Section 6.5

1.) $F(x) = kx$, where x is displacement;
$\text{Work} = \int_a^b F(x)\,dx$ so $1800\text{ J} = \int_0^3 kx\,dx \rightarrow$
$1800 = k\frac{x^2}{2}\Big|_0^3 = \frac{9}{2}k \rightarrow k = 400\ \text{N/m}.$

2.) $F(x) = kx \longrightarrow 800\text{ lb.} = k\cdot(4\text{ in}) \rightarrow$

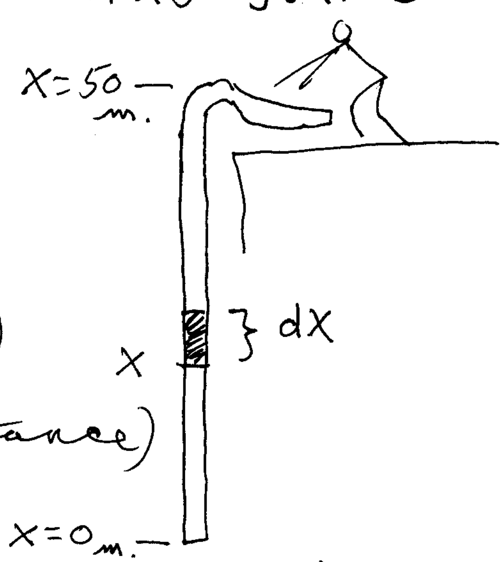
a.) $k = 200\text{ lb./in.}$

b.) $\text{Work} = \int_0^2 F(x)\,dx = \int_0^2 200x\,dx$
$= 100x^2\Big|_0^2 = 400\text{ in.-lbs.}$

c.) $1600 = kx \longrightarrow 1600 = (200)x \longrightarrow$
$x = 8\text{ in.}$

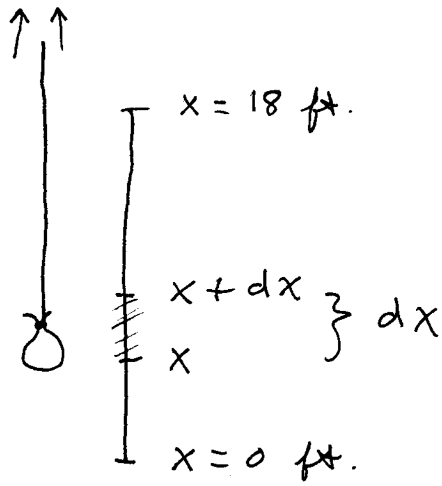
4.) $F(x) = kx \rightarrow 90\text{ N} = k\cdot(1\text{ m.}) \rightarrow k = 90$ so
$F(x) = kx$; ~~then~~ $\text{Work} = \int_0^5 F(x)\,dx$
$= \int_0^5 90x\,dx = 45x^2\Big|_0^5 = 1125$ joules

7.) work required to lift small piece of rope to top is
$\approx$ (weight) (distance)
$=$ (density)(length) (distance)
$= (0.624)(dx)(50-x)$
$= (0.624)(50-x)\,dx$; total work is



$$\text{Work} = \int_0^{50} (0.624)(50-x)\,dx$$
$$= (0.624)\left(50x - \frac{x^2}{2}\right)\Big|_0^{50} = (0.624)(2500-1250)$$
$$= 780\ \text{J.}$$

8.) Bag loses 4 lbs./ft.; work required to lift bag from x ~~to~~ $x+dx$ is



$\approx$ (weight)$\cdot$(distance)

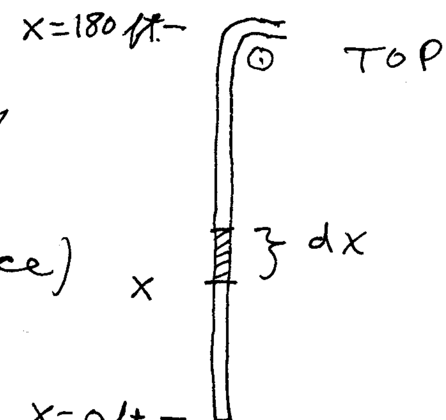
$= (144 - 4x)\,dx$;

so total work is

$$\text{Work} = \int_0^{18} (144 - 4x)\,dx$$

$$= (144x - 2x^2)\Big|_0^{18} = 2592 - 648 = 1944 \text{ ft.-lbs.}$$

9.) work to move small piece of cable ~~to~~ top is



$\approx$ (weight)(distance)

= (density)(length) (distance)

$= (4.5)(dx)(180 - x)$

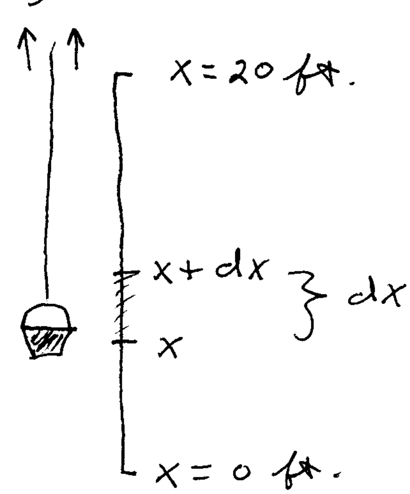
$= (4.5)(180 - x)\,dx$;

total work ~~to~~ lift cable is

$$\text{Work} = \int_0^{180} (4.5)(180 - x)\,dx$$

$$= (4.5)\left(180x - \frac{x^2}{2}\right)\Big|_0^{180} = 4.5\,(16{,}200)$$

$= 72{,}900$ ft.-lbs.



11.) bucket weighs 5 lbs.; starts with 16 lbs. of H_2O in it, but leaks at rate of $\dfrac{16 \text{ lbs.}}{20 \text{ ft.}} = \dfrac{4}{5} \dfrac{\text{lb.}}{\text{ft.}}$;

work to move bucket from

x to $x + dx$ is

$\approx$ (weight) (distance)

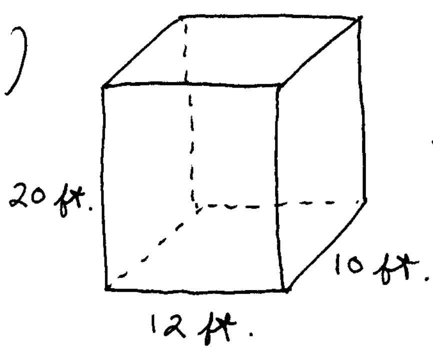
$= \left(21 - \frac{4}{5}x\right)(dx)$; total work

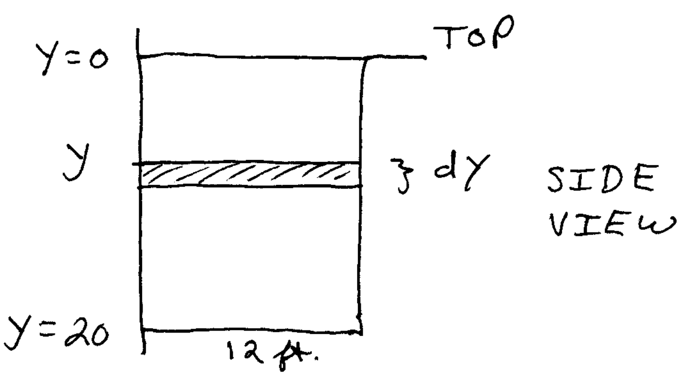
to lift bucket is

$$\text{Work} = \int_0^{20} \left(21 - \frac{4}{5}x\right) dx$$

$$= \left(21x - \frac{\overset{2}{\cancel{4}}}{5} \cdot \frac{x^2}{\cancel{2}}\right)\Big|_0^{20} = 420 - 160 = 260 \text{ ft.-lbs.}$$

13.) a.)





work to move thin slice of H_2O to top is

$\approx$ (weight) (distance)

$=$ (density)(volume) (distance)

$= (62.4) \cdot ((10)(12)(dy)) \cdot (y)$

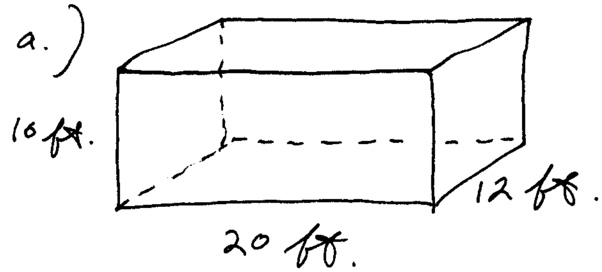
$= 7488\, y\, dy$; so total work to

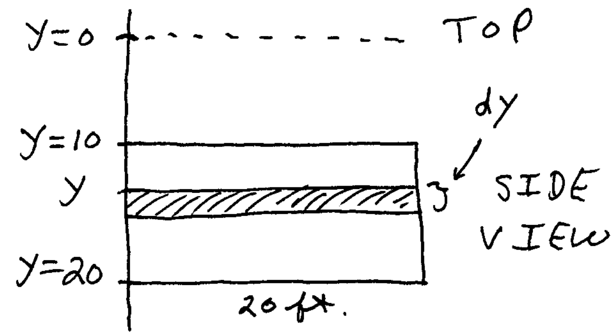
empty the tank is

$$\text{Work} = \int_0^{20} 7488\, y\, dy = 3744\, y^2 \Big|_0^{20}$$

$= 1{,}497{,}600$ ft.-lbs.

14.) a.)

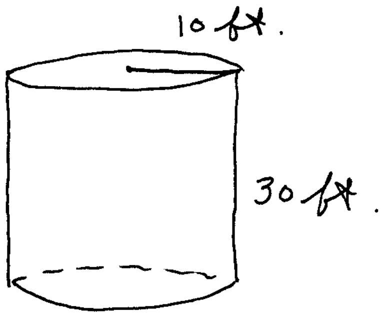


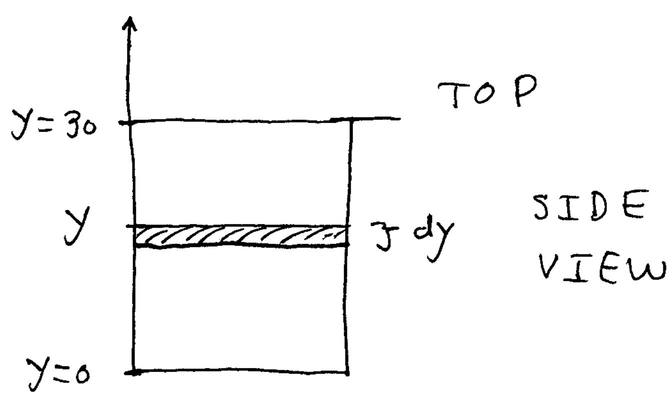


work to move thin slice of H_2O to top is
$\approx$ (weight) (distance)
= (density)(volume) (distance)
$= (62.4)\left((20)(12)(dy)\right)\cdot(y)$; so total

work to move H_2O to top is
$$\text{Work} = \int_{10}^{20} 14{,}976\, y\, dy = 7488\, y^2 \Big|_{10}^{20}$$
$$= 7488\,(400-100) = 2{,}246{,}400 \text{ ft.-lbs.}$$

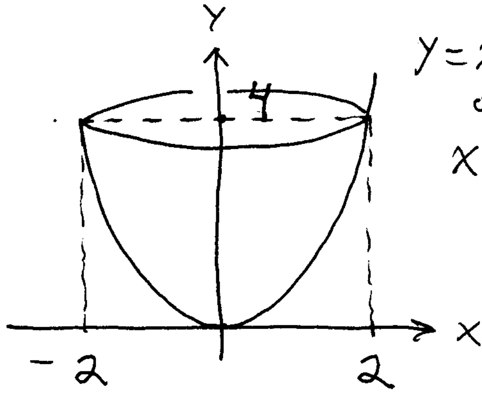
17.)



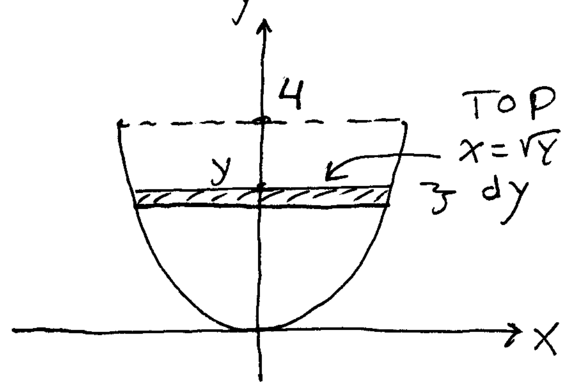


work to move slice of kerosene to top is
$\approx$ (weight)(distance)
= (density)(volume)(distance)
$= (51.2)\left(\pi (10)^2 \cdot dy\right)(30-y)$
$= 5120\pi\,(30-y)\,dy$; so work
to empty tank is

$$\text{Work} = \int_0^{30} 5120\pi\,(30-y)\,dy$$
$$= 5120\pi\left(30y - \frac{y^2}{2}\right)\Big|_0^{30}$$
$$= 5120\pi\,(900-450) = 2{,}304{,}000\,\pi \text{ ft.-lbs.}$$

19.) 

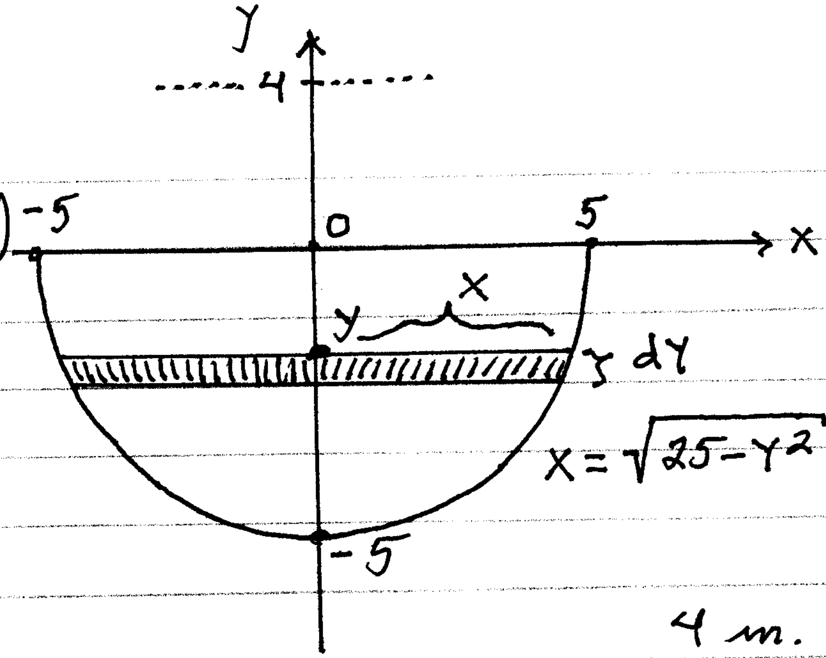
$y = x^2$ or $x = \sqrt{y}$



work to move circular slice of H_2O to top is
$\approx$ (weight)(distance)
= (density)(volume)(distance)
$= (73)\left(\pi(\sqrt{y})^2 \cdot dy\right)(4-y)$
$= 73 \cdot \pi\,(4y - y^2)\,dy$;

so total work to empty pipe is
$$\text{Work} = \int_0^4 73\,\pi\,(4y-y^2)\,dy$$
$$= 73\,\pi\left(2y^2 - \frac{y^3}{3}\right)\Big|_0^4$$
$$= 73\,\pi\left(32 - \frac{64}{3}\right)$$
$$= (73)\,\frac{32}{3}\,\pi \approx 2446.25 \text{ ft.-lbs.}$$

21.)



work required to lift thin layer of water 4 m. above top of tank

is $\approx$ (weight)(distance)

$= $ (volume)(density)(distance)

$= \pi (X)^2 \cdot dY \cdot 9800 \cdot ((-Y)+4)$

$= 9800\,\pi\,(4-Y)\cdot\left(\sqrt{25-Y^2}\right)^2\,dy$

$= 9800\,\pi\,(4-Y)(25-Y^2)\,dy$;

total work is

$$\text{Work} = \int_{-5}^{0} 9800\pi\,(4-Y)(25-Y^2)\,dy \text{ J.}$$

24.) $K.E. = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$(2\,oz)\left(\frac{1\text{ lb.}}{16\text{ oz.}}\right) = \frac{1}{8}\text{ lb.}$; $F = mg \rightarrow$

$\frac{1}{8} = m \cdot 32 \rightarrow m = \frac{1}{256}$ slugs ; $v_1 = 0$ ft./sec. ,

$v_2 = 160$ ft./sec. ; then

$$K.E. = \frac{1}{2}\left(\frac{1}{256}\right)(160)^2 - \frac{1}{2}\left(\frac{1}{256}\right)(0)^2 = 50 \text{ ft.-lbs.}$$

25.) $F = mg \rightarrow 0.3125 = m \cdot (32) \rightarrow$

$m = \frac{0.3125}{32}$ slugs ; $v_1 = 0$ ft./sec. ,

$$v_2 = \left(90 \frac{mi}{hr.}\right)\left(\frac{1 \text{ hr.}}{3600 \text{ sec.}}\right)\left(\frac{5280 \text{ ft.}}{1 \text{ mi.}}\right) = 132 \frac{ft.}{sec.} \text{ ; then}$$

$$K.E. = \frac{1}{2}\left(\frac{0.3125}{32}\right)(132)^2 \approx 85.1 \text{ ft.-lbs.}$$

26.) $(1.6 \text{ oz})\left(\frac{1 \text{ lb.}}{16 \text{ oz.}}\right) = \frac{1}{10}$ lb. ; $F = mg \rightarrow$

$\frac{1}{10} = m(32) \rightarrow m = \frac{1}{320}$ slugs ; $v_1 = 0$ ft./sec. ,

$v_2 = 280 \frac{ft.}{sec.}$, then

$$K.E. = \frac{1}{2}\left(\frac{1}{320}\right)(280)^2 = 122.5 \text{ ft.-lbs.}$$