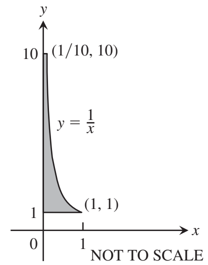
21. (a) $\begin{vmatrix} \cos v & -u \sin v \\ \sin v & u \cos v \end{vmatrix} = u\cos^2 v + u \sin^2 v = u$

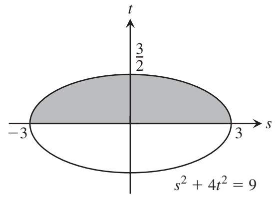
(b) $\begin{vmatrix} \sin v & u \cos v \\ \cos v & -u \sin v \end{vmatrix} = -u\sin^2 v - u \cos^2 v = -u$

27. $\frac{3}{2}\ln 2$

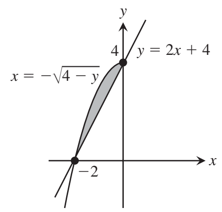
Practice Exercises, pp. 944–946

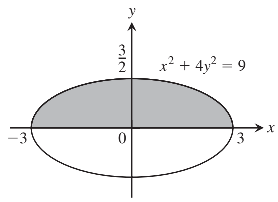
1. $9e - 9$ **3.** $9/2$





5. $\int_{-2}^{0}\int_{2x+4}^{4-x^2} dy \, dx = \frac{4}{3}$ **7.** $\int_{-3}^{3}\int_0^{(1/2)\sqrt{9-x^2}} y \, dy \, dx = \frac{9}{2}$





9. $\sin 4$ **11.** $\frac{\ln 17}{4}$ **13.** $4/3$ **15.** $4/3$ **17.** $1/4$

19. π **21.** $\frac{\pi - 2}{4}$ **23.** 0 **25.** $8/35$ **27.** $\pi/2$

29. $\frac{2(31 - 3^{5/2})}{3}$

31. (a) $\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} 3 \, dz \, dx \, dy$

(b) $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} 3\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$ **(c)** $2\pi(8 - 4\sqrt{2})$

33. $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sec\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \frac{\pi}{3}$

35. $\int_0^{1}\int_{\sqrt{1-x^2}}^{\sqrt{3-x^2}}\int_1^{\sqrt{4-x^2-y^2}} z^2 xy \, dz \, dy \, dx + \int_1^{\sqrt{3}}\int_0^{\sqrt{3-x^2}}\int_1^{\sqrt{4-x^2-y^2}} z^2 xy \, dz \, dy \, dx$

37. (a) $\frac{8\pi(4\sqrt{2} - 5)}{3}$ **(b)** $\frac{8\pi(4\sqrt{2} - 5)}{3}$

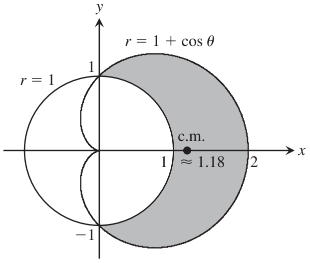
39. $I_z = \frac{8\pi\delta(b^5 - a^5)}{15}$

41. $\bar{x} = \bar{y} = \frac{1}{2 - \ln 4}$ **43.** $I_0 = 104$ **45.** $I_x = 2\delta$

47. $M = 4, M_x = 0, M_y = 0$ **49.** $\bar{x} = \frac{3\sqrt{3}}{\pi}, \bar{y} = 0$

51. **(a)** $\bar{x} = \dfrac{15\pi + 32}{6\pi + 48}, \bar{y} = 0$

(b)



Additional and Advanced Exercises, pp. 947–948

1. **(a)** $\displaystyle\int_{-3}^{2}\int_{x}^{6-x^2} x^2\,dy\,dx$ **(b)** $\displaystyle\int_{-3}^{2}\int_{x}^{6-x^2}\int_{0}^{x^2} dz\,dy\,dx$

(c) $125/4$

3. 2π **5.** $3\pi/2$

7. **(a)** Hole radius $= 1$, sphere radius $= 2$ **(b)** $4\sqrt{3}\pi$

9. $\pi/4$ **11.** $\ln\left(\dfrac{b}{a}\right)$ **15.** $1/\sqrt[4]{3}$

17. Mass $= a^2\cos^{-1}\left(\dfrac{b}{a}\right) - b\sqrt{a^2 - b^2}$,

$I_0 = \dfrac{a^4}{2}\cos^{-1}\left(\dfrac{b}{a}\right) - \dfrac{b^3}{2}\sqrt{a^2 - b^2} - \dfrac{b^3}{6}(a^2 - b^2)^{3/2}$

19. $\dfrac{1}{ab}(e^{a^2b^2} - 1)$ **21.** **(b)** 1 **(c)** 0

25. $h = \sqrt{20}$ in., $h = \sqrt{60}$ in. **27.** $2\pi\left[\dfrac{1}{3} - \left(\dfrac{1}{3}\right)\dfrac{\sqrt{2}}{2}\right]$

Chapter 16

Section 16.1, pp. 955–957

1. Graph (c) **3.** Graph (g) **5.** Graph (d) **7.** Graph (f)

9. $\sqrt{2}$ **11.** $\dfrac{13}{2}$ **13.** $3\sqrt{14}$ **15.** $\dfrac{1}{6}(5\sqrt{5} + 9)$

17. $\sqrt{3}\ln\left(\dfrac{b}{a}\right)$ **19.** **(a)** $4\sqrt{5}$ **(b)** $\dfrac{1}{12}(17^{3/2} - 1)$

21. $\dfrac{15}{32}(e^{16} - e^{64})$ **23.** $\dfrac{1}{27}(40^{3/2} - 13^{3/2})$

25. $\dfrac{1}{6}(5^{3/2} + 7\sqrt{2} - 1)$ **27.** $\dfrac{10\sqrt{5} - 2}{3}$ **29.** 8

31. $\dfrac{1}{6}(17^{3/2} - 1)$ **33.** $2\sqrt{2} - 1$

35. **(a)** $4\sqrt{2} - 2$ **(b)** $\sqrt{2} + \ln(1 + \sqrt{2})$ **37.** $I_z = 2\pi\delta a^3$

39. **(a)** $I_z = 2\pi\sqrt{2}\delta$ **(b)** $I_z = 4\pi\sqrt{2}\delta$ **41.** $I_x = 2\pi - 2$

Section 16.2, pp. 967–969

1. $\nabla f = -(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})(x^2 + y^2 + z^2)^{-3/2}$

3. $\nabla g = -\left(\dfrac{2x}{x^2 + y^2}\right)\mathbf{i} - \left(\dfrac{2y}{x^2 + y^2}\right)\mathbf{j} + e^z\mathbf{k}$

5. $\mathbf{F} = -\dfrac{kx}{(x^2 + y^2)^{3/2}}\mathbf{i} - \dfrac{ky}{(x^2 + y^2)^{3/2}}\mathbf{j}$, any $k > 0$

7. **(a)** $9/2$ **(b)** $13/3$ **(c)** $9/2$

9. **(a)** $1/3$ **(b)** $-1/5$ **(c)** 0

11. **(a)** 2 **(b)** $3/2$ **(c)** $1/2$

13. $-15/2$ **15.** 36 **17.** **(a)** $-5/6$ **(b)** 0 **(c)** $-7/12$

19. $1/2$ **21.** $-\pi$ **23.** $69/4$ **25.** $-39/2$ **27.** $25/6$

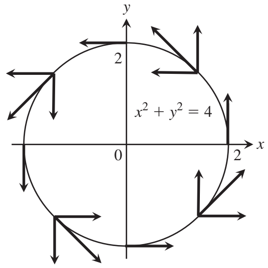
29. **(a)** $\text{Circ}_1 = 0$, $\text{circ}_2 = 2\pi$, $\text{flux}_1 = 2\pi$, $\text{flux}_2 = 0$

(b) $\text{Circ}_1 = 0$, $\text{circ}_2 = 8\pi$, $\text{flux}_1 = 8\pi$, $\text{flux}_2 = 0$

31. Circ $= 0$, flux $= a^2\pi$ **33.** Circ $= a^2\pi$, flux $= 0$

35. **(a)** $-\dfrac{\pi}{2}$ **(b)** 0 **(c)** 1 **37.** **(a)** 32 **(b)** 32 **(c)** 32

39.



41. **(a)** $\mathbf{G} = -y\mathbf{i} + x\mathbf{j}$ **(b)** $\mathbf{G} = \sqrt{x^2 + y^2}\,\mathbf{F}$

43. $\mathbf{F} = -\dfrac{x\mathbf{i} + y\mathbf{j}}{\sqrt{x^2 + y^2}}$ **47.** 48 **49.** π **51.** 0 **53.** $\dfrac{1}{2}$

Section 16.3, pp. 978–980

1. Conservative **3.** Not conservative **5.** Not conservative

7. $f(x, y, z) = x^2 + \dfrac{3y^2}{2} + 2z^2 + C$ **9.** $f(x, y, z) = xe^{y+2z} + C$

11. $f(x, y, z) = x\ln x - x + \tan(x + y) + \dfrac{1}{2}\ln(y^2 + z^2) + C$

13. 49 **15.** -16 **17.** 1 **19.** $9\ln 2$ **21.** 0 **23.** -3

27. $\mathbf{F} = \nabla\left(\dfrac{x^2 - 1}{y}\right)$ **29.** **(a)** 1 **(b)** 1 **(c)** 1

31. **(a)** 2 **(b)** 2 **33.** **(a)** $c = b = 2a$ **(b)** $c = b = 2$

35. It does not matter what path you use. The work will be the same on any path because the field is conservative.

37. The force $\mathbf{F}$ is conservative because all partial derivatives of M, N, and P are zero. $f(x, y, z) = ax + by + cz + C$; $A = (xa, ya, za)$ and $B = (xb, yb, zb)$. Therefore, $\int \mathbf{F}\cdot d\mathbf{r} = f(B) - f(A) = a(xb - xa) + b(yb - ya) + c(zb - za) = \mathbf{F}\cdot\overrightarrow{AB}$.

Section 16.4, pp. 990–992

1. Flux $= 0$, circ $= 2\pi a^2$ **3.** Flux $= -\pi a^2$, circ $= 0$

5. Flux $= 2$, circ $= 0$ **7.** Flux $= -9$, circ $= 9$

9. Flux $= -11/60$, circ $= -7/60$

11. Flux $= 64/9$, circ $= 0$ **13.** Flux $= 1/2$, circ $= 1/2$

15. Flux $= 1/5$, circ $= -1/12$ **17.** 0 **19.** $2/33$ **21.** 0

23. -16π **25.** πa^2 **27.** $3\pi/8$

29. **(a)** 0 if C is traversed counterclockwise

(b) $(h - k)$(area of the region) **39.** **(a)** 0

Section 16.5, pp. 1001–1003

1. $\mathbf{r}(r, \theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + r^2\mathbf{k}$, $0 \le r \le 2$, $0 \le \theta \le 2\pi$

3. $\mathbf{r}(r, \theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + (r/2)\mathbf{k}$, $0 \le r \le 6$, $0 \le \theta \le \pi/2$

5. $\mathbf{r}(r, \theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + \sqrt{9 - r^2}\,\mathbf{k}$, $0 \le r \le 3\sqrt{2}/2$, $0 \le \theta \le 2\pi$; Also: $\mathbf{r}(\phi, \theta) = (3\sin\phi\cos\theta)\mathbf{i} + (3\sin\phi\sin\theta)\mathbf{j} + (3\cos\phi)\mathbf{k}$, $0 \le \phi \le \pi/4$, $0 \le \theta \le 2\pi$

7. $\mathbf{r}(\phi, \theta) = (\sqrt{3}\sin\phi\cos\theta)\mathbf{i} + (\sqrt{3}\sin\phi\sin\theta)\mathbf{j} + (\sqrt{3}\cos\phi)\mathbf{k}$, $\pi/3 \le \phi \le 2\pi/3$, $0 \le \theta \le 2\pi$

9. $\mathbf{r}(x, y) = x\mathbf{i} + y\mathbf{j} + (4 - y^2)\mathbf{k}$, $0 \le x \le 2$, $-2 \le y \le 2$

11. $\mathbf{r}(u, v) = u\mathbf{i} + (3\cos v)\mathbf{j} + (3\sin v)\mathbf{k}$, $0 \le u \le 3$, $0 \le v \le 2\pi$

13. (a) $\mathbf{r}(r, \theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + (1 - r\cos\theta - r\sin\theta)\mathbf{k}$, $0 \le r \le 3$, $0 \le \theta \le 2\pi$
(b) $\mathbf{r}(u, v) = (1 - u\cos v - u\sin v)\mathbf{i} + (u\cos v)\mathbf{j} + (u\sin v)\mathbf{k}$, $0 \le u \le 3$, $0 \le v \le 2\pi$

15. $\mathbf{r}(u, v) = (4\cos^2 v)\mathbf{i} + u\mathbf{j} + (4\cos v\sin v)\mathbf{k}$, $0 \le u \le 3$, $-(\pi/2) \le v \le (\pi/2)$; Another way: $\mathbf{r}(u, v) = (2 + 2\cos v)\mathbf{i} + u\mathbf{j} + (2\sin v)\mathbf{k}$, $0 \le u \le 3$, $0 \le v \le 2\pi$

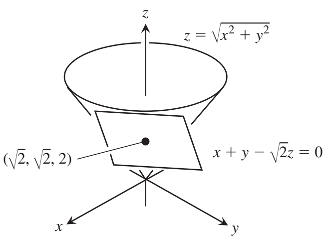
17. $\displaystyle\int_0^{2\pi}\int_0^1 \frac{\sqrt{5}}{2} r\,dr\,d\theta = \frac{\pi\sqrt{5}}{2}$

19. $\displaystyle\int_0^{2\pi}\int_1^3 r\sqrt{5}\,dr\,d\theta = 8\pi\sqrt{5}$ **21.** $\displaystyle\int_0^{2\pi}\int_1^4 1\,du\,dv = 6\pi$

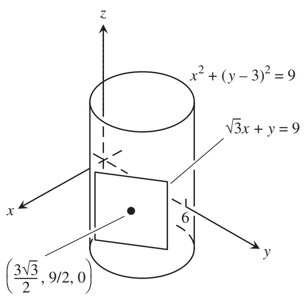
23. $\displaystyle\int_0^{2\pi}\int_0^1 u\sqrt{4u^2 + 1}\,du\,dv = \frac{(5\sqrt{5} - 1)}{6}\pi$

25. $\displaystyle\int_0^{2\pi}\int_{\pi/4}^{\pi} 2\sin\phi\,d\phi\,d\theta = (4 + 2\sqrt{2})\pi$

27.



29.



33. (b) $\displaystyle A = \int_0^{2\pi}\int_0^{\pi} [a^2b^2\sin^2\phi\cos^2\phi + b^2c^2\cos^4\phi\cos^2\theta + a^2c^2\cos^4\phi\sin^2\theta]^{1/2}\,d\phi\,d\theta$

35. $x_0x + y_0y = 25$ **37.** $13\pi/3$ **39.** 4

41. $6\sqrt{6} - 2\sqrt{2}$ **43.** $\pi\sqrt{c^2 + 1}$

45. $\frac{\pi}{6}(17\sqrt{17} - 5\sqrt{5})$ **47.** $3 + 2\ln 2$

49. $\frac{\pi}{6}(13\sqrt{13} - 1)$ **51.** $5\pi\sqrt{2}$ **53.** $\frac{2}{3}(5\sqrt{5} - 1)$

Section 16.6, pp. 1012–1014

1. $\displaystyle\iint_S x\,d\sigma = \int_0^3\int_0^2 u\sqrt{4u^2 + 1}\,du\,dv = \frac{17\sqrt{17} - 1}{4}$

3. $\displaystyle\iint_S x^2\,d\sigma = \int_0^{2\pi}\int_0^{\pi} \sin^3\phi\cos^2\theta\,d\phi\,d\theta = \frac{4\pi}{3}$

5. $\displaystyle\iint_S z\,d\sigma = \int_0^1\int_0^1 (4 - u - v)\sqrt{3}\,dv\,du = 3\sqrt{3}$
(for $x = u$, $y = v$)

7. $\displaystyle\iint_S x^2\sqrt{5 - 4z}\,d\sigma = \int_0^1\int_0^{2\pi} u^2\cos^2 v \cdot \sqrt{4u^2 + 1} \cdot u\sqrt{4u^2 + 1}\,dv\,du = \int_0^1\int_0^{2\pi} u^3(4u^2 + 1)\cos^2 v\,dv\,du = \frac{11\pi}{12}$

9. $9a^3$ **11.** $\frac{abc}{4}(ab + ac + bc)$ **13.** 2

15. $\frac{1}{30}(\sqrt{2} + 6\sqrt{6})$ **17.** $\sqrt{6}/30$ **19.** -32 **21.** $\frac{\pi a^3}{6}$

23. $13a^4/6$ **25.** $2\pi/3$ **27.** $-73\pi/6$ **29.** 18

31. $\frac{\pi a^3}{6}$ **33.** $\frac{\pi a^2}{4}$ **35.** $\frac{\pi a^3}{2}$ **37.** -32 **39.** -4

41. $3a^4$ **43.** $\left(\frac{a}{2}, \frac{a}{2}, \frac{a}{2}\right)$

45. $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{14}{9}\right)$, $I_z = \frac{15\pi\sqrt{2}}{2}\delta$

47. (a) $\frac{8\pi}{3}a^4\delta$ **(b)** $\frac{20\pi}{3}a^4\delta$

Section 16.7, pp. 1025–1026

1. 4π **3.** $-5/6$ **5.** 0 **7.** -6π **9.** $2\pi a^2$
11. $-\pi$ **13.** 12π **15.** $-\pi/4$ **17.** -15π **19.** -8π
27. $16I_y + 16I_x$

Section 16.8, pp. 1037–1039

1. 0 **3.** 0 **5.** -16 **7.** -8π **9.** 3π **11.** $-40/3$
13. 12π **15.** $12\pi(4\sqrt{2} - 1)$ **19.** No
21. The integral's value never exceeds the surface area of S.
23. 184/35

Practice Exercises, pp. 1040–1042

1. Path 1: $2\sqrt{3}$; path 2: $1 + 3\sqrt{2}$ **3.** $4a^2$ **5.** 0
7. $8\pi\sin(1)$ **9.** 0 **11.** $\pi\sqrt{3}$

13. $2\pi\left(1 - \frac{1}{\sqrt{2}}\right)$ **15.** $\frac{abc}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$ **17.** 50

19. $\mathbf{r}(\phi, \theta) = (6\sin\phi\cos\theta)\mathbf{i} + (6\sin\phi\sin\theta)\mathbf{j} + (6\cos\phi)\mathbf{k}$, $\frac{\pi}{6} \le \phi \le \frac{2\pi}{3}$, $0 \le \theta \le 2\pi$

21. $\mathbf{r}(r, \theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + (1 + r)\mathbf{k}$, $0 \le r \le 2$, $0 \le \theta \le 2\pi$

23. $\mathbf{r}(u, v) = (u\cos v)\mathbf{i} + 2u^2\mathbf{j} + (u\sin v)\mathbf{k}$, $0 \le u \le 1$, $0 \le v \le \pi$

25. $\sqrt{6}$ **27.** $\pi[\sqrt{2} + \ln(1 + \sqrt{2})]$ **29.** Conservative

31. Not conservative **33.** $f(x, y, z) = y^2 + yz + 2x + z$

35. Path 1: 2; path 2: 8/3 **37. (a)** $1 - e^{-2\pi}$ **(b)** $1 - e^{-2\pi}$

39. 0 **41. (a)** $4\sqrt{2} - 2$ **(b)** $\sqrt{2} + \ln(1 + \sqrt{2})$

43. $(\bar{x}, \bar{y}, \bar{z}) = \left(1, \frac{16}{15}, \frac{2}{3}\right)$; $I_x = \frac{232}{45}$, $I_y = \frac{64}{15}$, $I_z = \frac{56}{9}$

45. $\bar{z} = \frac{3}{2}$, $I_z = \frac{7\sqrt{3}}{3}$ **47.** $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 49/12)$, $I_z = 640\pi$

49. Flux: 3/2; circ: $-1/2$ **53.** 3 **55.** $\frac{2\pi}{3}(7 - 8\sqrt{2})$

57. 0 **59.** π

Additional and Advanced Exercises, pp. 1042–1044

1. 6π **3.** $2/3$

5. (a) $\mathbf{F}(x, y, z) = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$
(b) $\mathbf{F}(x, y, z) = z\mathbf{i} + y\mathbf{k}$ **(c)** $\mathbf{F}(x, y, z) = z\mathbf{i}$

7. $\frac{16\pi R^3}{3}$ **9.** $a = 2, b = 1$. The minimum flux is -4.

11. (b) $\frac{16}{3}g$

(c) Work $= \left(\int_C gxy\, ds\right)\bar{y} = g\int_C xy^2\, ds = \frac{16}{3}g$

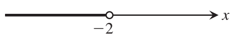
13. (c) $\frac{4}{3}\pi w$ **19.** False if $\mathbf{F} = y\mathbf{i} + x\mathbf{j}$

Appendices

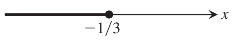
Appendix 1, p. AP-6

1. $0.\bar{1}, 0.\bar{2}, 0.\bar{3}, 0.\bar{8}, 0.\bar{9}$ or 1

3. $x < -2$



5. $x \le -\frac{1}{3}$

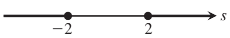


7. $3, -3$ **9.** $7/6, 25/6$

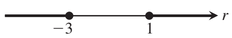
11. $-2 \le t \le 4$

13. $0 \le z \le 10$

15. $(-\infty, -2] \cup [2, \infty)$

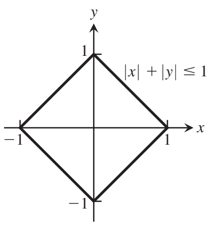


17. $(-\infty, -3] \cup [1, \infty)$



19. $(-3, -2) \cup (2, 3)$ **21.** $(0, 1)$ **23.** $(-\infty, 1]$

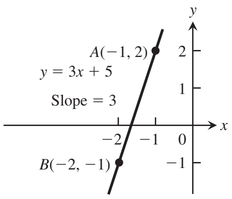
27. The graph of $|x| + |y| \le 1$ is the interior and boundary of the "diamond-shaped" region.



Appendix 3, pp. AP-17–AP-18

1. $2, -4; 2\sqrt{5}$ **3.** Unit circle

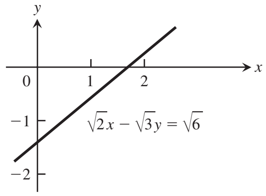
5. $m_\perp = -\frac{1}{3}$



7. (a) $x = -1$ **(b)** $y = 4/3$ **9.** $y = -x$

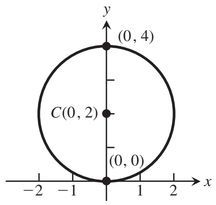
11. $y = -\frac{5}{4}x + 6$ **13.** $y = 4x + 4$ **15.** $y = -\frac{x}{2} + 12$

17. x-intercept $= \sqrt{3}$, y-intercept $= -\sqrt{2}$

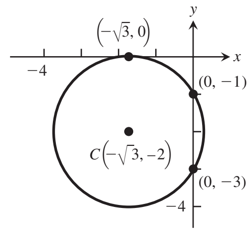


19. $(3, -3)$

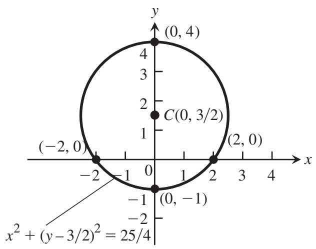
21. $x^2 + (y - 2)^2 = 4$



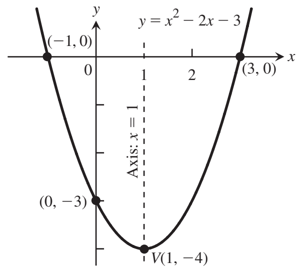
23. $\left(x + \sqrt{3}\right)^2 + (y + 2)^2 = 4$



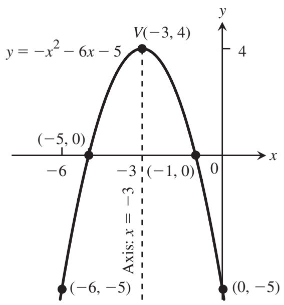
25. $x^2 + (y - 3/2)^2 = 25/4$



27.



29.



31. Exterior points of a circle of radius $\sqrt{7}$, centered at the origin

33. The washer between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ (points with distance from the origin between 1 and 2)

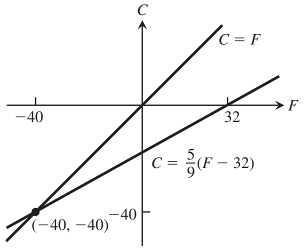
35. $(x + 2)^2 + (y - 1)^2 < 6$

37. $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$, $\left(-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$

39. $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{3}\right)$, $\left(\frac{1}{\sqrt{3}}, -\frac{1}{3}\right)$

41. (a) ≈ -2.5 degrees/inch **(b)** ≈ -16.1 degrees/inch
(c) ≈ -8.3 degrees/inch **43.** 5.97 atm

45. Yes: $C = F = -40°$



51. $k = -8, \quad k = 1/2$

Appendix 7, pp. AP-34–AP-35

1. **(a)** $(14, 8)$ **(b)** $(-1, 8)$ **(c)** $(0, -5)$

3. **(a)** By reflecting z across the real axis
(b) By reflecting z across the imaginary axis
(c) By reflecting z across the real axis and then multiplying the length of the vector by $1/|z|^2$

5. **(a)** Points on the circle $x^2 + y^2 = 4$
(b) Points inside the circle $x^2 + y^2 = 4$
(c) Points outside the circle $x^2 + y^2 = 4$

7. Points on a circle of radius 1, center $(-1, 0)$

9. Points on the line $y = -x$ **11.** $4e^{2\pi i/3}$ **13.** $1e^{2\pi i/3}$

15. $\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

17. $1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ **19.** $2i, -\sqrt{3} - i, \sqrt{3} - i$

21. $\frac{\sqrt{6}}{2} \pm \frac{\sqrt{2}}{2}i, -\frac{\sqrt{6}}{2} \pm \frac{\sqrt{2}}{2}i$ **23.** $1 \pm \sqrt{3}i, -1 \pm \sqrt{3}i$