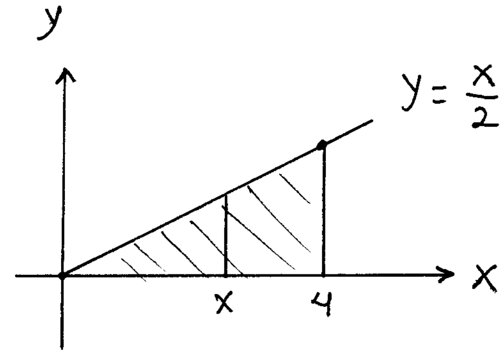
Section 6.4

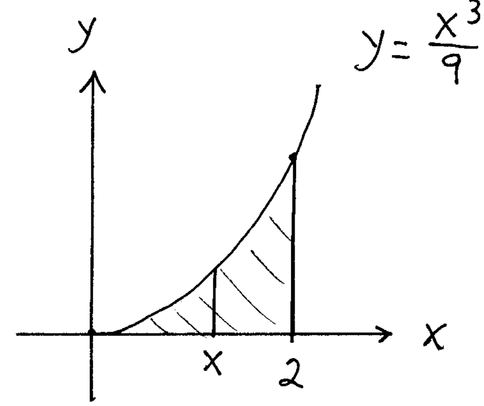


9.) $\text{Area} = 2\pi \int_0^4 f(x)\cdot\sqrt{1+(f'(x))^2}\,dx$

$= 2\pi \int_0^4 \left(\frac{x}{2}\right)\cdot\sqrt{1+\left(\frac{1}{2}\right)^2}\,dx$

$= \pi \int_0^4 x\cdot\sqrt{\frac{5}{4}}\,dx$

$= \frac{\sqrt{5}}{2}\pi\cdot\frac{x^2}{2}\Big|_0^4 = 4\sqrt{5}\,\pi$



13.) $\text{Area} = 2\pi \int_0^2 f(x)\cdot\sqrt{1+(f'(x))^2}\,dx$

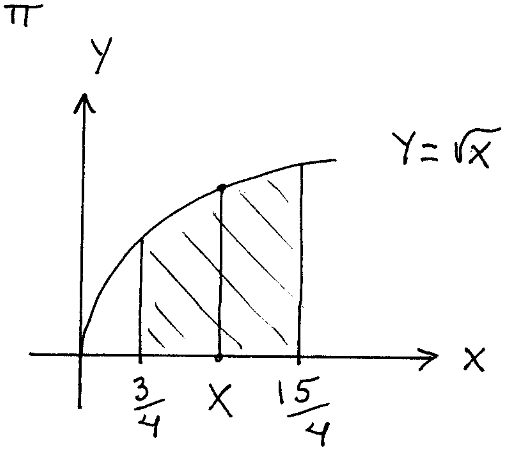
$= 2\pi \int_0^2 \frac{x^3}{9}\cdot\sqrt{1+\left(\frac{x^2}{3}\right)^2}\,dx$

$= \frac{2}{9}\pi \int_0^2 x^3\cdot\sqrt{1+\frac{1}{9}x^4}\,dx$

$= \cancel{\frac{2}{9}}\pi\cdot\cancel{\frac{9}{4}}\frac{\left(1+\frac{1}{9}x^4\right)^{3/2}}{3/\cancel{2}}\Big|_0^2$

$= \frac{\pi}{3}\left(\left(1+\frac{16}{9}\right)^{3/2}-1^{3/2}\right) = \frac{\pi}{3}\left(\left(\frac{25}{9}\right)^{3/2}-1\right)$

$= \frac{\pi}{3}\left(\frac{125}{27}-1\right) = \frac{\pi}{3}\cdot\frac{98}{27} = \frac{98}{81}\pi$



14.) $\text{Area} = 2\pi \int_{\frac{3}{4}}^{\frac{15}{4}} f(x)\cdot\sqrt{1+(f'(x))^2}\,dx$

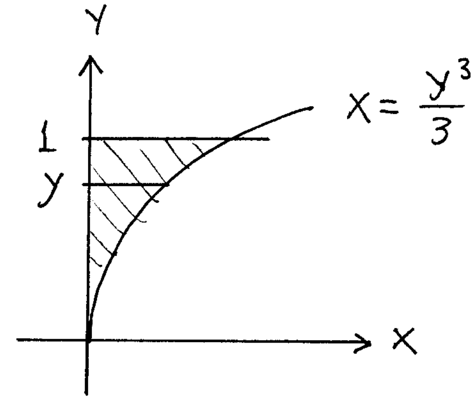
$= 2\pi \int_{3/4}^{15/4} \sqrt{x}\cdot\sqrt{1+\left(\frac{1}{2\sqrt{x}}\right)^2}\,dx$

$= 2\pi \int_{3/4}^{15/4} \sqrt{x}\cdot\sqrt{1+\frac{1}{4x}}\,dx$

$= 2\pi \int_{\frac{3}{4}}^{\frac{15}{4}} \sqrt{x}\cdot\sqrt{\frac{4x+1}{4x}}\,dx = \cancel{2}\pi \int_{\frac{3}{4}}^{\frac{15}{4}} \cancel{\sqrt{x}}\cdot\frac{\sqrt{4x+1}}{\cancel{2}\cancel{\sqrt{x}}}\,dx$

$$= \pi \cdot \frac{1}{4}\, \frac{(4x+1)^{3/2}}{3/2} \Bigg|_{3/4}^{15/4} = \frac{\pi}{6}\left(16^{3/2} - 4^{3/2}\right)$$

$$= \frac{\pi}{6}(64 - 8) = \frac{56}{6}\pi = \frac{28}{3}\pi$$



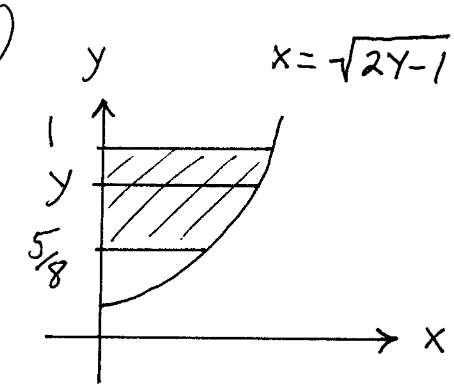
17.) $$\text{Area} = 2\pi \int_0^1 g(y) \cdot \sqrt{1 + (g'(y))^2}\, dy$$

$$= 2\pi \int_0^1 \left(\frac{y^3}{3}\right) \cdot \sqrt{1 + (y^2)^2}\, dy$$

$$= \frac{2}{3}\pi \int_0^1 y^3 \cdot \sqrt{1 + y^4}\, dy$$

$$= \frac{\cancel{2}}{3}\pi \cdot \frac{1}{\cancel{4}}\, \frac{(1+y^4)^{3/2}}{3/\cancel{2}} \Bigg|_0^1$$

$$= \frac{\pi}{9} \cdot \left(2^{3/2} - 1^{3/2}\right) = \frac{\pi}{9}\left(2^{3/2} - 1\right)$$



20.) $$\text{Area} = 2\pi \int_{5/8}^1 g(y) \cdot \sqrt{1 + (g'(y))^2}\, dy$$

$$= 2\pi \int_{5/8}^1 \sqrt{2y-1} \cdot \sqrt{1 + \left(\frac{1}{\cancel{2}}(2y-1)^{-1/2}(\cancel{2})\right)^2}\, dy$$

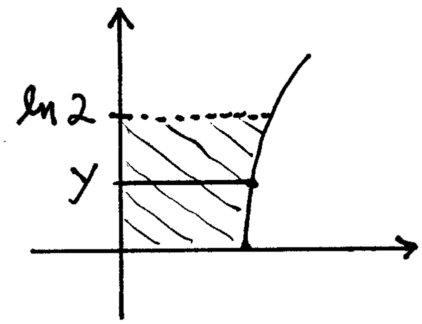
$$= 2\pi \int_{5/8}^1 \sqrt{2y-1} \cdot \sqrt{1 + \frac{1}{2y-1}}\, dy$$

$$= 2\pi \int_{5/8}^1 \sqrt{2y-1} \cdot \sqrt{\frac{2y}{2y-1}}\, dy = 2\sqrt{2}\pi \int_{5/8}^1 \sqrt{y}\, dy$$

$$= 2\sqrt{2}\pi\, \frac{y^{3/2}}{3/2} \Bigg|_{5/8}^1 = \frac{4}{3}\sqrt{2}\pi\left(1^{3/2} - \left(\frac{5}{8}\right)^{3/2}\right)$$

$$= \frac{4}{3}\sqrt{2}\pi\left(1 - \left(\frac{5}{8}\right)^{3/2}\right)$$

$$x = \frac{1}{2}(e^{y} + e^{-y})$$



21.) $$\text{Area} = 2\pi \int_0^{\ln 2} g(y)\sqrt{1 + (g'(y))^2}\, dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^{y} + e^{-y})\sqrt{1 + \left(\frac{1}{2}(e^{y} - e^{-y})\right)^2}\, dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^{y} + e^{-y})\sqrt{1 + \frac{1}{4}(e^{2y} - 2 + e^{-2y})}\, dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^{y} + e^{-y})\sqrt{\frac{1}{4}e^{2y} + \frac{1}{2} + \frac{1}{4}e^{-2y}}\, dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^{y} + e^{-y})\sqrt{\left(\frac{1}{2}e^{y}\right)^2 + \frac{1}{2} + \left(\frac{1}{2}e^{-y}\right)^2}\, dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^{y} + e^{-y})\sqrt{\left(\frac{1}{2}e^{y} + \frac{1}{2}e^{-y}\right)^2}\, dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2}(e^{y} + e^{-y})\left(\frac{1}{2}e^{y} + \frac{1}{2}e^{-y}\right) dy$$

$$= 2\pi \int_0^{\ln 2} \frac{1}{2} \cdot \frac{1}{2} \cdot (e^{y} + e^{-y})^2\, dy$$

$$= \frac{\pi}{2} \int_0^{\ln 2} (e^{2y} + 2 + e^{-2y})\, dy$$

$$= \frac{\pi}{2}\left(\frac{1}{2}e^{2y} + 2y - \frac{1}{2}e^{-2y}\right)\Big|_0^{\ln 2}$$

$$= \frac{\pi}{2} \cdot \frac{1}{2}\left(e^{2y} + 4y - e^{-2y}\right)\Big|_0^{\ln 2}$$

$$= \frac{\pi}{4}\left(e^{2\ln 2} + 4\ln 2 - e^{-2\ln 2}\right) - \frac{\pi}{4}(1 + 0 - 1)$$

$$= \frac{\pi}{4}\left(e^{\ln 4} + 4\ln 2 - e^{\ln \frac{1}{4}}\right)$$

$$= \frac{\pi}{4}\left(4 + 4\ln 2 - \frac{1}{4}\right)$$

$$= \frac{\pi}{4}\left(\frac{15}{4} + 4\ln 2\right)$$

$$= \frac{15}{16}\pi + \pi \ln 2$$

24.) a.) $L = \int_1^2 \sqrt{1 + \frac{1}{y^4}}\, dy = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$\rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{y^4} \rightarrow \frac{dx}{dy} = \frac{1}{y^2} \rightarrow$

$x = \frac{-1}{y} + c$ and $x = 0,\ y = 1 \rightarrow$

$0 = -1 + c \rightarrow c = 1 \rightarrow x = \frac{-1}{y} + 1$.

25.) $\text{Area} = 2\pi \int_{-a}^{a} f(x) \cdot \sqrt{1 + (f'(x))^2}\, dx$

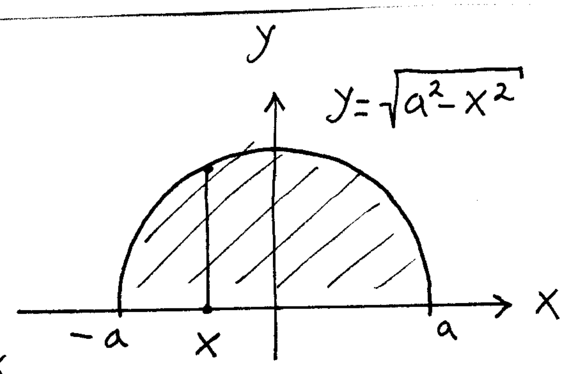
$= 2\pi \int_{-a}^{a} \sqrt{a^2 - x^2} \cdot \sqrt{1 + \left(\frac{1}{2}(a^2 - x^2)^{-1/2} \cdot (-2x)\right)^2}\, dx$

$= 2\pi \int_{-a}^{a} \sqrt{a^2 - x^2}\, \sqrt{1 + \frac{x^2}{a^2 - x^2}}\, dx$

$= 2\pi \int_{-a}^{a} \sqrt{a^2 - x^2} \cdot \sqrt{\frac{a^2}{a^2 - x^2}}\, dx$

$= 2\pi \int_{-a}^{a} \cancel{\sqrt{a^2 - x^2}} \cdot \frac{\sqrt{a^2}}{\cancel{\sqrt{a^2 - x^2}}}\, dx = 2\pi a \int_{-a}^{a} 1\, dx$

$= 2\pi a \cdot x \Big|_{-a}^{a} = 2\pi a\,(a - (-a)) = 4\pi a^2$



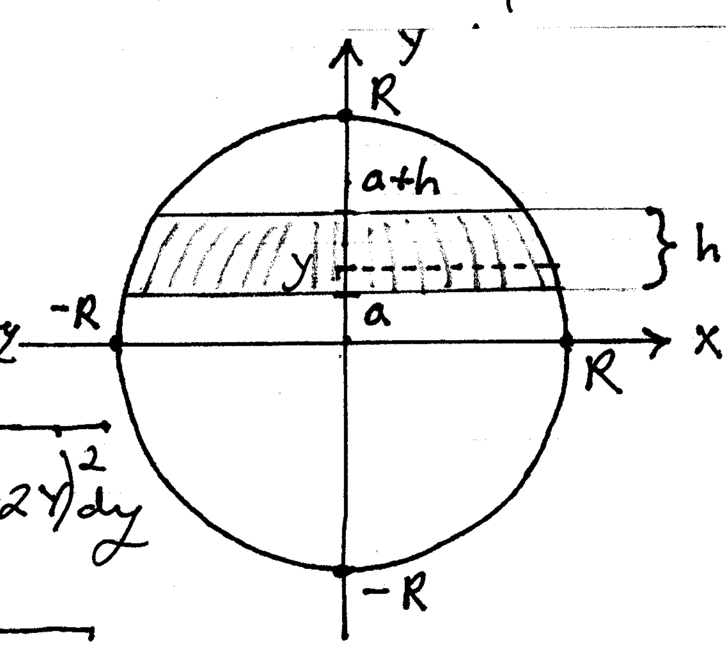
29.) $x^2 + y^2 = R^2 \rightarrow$

$x = \sqrt{R^2 - y^2}$; then

$\text{Area} = 2\pi \int_{a}^{a+h} g(y) \sqrt{1 + (g'(y))^2}\, dy$

$= 2\pi \int_{a}^{a+h} \sqrt{R^2 - y^2}\, \sqrt{1 + \left(\frac{1}{2}(R^2 - y^2)^{-1/2} \cdot -2y\right)^2}\, dy$

$= 2\pi \int_{a}^{a+h} \sqrt{R^2 - y^2}\, \sqrt{1 + \frac{y^2}{R^2 - y^2}}\, dy$



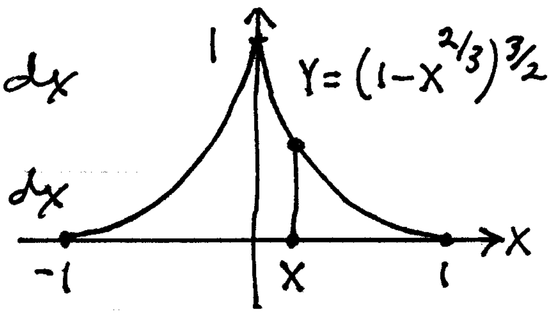
$$= 2\pi \int_a^{a+h} \sqrt{R^2-y^2}\,\sqrt{\frac{R^2-y^2}{R^2-y^2}+\frac{y^2}{R^2-y^2}}\,dy$$

$$= 2\pi \int_a^{a+h} \sqrt{R^2-y^2}\cdot\sqrt{\frac{R^2}{R^2-y^2}}\,dy$$

$$= 2\pi \int_a^{a+h} R\,dy = 2\pi\cdot Ry\Big|_a^{a+h}$$

$$= 2\pi R\,[(a+h)-a] = 2\pi R h$$

32.) $\text{Area} = 2\pi \int_{-1}^{1} f(x)\sqrt{1+(f'(x))^2}\,dx$



$$= 2\cdot 2\pi \int_0^1 (1-x^{2/3})^{3/2}\sqrt{1+\left(\frac{3}{2}(1-x^{2/3})^{1/2}\cdot\frac{-2}{3}x^{-1/3}\right)^2}\,dx$$

$$= 4\pi \int_0^1 (1-x^{2/3})^{3/2}\sqrt{1+\frac{1-x^{2/3}}{x^{2/3}}}\,dx$$

$$= 4\pi \int_0^1 (1-x^{2/3})^{3/2}\sqrt{\frac{x^{2/3}}{x^{2/3}}+\frac{1-x^{2/3}}{x^{2/3}}}\,dx$$

$$= 4\pi \int_0^1 (1-x^{2/3})^{3/2}\sqrt{\frac{1}{x^{2/3}}}\,dx$$

$$= 4\pi \int_0^1 \frac{(1-x^{2/3})^{3/2}}{x^{2/3}}\,dx \qquad (\text{let } u = 1-x^{2/3} \xrightarrow{D}$$

$$du = -\frac{2}{3}x^{-1/3}dx \rightarrow -\frac{3}{2}du = \frac{1}{x^{1/3}}dx \;)$$

$$= 4\pi \cdot -\frac{3}{2}\int_{x=0}^{x=1} u^{3/2}du = -6\pi \cdot \frac{2}{5}u^{5/2}\Big|_{x=0}^{x=1}$$

$$= -\frac{12}{5}\pi\left(1-x^{2/3}\right)^{5/2}\Big|_{x=0}^{x=1}$$

$$= -\frac{12}{5}\pi(0) - -\frac{12}{5}\pi(1) = \frac{12}{5}\pi$$