

Section 11.1

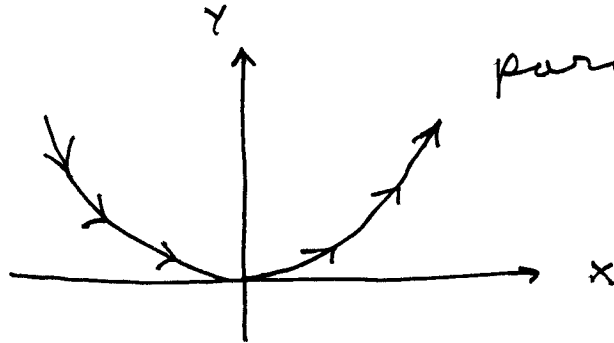
1.) $\begin{cases} x = 3t \\ y = 9t^2 \end{cases}$ for $-\infty < t < \infty$;

$t = \frac{x}{3} \xrightarrow{\text{sub}} y = 9t^2 = 9\left(\frac{x}{3}\right)^2 = 9 \cdot \frac{x^2}{9} = x^2$

$\rightarrow \boxed{y = x^2}$

parabola

left
to right



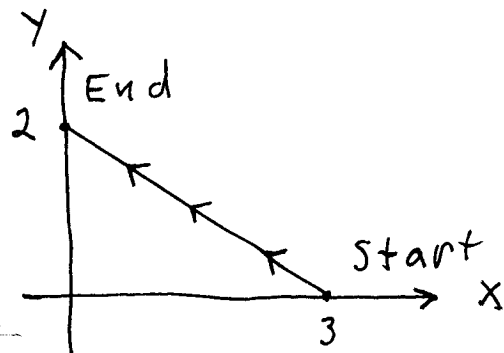
4.) $\begin{cases} x = 3 - 3t \\ y = 2t \end{cases}$ for $0 \leq t \leq 1$;

$x = 3 - 3t \rightarrow 3t = 3 - x \rightarrow t = 1 - \frac{1}{3}x \xrightarrow{\text{sub}}$

$y = 2t = 2\left(1 - \frac{1}{3}x\right) = 2 - \frac{2}{3}x \rightarrow$

$\boxed{y = 2 - \frac{2}{3}x}$

line



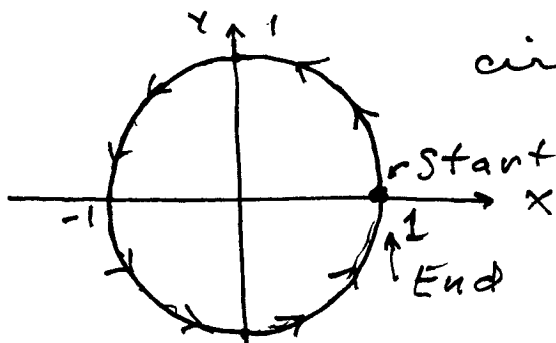
5.) $\begin{cases} x = \cos 2t \\ y = \sin 2t \end{cases}$ for $0 \leq t \leq \pi$;

$x^2 + y^2 = \cos^2 2t + \sin^2 2t = 1 \rightarrow$

$\boxed{x^2 + y^2 = 1}$

circle

once
around,
counter-

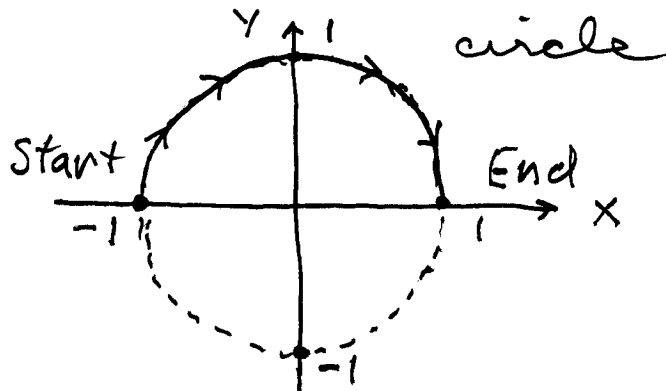


$$6.) \begin{cases} x = \cos(\pi - t) \\ y = \sin(\pi - t) \end{cases} \quad \text{for } 0 \leq t \leq \pi ;$$

$$x^2 + y^2 = \cos^2(\pi - t) + \sin^2(\pi - t) = 1 \rightarrow$$

$$\boxed{x^2 + y^2 = 1}$$

$\frac{1}{2}$ of circle,
clockwise



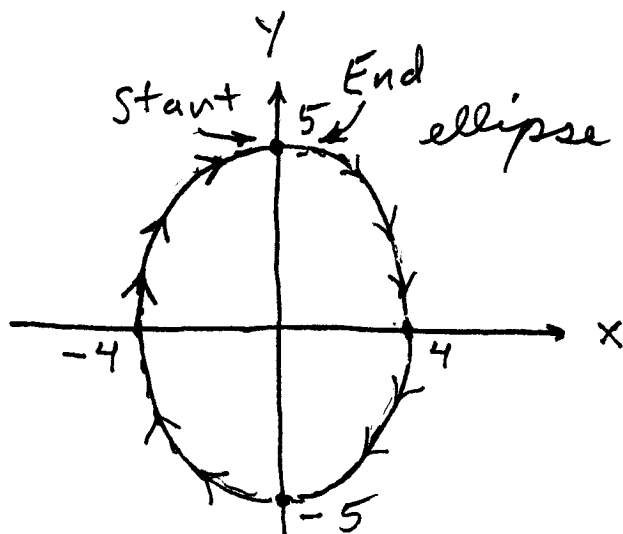
$$8.) \begin{cases} x = 4 \sin t \\ y = 5 \cos t \end{cases} \quad \text{for } 0 \leq t \leq 2\pi ;$$

$$\frac{x}{4} = \sin t \quad \text{and} \quad \frac{y}{5} = \cos t \rightarrow$$

$$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = \sin^2 t + \cos^2 t = 1 \rightarrow$$

$$\boxed{\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1}$$

once around,
clockwise

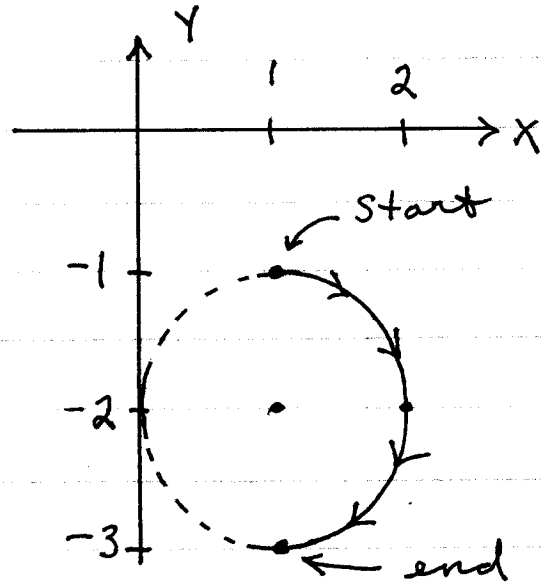


$$10.) \begin{cases} X=1+\sin t \\ Y=\cos t-2 \end{cases} \rightarrow \begin{cases} X-1=\sin t \\ Y+2=\cos t \end{cases} \quad \text{for } 0 \leq t \leq \pi$$

$$\rightarrow (X-1)^2 + (Y+2)^2 = \sin^2 t + \cos^2 t = 1$$

$$\rightarrow \boxed{(X-1)^2 + (Y+2)^2 = 1}$$

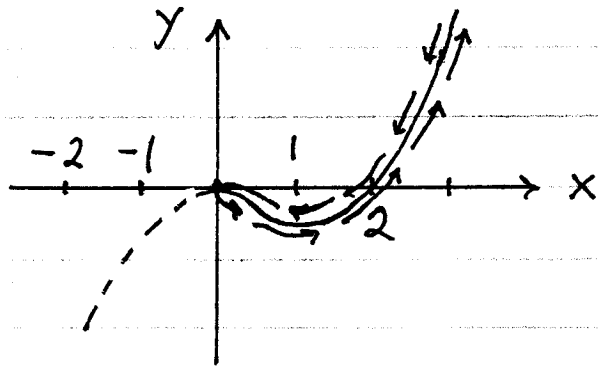
$\frac{1}{2}$ of circle,
clockwise



$$11.) \begin{cases} X=t^2 \\ Y=t^6-2t^4 = (t^2)^3 - 2(t^2)^2 \end{cases} \quad \text{for } -\infty < t < \infty$$

$$\rightarrow \boxed{Y = X^3 - 2X^2} = X^2(X-2)$$

cubic polynomial,
changes direction
at (0,0)



$$12.) \begin{cases} X = \frac{t}{t-1} \\ Y = \frac{t-2}{t+1} \end{cases} \quad \text{for } -1 < t < 1$$

; then $X(t-1) = t \rightarrow$

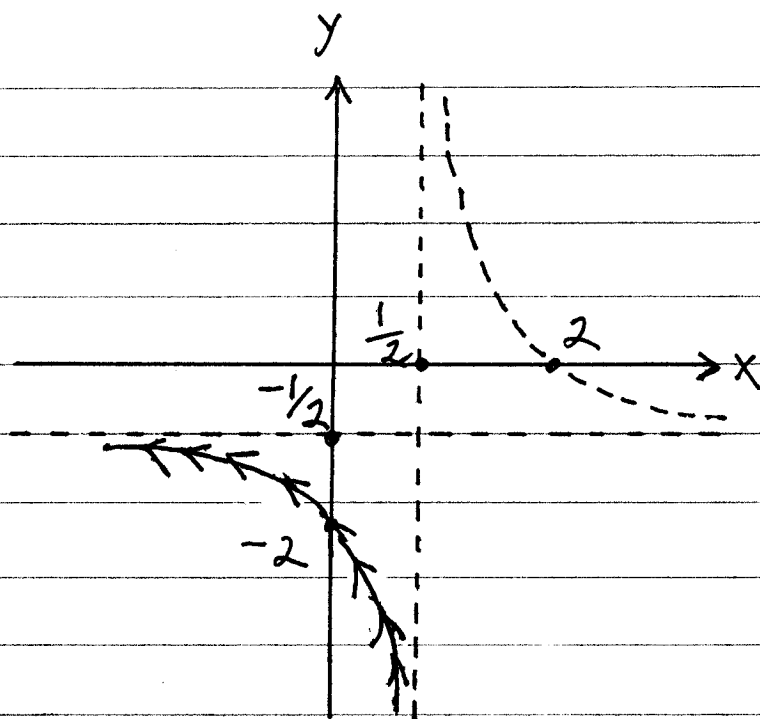
$$Xt - X = t \rightarrow Xt - t = X \rightarrow t(X-1) = X$$

$$\rightarrow t = \frac{X}{X-1} \rightarrow \text{(SUB)} \rightarrow Y = \frac{\left(\frac{X}{X-1}\right) - 2}{\left(\frac{X}{X-1}\right) + 1} \cdot \frac{X-1}{X-1}$$

$$\rightarrow Y = \frac{X - 2(X-1)}{X + (X-1)}$$

$$\rightarrow Y = \frac{2-X}{2X-1}$$

hyperbola,
left half only

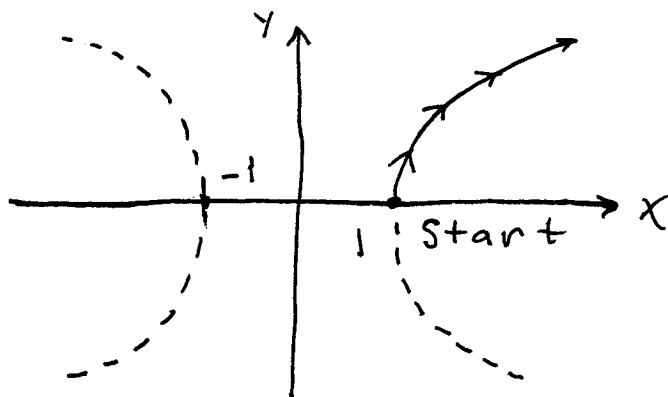


$$14.) \begin{cases} X = \sqrt{t+1} \\ Y = \sqrt{t} \end{cases} \quad \text{for } t \geq 0$$

$$\begin{cases} X^2 = t+1 \\ Y^2 = t \end{cases} \quad \rightarrow \quad X^2 = Y^2 + 1$$

$$\boxed{X^2 - Y^2 = 1}$$

(hyperbola)
with
 $X \geq 0, Y \geq 0$



$$16.) \begin{cases} X = -\sec t \\ Y = \tan t \end{cases}$$

$$\text{for } -\frac{\pi}{2} < t < \frac{\pi}{2}$$

$$\begin{cases} X^2 = \sec^2 t \\ Y^2 = \tan^2 t \end{cases}$$

$$1 + \tan^2 t = \sec^2 t \rightarrow$$

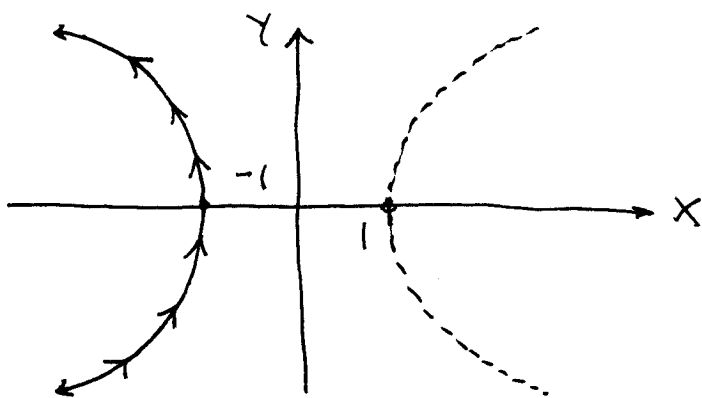
$$1 + Y^2 = X^2 \rightarrow$$

$$x^2 - y^2 = 1$$

(hyperbola)

$$x < 0$$

$$-\infty < y < \infty$$

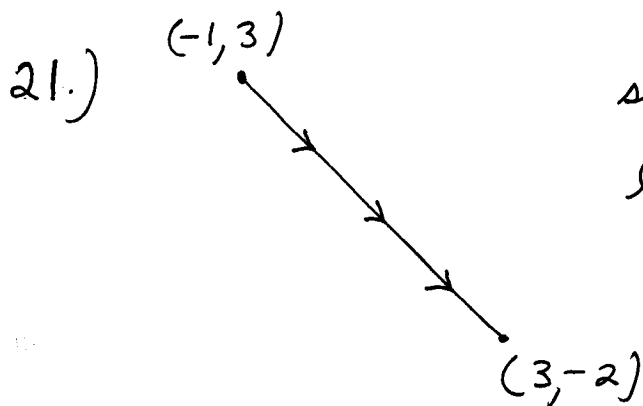


19.) a.)
$$\begin{cases} x = a \sin(t + \frac{\pi}{2}) \\ y = a \cos(t + \frac{\pi}{2}) \end{cases} \text{ for } 0 \leq t \leq 2\pi$$

b.)
$$\begin{cases} x = a \cos t \\ y = a \sin t \end{cases} \text{ for } 0 \leq t \leq 2\pi$$

c.)
$$\begin{cases} x = a \sin(t + \frac{\pi}{2}) \\ y = a \cos(t + \frac{\pi}{2}) \end{cases} \text{ for } 0 \leq t \leq 4\pi$$

d.)
$$\begin{cases} x = a \cos t \\ y = a \sin t \end{cases} \text{ for } 0 \leq t \leq 4\pi$$



$$\text{slope} = \frac{3 - (-2)}{-1 - 3} = \frac{-5}{4};$$

$$\text{line } y - 3 = \frac{-5}{4}(x - (-1)) \rightarrow$$

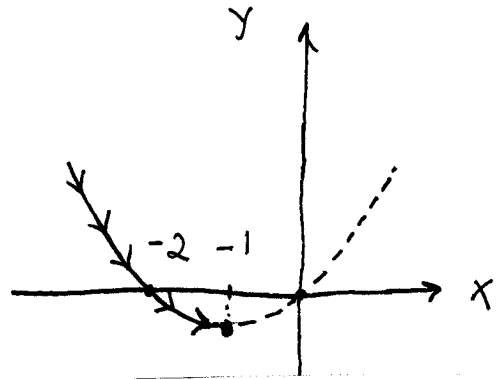
$$y - 3 = \frac{-5}{4}x - \frac{5}{4} \rightarrow$$

$$y = \frac{-5}{4}x + \frac{7}{4}$$

$$\begin{cases} x = t \\ y = \frac{-5}{4}t + \frac{7}{4} \end{cases} \text{ for } -1 \leq t \leq 3$$

$$24.) \quad Y = x^2 + 2x = x(x+2)$$

$$\begin{cases} x = t \\ y = t^2 + 2t \end{cases} \quad \text{for } t \leq -1$$



27.) SEE NEXT PAGE

28.) SEE NEXT PAGE

$$31.) \quad \tan \theta = \frac{y}{x}$$

$$= \frac{2 - \frac{1}{2}x}{x} = \frac{2}{x} - \frac{1}{2} \rightarrow$$

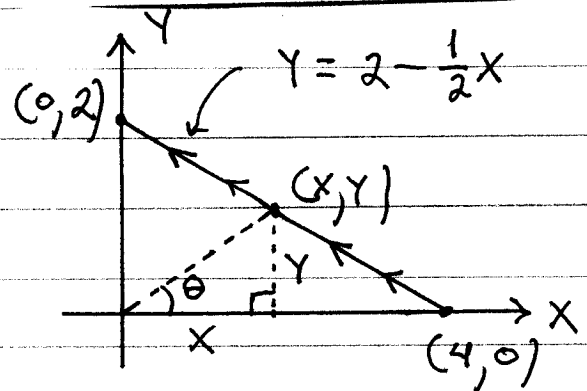
$$\frac{2}{x} = \frac{1}{2} + \tan \theta \rightarrow$$

$$\frac{x}{2} = \frac{1}{\frac{1}{2} + \tan \theta} \rightarrow x = \frac{2}{\frac{1}{2} + \tan \theta} \cdot \frac{2}{2} = \frac{4}{1 + 2 \tan \theta} ;$$

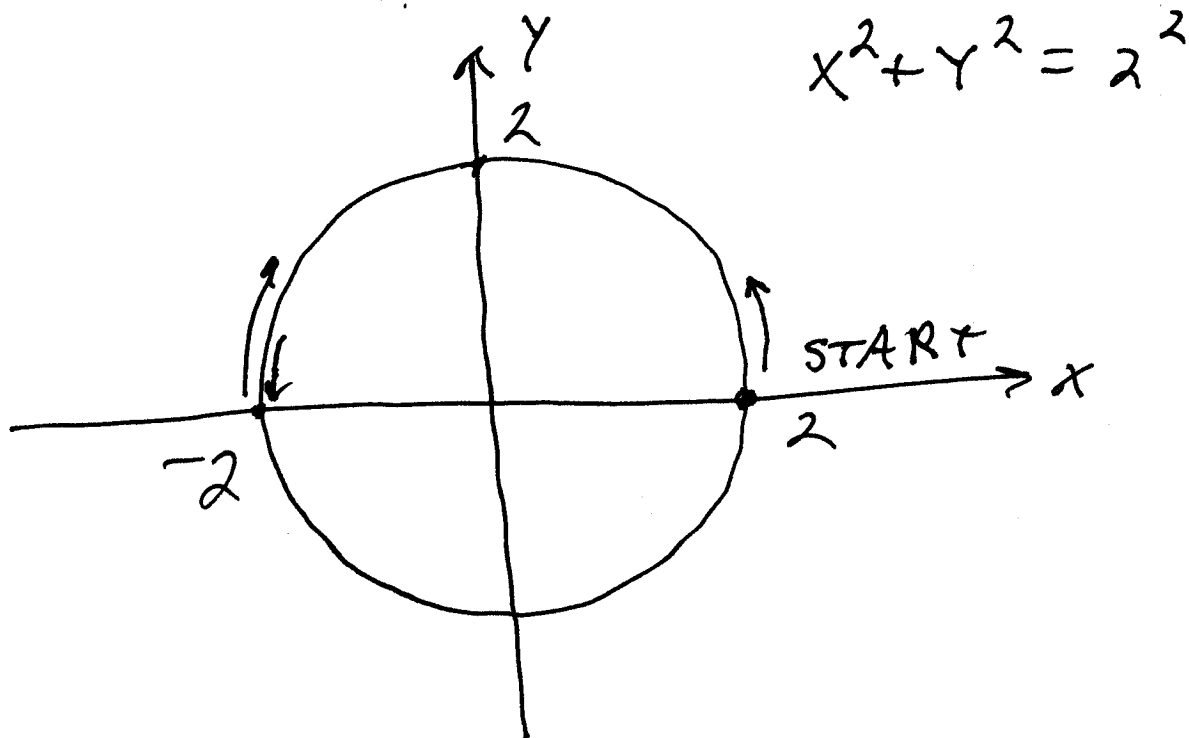
$$y = 2 - \frac{1}{2}x = 2 - \frac{1}{2} \cdot \frac{4}{1 + 2 \tan \theta} = \frac{2}{1} - \frac{2}{1 + 2 \tan \theta}$$

$$= \frac{2 + 4 \tan \theta - 2}{1 + 2 \tan \theta} = \frac{4 \tan \theta}{1 + 2 \tan \theta} \quad \text{so}$$

$$\begin{cases} x = \frac{4}{1 + 2 \tan \theta} \\ y = \frac{4 \tan \theta}{1 + 2 \tan \theta} \end{cases} \quad \text{for } 0 \leq \theta \leq \frac{\pi}{2}$$

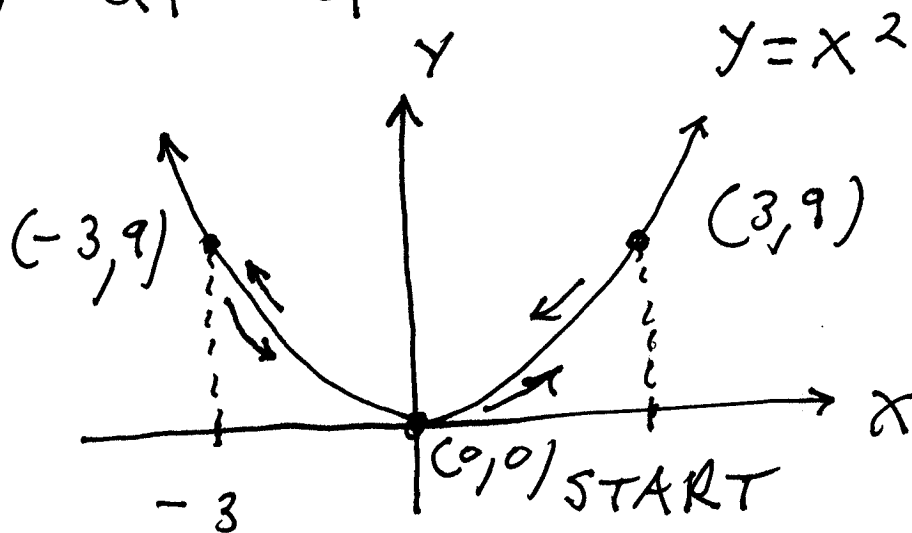


27.)



$$\begin{cases} x = 2 \cos t \\ y = 2 |\sin t| \end{cases} \text{ for } 0 \leq t \leq 4\pi$$

28.)

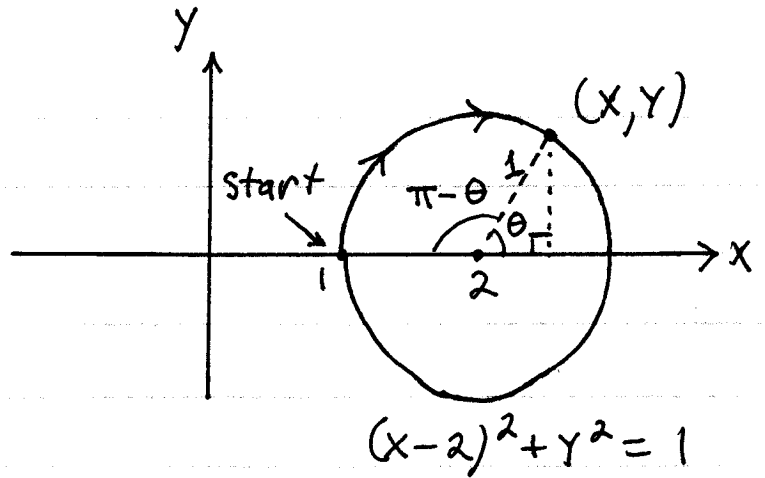


$$\begin{cases} x = 3 \sin t \\ y = 9 \sin^2 t \end{cases} \text{ for } t \geq 0$$

33.)

$$\begin{cases} x = 2 + \cos(\pi - \theta) \\ y = \sin(\pi - \theta) \end{cases}$$

for $0 \leq \theta \leq 2\pi$



34.) $\tan \theta = \frac{y}{x+2}$

$$= \frac{\sqrt{1-x^2}}{x+2} \rightarrow$$

$$\tan \theta = \frac{\sqrt{1-x^2}}{x+2} \rightarrow$$

$$\tan^2 \theta = \left(\frac{\sqrt{1-x^2}}{x+2} \right)^2 = \frac{1-x^2}{x^2+4x+4} \rightarrow$$

A

$$A(x^2+4x+4) = 1-x^2 \rightarrow Ax^2+4Ax+4A+x^2-1=0 \rightarrow$$

$$(A+1)x^2+(4A)x+(4A-1)=0 \rightarrow$$

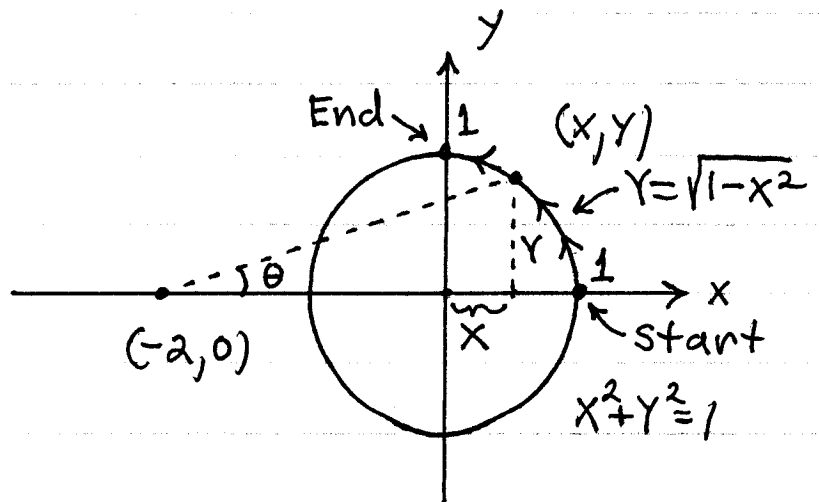
$$x = \frac{-4A \pm \sqrt{(4A)^2 - 4(A+1)(4A-1)}}{2(A+1)}$$

$$= \frac{-4A \pm \sqrt{16A^2 - 4(4A^2+3A-1)}}{2(A+1)}$$

$$= \frac{-4A \pm \sqrt{16A^2 - 16A^2 - 12A + 4}}{2(A+1)}$$

$$= \frac{-4A \pm \sqrt{4(1-3A)}}{2(A+1)} = \frac{2(-2A) \pm 2\sqrt{1-3A}}{2(A+1)}$$

x is ≥ 0



$$\rightarrow X = \frac{-2A + \sqrt{1-3A}}{A+1}; \text{ then}$$

$$Y = \sqrt{1-X^2} = \sqrt{1 - \left(\frac{\sqrt{1-3A} - 2A}{A+1}\right)^2}$$

$$= \sqrt{\frac{(A+1)^2}{(A+1)^2} - \frac{1-3A-4A\sqrt{1-3A}+4A^2}{(A+1)^2}}$$

$$= \sqrt{\frac{A^2+2A+1-1+3A+4A\sqrt{1-3A}-4A^2}{(A+1)}}$$

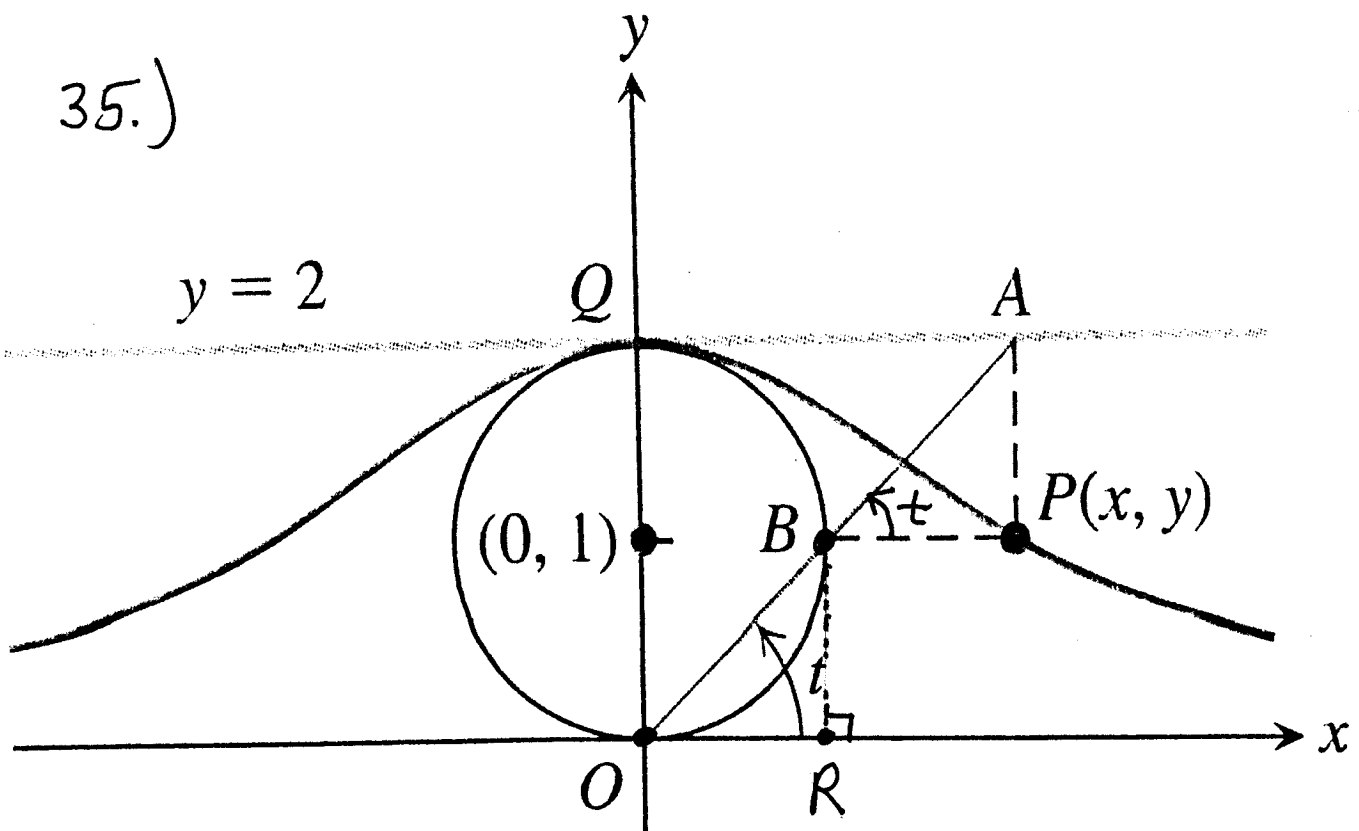
$$= \frac{\sqrt{5A-3A^2+4A\sqrt{1-3A}}}{A+1}; \text{ thus,}$$

$$\left\{ \begin{array}{l} X = \frac{-2\tan^2\theta + \sqrt{1-3\tan^2\theta}}{\tan^2\theta + 1} \end{array} \right.$$

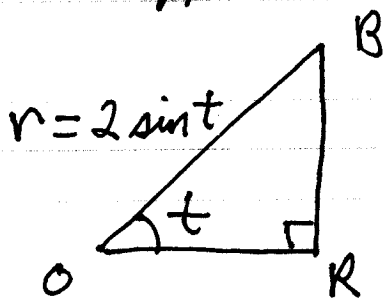
$$Y = \frac{\sqrt{5\tan^2\theta - 3\tan^4\theta + 4\tan^2\theta\sqrt{1-3\tan^2\theta}}}{\tan^2\theta + 1}$$

$$\text{for } 0 \leq \theta \leq \arctan\left(\frac{1}{2}\right)$$

35.)



Construct right $\triangle OBR$, which is similar to right $\triangle BAP$. By polar form of circle we know the hypotenuse of $\triangle OBR$ is $r = 2 \sin t$:



$$\cos t = \frac{|OR|}{2 \sin t} \rightarrow$$

$$|OR| = 2 \sin t \cos t ;$$

$$\sin t = \frac{|BR|}{2 \sin t} \rightarrow |BR| = 2 \sin^2 t ,$$

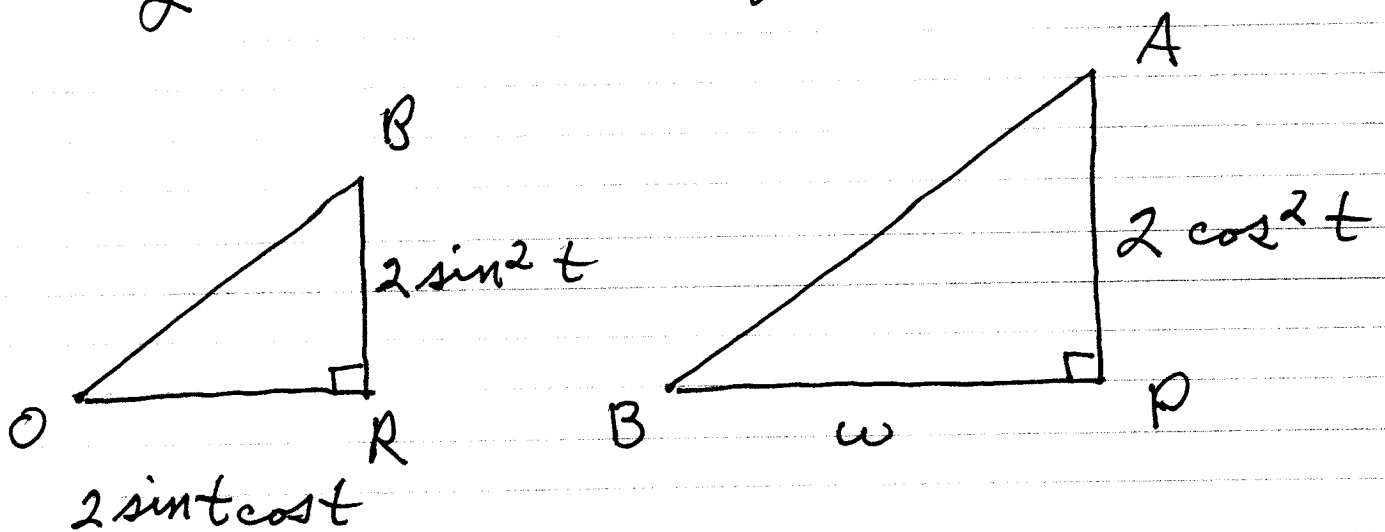
thus, $\boxed{y = 2 \sin^2 t}$ (since

points B and P have the same y-value.)

It follows that the height of $\triangle BAP$ is

$$2 - |BR| = 2 - 2 \sin^2 t \\ = 2(1 - \sin^2 t) = 2 \cos^2 t;$$

by similar triangles:



$$\frac{w}{2 \cos^2 t} = \frac{2 \sin t \cos t}{2 \sin^2 t} \rightarrow$$

$$w = \frac{2 \sin t \cos^3 t}{\sin^2 t} = 2 \frac{\cos^3 t}{\sin t};$$

now variable x is

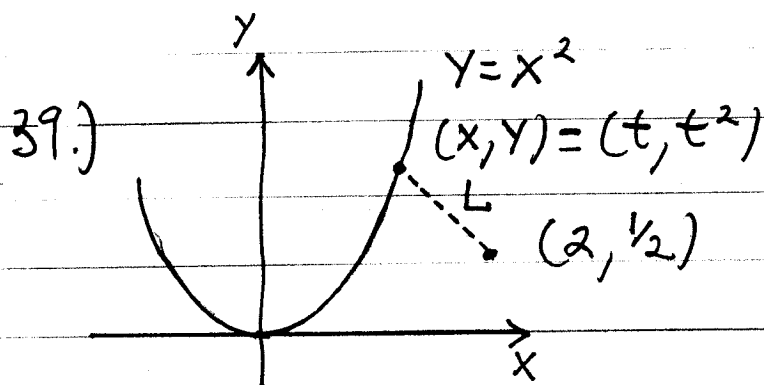
$$x = |OR| + |BP| = 2 \sin t \cos t + \frac{2 \cos^3 t}{\sin t} \\ = \frac{2 \sin^2 t \cos t}{\sin t} + \frac{2 \cos^3 t}{\sin t}$$

$$= \frac{2 \cos t (\sin^2 t + \cos^2 t)}{\sin t}$$

$$= \frac{2 \cos t}{\sin t} (1) = 2 \cot t ;$$

thus parameterization is

$$\begin{cases} x = 2 \cot t \\ y = 2 \sin^2 t \end{cases} \text{ for } 0 < t < \pi .$$



Minimize
distance

$$L = \sqrt{(t-2)^2 + (t^2 - \frac{1}{2})^2}$$

$$\xrightarrow{D} L' = \frac{1}{2} \left((t-2)^2 + (t^2 - \frac{1}{2})^2 \right)^{-\frac{1}{2}} [2(t-2) + 2(t^2 - \frac{1}{2}) \cdot 2t]$$

$$= \frac{1}{2} \cdot 2 \frac{[(t-2) + (t^2 - \frac{1}{2})2t]}{\sqrt{(t-2)^2 + (t^2 - \frac{1}{2})^2}} = 0 \rightarrow$$

$$\cancel{t} - 2 + 2t^3 - \cancel{t} = 0 \rightarrow 2t^3 = 2 \rightarrow t^3 = 1 \rightarrow t = 1:$$

$$\begin{array}{c} - \quad 0 \quad + \\ \hline \quad \quad \quad | \quad \quad \quad \\ \quad \quad \quad t=1 \end{array} \quad L'$$

point $\begin{cases} X=t=1 \\ Y=t^2=1 \end{cases}$ and min. distance

$$L = \sqrt{(1-2)^2 + (1 - \frac{1}{2})^2} = \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$