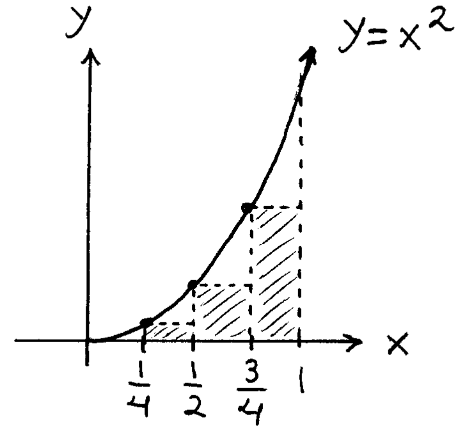
Section 5.1

1.) b.) Lower Sum :

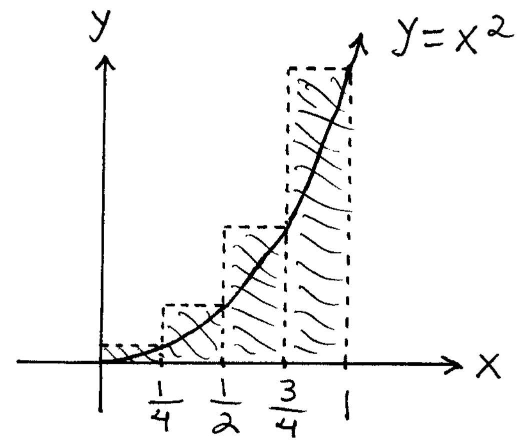


$(0)^2 \cdot \frac{1}{4} + \left(\frac{1}{4}\right)^2 \cdot \frac{1}{4} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4}$

$= \frac{1}{4}\left(\frac{1}{16} + \frac{1}{4} + \frac{9}{16}\right)$

$= \frac{1}{4} \cdot \frac{14}{16} = \frac{7}{32}$

d.) Upper Sum :

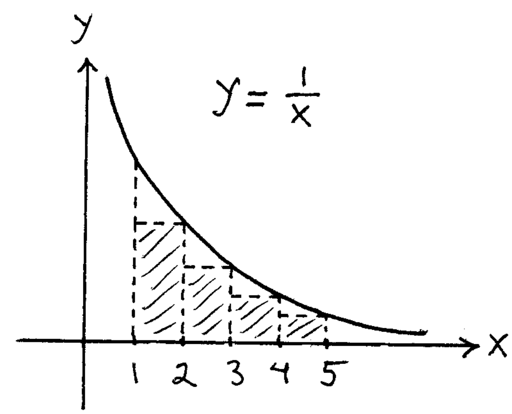


$\left(\frac{1}{4}\right)^2 \cdot \frac{1}{4} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4} + (1)^2 \cdot \frac{1}{4}$

$= \frac{1}{4}\left(\frac{1}{16} + \frac{1}{4} + \frac{9}{16} + 1\right)$

$= \frac{1}{4} \cdot \frac{30}{16} = \frac{15}{32}$

3.) b.) Lower Sum :

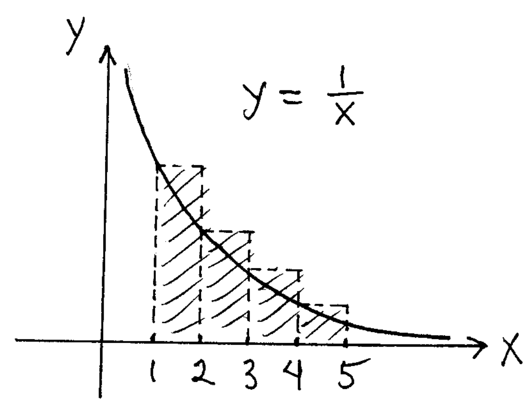


$\frac{1}{2} \cdot 1 + \frac{1}{3} \cdot 1 + \frac{1}{4} \cdot 1 + \frac{1}{5} \cdot 1$

$= \frac{30}{60} + \frac{20}{60} + \frac{15}{60} + \frac{12}{60}$

$= \frac{77}{60}$

d.) Upper Sum :



$1 \cdot 1 + \frac{1}{2} \cdot 1 + \frac{1}{3} \cdot 1 + \frac{1}{4} \cdot 1$

$= \frac{12}{12} + \frac{6}{12} + \frac{4}{12} + \frac{3}{12}$

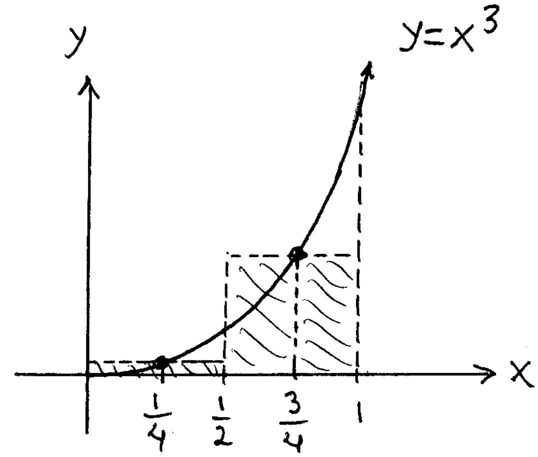
$= \frac{25}{12}$

6.) a.) midpoint Sum:

$$\left(\tfrac{1}{4}\right)^3 \cdot \tfrac{1}{2} + \left(\tfrac{3}{4}\right)^3 \cdot \tfrac{1}{2}$$

$$= \tfrac{1}{2}\left(\tfrac{1}{64} + \tfrac{27}{64}\right)$$

$$= \tfrac{1}{2} \cdot \tfrac{28}{64} = \tfrac{14}{64} = \tfrac{7}{32}$$

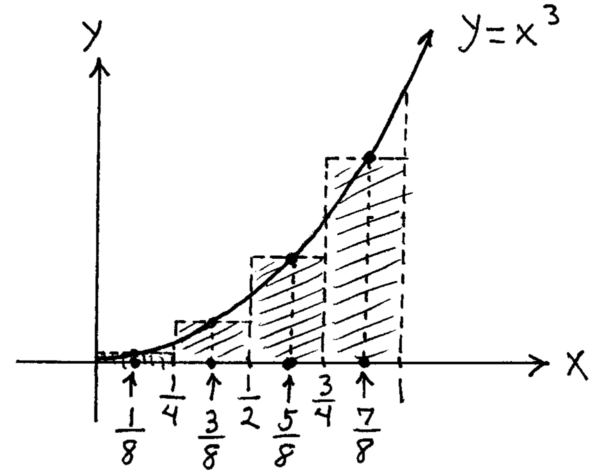


b.) midpoint Sum:

$$\left(\tfrac{1}{8}\right)^3 \cdot \tfrac{1}{4} + \left(\tfrac{3}{8}\right)^3 \cdot \tfrac{1}{4} + \left(\tfrac{5}{8}\right)^3 \cdot \tfrac{1}{4} + \left(\tfrac{7}{8}\right)^3 \cdot \tfrac{1}{4}$$

$$= \tfrac{1}{4}\left(\tfrac{1}{512} + \tfrac{27}{512} + \tfrac{125}{512} + \tfrac{343}{512}\right)$$

$$= \tfrac{1}{4} \cdot \tfrac{496}{512} = \tfrac{124}{512} = \tfrac{31}{128}$$



10.)

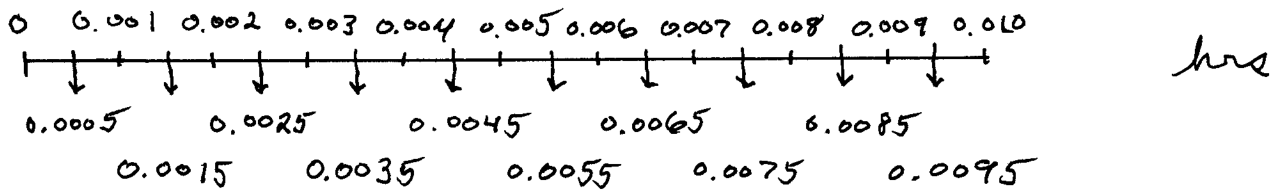
minutes	0	5	10	15	20	25	30	35	40	45	50	55	60
seconds	0	300	600	900	1200	1500	1800	2100	2400	2700	3000	3300	3600

Distance = Velocity × Time
(m.) (m./sec.) (sec.)

a.) Distance $\approx (1)(300) + (1.2)(300) + (1.7)(300) + (2.0)(300) + (1.8)(300) + (1.6)(300) + (1.4)(300) + (1.2)(300) + (1.0)(300) + (1.8)(300) + (1.5)(300) + (1.2)(300) = 5220$ m.

b.) Distance $\approx (1.2)(300) + (1.7)(300) + (2.0)(300) + (1.8)(300) + (1.6)(300) + (1.4)(300) + (1.2)(300) + (1.0)(300) + (1.8)(300) + (1.5)(300) + (1.2)(300) + (0)(300) = 4920$ m.

12.) Use midpoints of subintervals to determine velocities, which can be estimated from the graph:

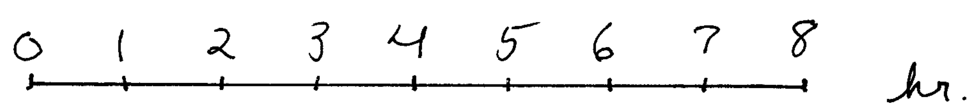


a.) Distance = Velocity × Time
(mi.) (mph) (hr.)

Distance $\approx$ (20)(0.001) + (50)(0.001) + (72)(0.001) + (90)(0.001) + (103)(0.001) + (112)(0.001) + (120)(0.001) + (128)(0.001) + (134)(0.001) + (140)(0.001) = 0.969 mi.

b.) at approx. $t = 0.006$ hrs. the car reaches its halfway point of 0.4845 mi ($\frac{1}{2}$ of area under graph); its speed at that time is approx. 118 mph.

19.)



a.) Leakage = Rate × Time
(gal.) (gal./hr.) (hr.)

i.) Lower Estimate (left-hand endpoints):

Leakage $\approx$ (50)(1) + (70)(1) + (97)(1) + (136)(1) + (190)(1) = 543 gal.

ii.) Upper Estimate (right-hand endpoints):

Leakage $\approx (70)(1) + (97)(1) + (136)(1) + (190)(1) + (265)(1) = 758$ gal.

b.) i.) Lower Estimate:

Leakage $\approx (50)(1) + (70)(1) + (97)(1) + (136)(1) + (190)(1) + (265)(1) + (369)(1) + (516)(1) = 1693$ gal.

ii.) Upper Estimate:

Leakage $\approx (70)(1) + (97)(1) + (136)(1) + (190)(1) + (265)(1) + (369)(1) + (516)(1) + (720)(1) = 2363$ gal.

c.) <u>after 8 hrs</u>. leakage rate is constant at 720 gal./hr. ;

i.) worst case (shortest time):

$25{,}000 - 2363 = 22{,}637$ gal. ;

$$\frac{22{,}637 \text{ gal.}}{720 \frac{\text{gal.}}{\text{hr.}}} \approx 31.44 \text{ more hours}$$

ii.) best case (longest time):

$25{,}000 - 1693 = 23{,}307$ gal. ;

$$\frac{23{,}307 \text{ gal.}}{720 \frac{\text{gal.}}{\text{hr.}}} \approx 32.37 \text{ more hours}$$

Section 5.2

2.) $\sum_{k=1}^{3} \frac{k-1}{k} = \frac{0}{1} + \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$

3.) $\sum_{k=1}^{4} \cos k\pi = \cos \pi + \cos 2\pi + \cos 3\pi + \cos 4\pi$

$= (-1) + (1) + (-1) + (1) = 0$

4.) $\sum_{k=1}^{5} \sin k\pi = \sin \pi + \sin 2\pi + \sin 3\pi + \sin 4\pi$

$= (0) + (0) + (0) + (0) = 0$

5.) $\sum_{k=1}^{3} (-1)^{k+1} \cdot \sin \frac{\pi}{k} = (1) \cdot \sin \pi + (-1) \cdot \sin \frac{\pi}{2} + (1) \cdot \sin \frac{\pi}{3}$

$= (1)(0) + (-1)(1) + (1)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2} - 1$

7.) all of them.

8.) a.) yes b.) yes c.) no

11.) $1+2+3+4+5+6 = \sum_{i=1}^{6} i$

12.) $1+4+9+16 = 1^2+2^2+3^2+4^2$

$= \sum_{i=1}^{4} i^2$

16.) $-\frac{1}{5} + \frac{2}{5} - \frac{3}{5} + \frac{4}{5} - \frac{5}{5} = \sum_{i=1}^{5} (-1)^i \cdot \frac{i}{5}$

17.) a.) $\sum_{k=1}^{n} 3a_k = 3 \cdot \sum_{k=1}^{n} a_k = 3(-5) = -15$

b.) $\sum_{k=1}^{n} \frac{b_k}{6} = \frac{1}{6} \cdot \sum_{k=1}^{n} b_k = \frac{1}{6}(6) = 1$

c.) $\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$

$= (-5) + (6) = 1$

d.) $\sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$

$= (-5) - (6) = -11$

e.) $\sum_{k=1}^{n} (b_k - 2a_k) = \sum_{k=1}^{n} b_k - 2 \cdot \sum_{k=1}^{n} a_k$

$= (6) - 2(-5) = 16$

20.) a.) $\sum_{k=1}^{13} k = 1+2+3+\cdots+13 = \frac{13(14)}{2} = 91$

b.) $\sum_{k=1}^{13} k^2 = 1^2+2^2+3^2+\cdots+13^2$

$= \frac{13(14)\cdot(26+1)}{6} = 819$

c.) $\sum_{k=1}^{13} k^3 = 1^3+2^3+3^3+\cdots+13^3$

$= \left(\frac{13(14)}{2}\right)^2 = 8281$

23.) $\sum_{k=1}^{6} (3-k^2) = \sum_{k=1}^{6} 3 - \sum_{k=1}^{6} k^2$

$= 3(6) - \frac{6(7)(13)}{6} = 18 - 91 = -73$

25.) $\sum_{k=1}^{5} k(3k+5) = \sum_{k=1}^{5} (3k^2+5k)$

$= 3 \cdot \sum_{k=1}^{5} k^2 + 5 \cdot \sum_{k=1}^{5} k$

$= 3 \cdot \frac{5(6)(11)}{6} + 5 \cdot \frac{5(6)}{2} = 240$

28.) $\left(\sum_{k=1}^{7} k\right)^2 - \sum_{k=1}^{7} \frac{k^3}{4} = \left(\frac{7(8)}{2}\right)^2 - \frac{1}{4} \cdot \sum_{k=1}^{7} k^3$

$= 784 - \frac{1}{4} \cdot \left(\frac{7(8)}{2}\right)^2 = 588$

29.) a.) $\sum_{k=1}^{7} 3 = 7(3) = 21$

b.) $\sum_{k=1}^{500} 7 = 500(7) = 3500$

c.) $\sum_{k=3}^{264} 10 = \sum_{k=1}^{264} 10 - \sum_{k=1}^{2} 10$

$= 264(10) - 2(10) = 2620$

30.) a.) $\sum_{k=9}^{36} k = \sum_{k=1}^{36} k - \sum_{k=1}^{8} k$

$= \frac{1}{2}(36)(36+1) - \frac{1}{2}(8)(8+1) = 630$

b.) $\sum_{k=3}^{17} k^2 = \sum_{k=1}^{17} k^2 - \sum_{k=1}^{2} k^2$

$= \frac{1}{6}(17)(17+1)(34+1) - (1^2+2^2)$

$= 1780$

c.) $\sum_{k=18}^{71} k(k-1) = \sum_{k=18}^{71} (k^2 - k)$

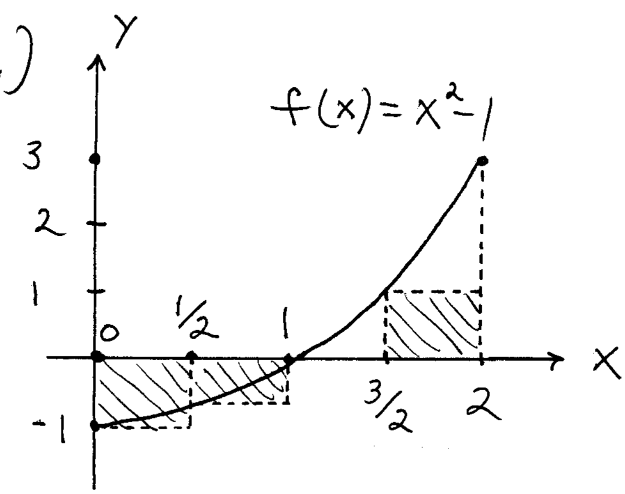
$= \sum_{k=18}^{71} k^2 - \sum_{k=18}^{71} k$

$= \left\{ \sum_{k=1}^{71} k^2 - \sum_{k=1}^{17} k^2 \right\} - \left\{ \sum_{k=1}^{71} k - \sum_{k=1}^{17} k \right\}$

$= \frac{1}{6}(71)(71+1)(142+1) - \frac{1}{6}(17)(18)(34+1)$

$- \left\{ \frac{1}{2}(71)(71+1) - \frac{1}{2}(17)(17+1) \right\} = 117{,}648$

33.) a.)



$\Delta x_i = 1/2$ for $i = 1, 2, 3, 4$;

$c_1 = 0$, $c_2 = 1/2$, $c_3 = 1$, $c_4 = 3/2$;

$\sum_{i=1}^{4} f(c_i) \cdot \Delta x_i = f(c_1) \cdot \Delta x_1 + f(c_2) \cdot \Delta x_2$

$+ f(c_3) \cdot \Delta x_3 + f(c_4) \cdot \Delta x_4$

$= f(0) \cdot \frac{1}{2} + f\left(\frac{1}{2}\right) \cdot \frac{1}{2} + f(1) \cdot \frac{1}{2} + f\left(\frac{3}{2}\right) \cdot \frac{1}{2}$

$= (-1) \cdot \frac{1}{2} + \left(\frac{-3}{4}\right) \cdot \frac{1}{2} + (0) \cdot \frac{1}{2} + \left(\frac{5}{4}\right) \cdot \frac{1}{2}$

$= \frac{-4}{8} + \frac{-3}{8} + 0 + \frac{5}{8}$

$= \frac{-2}{8} = \frac{-1}{4}$