Section 7.1

2.) $\int_{-1}^{0} \frac{3\,dx}{3x-2}$ (Let $u = 3x-2 \rightarrow$ $du = 3\,dx$; $x: -1 \rightarrow 0$ so $u: -5 \rightarrow -2$)

$= \int_{-5}^{-2} \frac{1}{u}\,du = \ln|u|\Big|_{-5}^{-2} = \ln 2 - \ln 5$

$= \ln(2/5)$

3.) $\int \frac{2y}{y^2-25}\,dy$ (Let $u = y^2-25 \rightarrow$ $du = 2y\,dy$)

$= \int \frac{1}{u}\,du = \ln|u| + c = \ln|y^2-25| + c$

5.) $\int \frac{3\sec^2 t}{6+3\tan t}\,dt$ (Let $u = 6+3\tan t \rightarrow$ $du = 3\cdot\sec^2 t\,dt$)

$= \int \frac{1}{u}\,du = \ln|u| + c = \ln|6+3\tan t| + c$

6.) $\int \frac{\sec y \tan y}{2+\sec y}\,dy$ (Let $u = 2+\sec y \rightarrow$ $du = \sec y \tan y\,dy$)

$= \int \frac{1}{u}\,du = \ln|u| + c = \ln|2+\sec y| + c$

7.) $\int \frac{1}{2\sqrt{x}+2x}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$

(Let $u = 1+\sqrt{x} \rightarrow du = \frac{1}{2\sqrt{x}}\,dx \rightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$)

$= \frac{1}{\cancel{2}}\int \frac{\cancel{2}}{u}\,du = \ln|u| + c = \ln|1+\sqrt{x}| + c$

8.) $\int \frac{\sec x\,dx}{\sqrt{\ln(\sec x + \tan x)}}$ (Let $u = \ln(\sec x + \tan x)$

$\rightarrow du = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}\,dx = \frac{\sec x\,\cancel{(\sec x + \tan x)}}{\cancel{\sec x + \tan x}}\,dx$)

$= \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + c =$

$2\left(\ln(\sec x + \tan x)\right)^{1/2} + c$

9.) $\int_{\ln 2}^{\ln 3} e^x\,dx = e^x \Big|_{\ln 2}^{\ln 3} = e^{\ln 3} - e^{\ln 2} = 3-2=1$

12.) $\int \frac{\ln(\ln x)}{x \ln x}\,dx$ (Let $u = \ln(\ln x) \rightarrow du = \frac{1}{\ln x}\cdot\frac{1}{x}\,dx$)

$= \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln(\ln x))^2 + C$

13.) $\int_{\ln 4}^{\ln 9} e^{x/2}\,dx$ (Let $u = \frac{x}{2} \rightarrow du = \frac{1}{2}\,dx$

$\rightarrow 2\,du = dx$

and $x: \ln 4 \rightarrow \ln 9$ so $u: \frac{1}{2}\ln 4 \rightarrow \frac{1}{2}\ln 9$)

$= 2\int_{\frac{1}{2}\ln 4}^{\frac{1}{2}\ln 9} e^u\,du = 2e^u \Big|_{\frac{1}{2}\ln 4}^{\frac{1}{2}\ln 9}$

$= 2e^{\frac{1}{2}\ln 9} - 2e^{\frac{1}{2}\ln 4} = 2e^{\ln 9^{1/2}} - 2e^{\ln 4^{1/2}}$

$= 2(3) - 2(2) = 2$

15.) $\int \frac{e^{\sqrt{r}}}{\sqrt{r}}\,dr$ (Let $u = \sqrt{r} \rightarrow du = \frac{1}{2\sqrt{r}}\,dr$

$\rightarrow 2\,du = \frac{1}{\sqrt{r}}\,dr$)

$= 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{r}} + C$

20.) $\int \frac{e^{-1/x^2}}{x^3}\,dx$ (Let $u = \frac{-1}{x^2} \rightarrow du = \frac{2}{x^3}\,dx \rightarrow$

$\frac{1}{2}\,du = \frac{1}{x^3}\,dx$)

$= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{-1/x^2} + C$

22.) $\int e^{\csc(\pi+t)}\csc(\pi+t)\cot(\pi+t)\,dt$

(Let $u = \csc(\pi+t) \rightarrow du = -\csc(\pi+t)\cot(\pi+t)\,dt$

$\rightarrow -du = \csc(\pi+t)\cot(\pi+t)\,dt$)

24.) $\int_0^{\sqrt{\ln \pi}} 2x e^{x^2}\cos(e^{x^2})\,dx$ (Let $u = e^{x^2} \rightarrow$

$du = 2xe^{x^2}dx$; $x: 0 \to \sqrt{\ln\pi}$ and
$u: e^0 \to e^{\ln\pi}$ or $u: 1 \to \pi$)

$= \int_1^{\pi} \cos u \, du = \sin u \Big|_1^{\pi} = \cancel{\sin}^{0}\pi - \sin 1$

$= -\sin 1$

25.) $\int \frac{e^r}{1+e^r} dr$ (Let $u = 1+e^r \to du = e^r dr$)

$= \int \frac{1}{u} du = \ln|u| + c = \ln|1+e^r| + c$

26.) $\int \frac{1}{1+e^x} dx = \int \frac{1}{1+e^x} \cdot \frac{e^{-x}}{e^{-x}} dx$

$= \int \frac{e^{-x}}{e^{-x}+1} dx$ (Let $u = e^{-x}+1 \to$
$du = -e^{-x}dx \to -du = e^{-x}dx$)

$= -\int \frac{1}{u} du = -\ln|u| + c = -\ln|e^{-x}+1| + c$

28.) $\int_{-2}^{0} 5^{-\theta} d\theta$ (Let $u = -\theta \to du = -d\theta \to$
$-du = d\theta$; $x: -2 \to 0$ so $u: 2 \to 0$)

$= -\int_2^0 5^u du = -\frac{5^u}{\ln 5}\Big|_2^0 = \frac{-5^0}{\ln 5} - \frac{-5^2}{\ln 5}$

$= \frac{-1}{\ln 5} + \frac{25}{\ln 5} = \frac{24}{\ln 5}$

32.) $\int_0^{\frac{\pi}{4}} \left(\frac{1}{3}\right)^{\tan t} \cdot \sec^2 t \, dt$ (Let $u = \tan t \to$
$du = \sec^2 t \, dt$; $t: 0 \to \frac{\pi}{4}$ so $u: 0 \to 1$)

$= \int_0^1 \left(\frac{1}{3}\right)^u du = \frac{(1/3)^u}{\ln(1/3)}\Big|_0^1 = \frac{\frac{1}{3}}{\ln(1/3)} - \frac{1}{\ln(1/3)}$

$= \frac{-2/3}{\ln(1/3)}$

33.) $\int_2^4 x^{2x}(1+\ln x)\,dx$ (Let $u = x^{2x} \rightarrow$
$\ln u = \ln x^{2x} = 2x \cdot \ln x \xrightarrow{D}$
$\frac{1}{u} \cdot \frac{du}{dx} = 2x \cdot \frac{1}{x} + 2\ln x \rightarrow du = u\,(2 + 2\ln x)\,dx$
$\rightarrow du = 2 \cdot x^{2x}(1+\ln x)\,dx \rightarrow \frac{1}{2}\,du = x^{2x}(1+\ln x)\,dx$;
$x: 2 \rightarrow 4$ so $u: 2^4 \rightarrow 4^8 = 2^{16}$)

$$= \int_{2^4}^{2^{16}} \frac{1}{2}\,du = \frac{1}{2}u \Big|_{2^4}^{2^{16}} = \frac{1}{2}(65{,}536 - 16)$$

$$= 32{,}760$$

36.) $$\int_1^e x^{(\ln 2)-1}\,dx = \frac{x^{\ln 2}}{\ln 2}\Big|_1^e = \frac{e^{\ln 2}}{\ln 2} - \frac{1^{\ln 2}}{\ln 2}$$

$$= \frac{2}{\ln 2} - \frac{1}{\ln 2} = \frac{1}{\ln 2}$$

39.) $$\int_1^4 \frac{\ln 2 \cdot \log_2 x}{x}\,dx = \ln 2 \int_1^4 \frac{\frac{\ln x}{\ln 2}}{x}\,dx$$

$= \int_1^4 \frac{\ln x}{x}\,dx$ (Let $u = \ln x \rightarrow du = \frac{1}{x}\,dx$;
$x: 1 \rightarrow 4$ so $u: 0 \rightarrow \ln 4$)

$$= \int_0^{\ln 4} u\,du = \frac{u^2}{2}\Big|_0^{\ln 4} = \frac{1}{2}(\ln 4)^2$$

42.) $$\int_{\frac{1}{10}}^{10} \frac{\log(10x)}{x}\,dx = \int_{\frac{1}{10}}^{10} \frac{\frac{\ln(10x)}{\ln 10}}{x}\,dx$$

$= \frac{1}{\ln 10}\int_{\frac{1}{10}}^{10} \frac{\ln(10x)}{x}\,dx$ (Let $u = \ln(10x) \rightarrow$
$du = \frac{1}{\cancel{10}x} \cdot \cancel{10}\,dx$; $x: \frac{1}{10} \rightarrow 10$ so $u: 0 \rightarrow \ln 100$)

$$= \frac{1}{\ln 10}\int_0^{\ln 100} u\,du = \frac{1}{\ln 10} \cdot \frac{u^2}{2}\Big|_0^{\ln 100}$$

$$= \frac{1}{\ln 10^2} (\ln 100)^2 = \ln 100$$

45.) $\int \frac{1}{x \log x} dx = \int \frac{1}{x \cdot \frac{\ln x}{\ln 10}} dx$

$= \ln 10 \int \frac{1}{x \ln x} dx$ (Let $u = \ln x \rightarrow du = \frac{1}{x} dx$)

$= \ln 10 \int \frac{1}{u} du = \ln 10 \cdot (\ln|u| + c)$

$= \ln 10 \, (\ln|\ln x| + c)$

46.) $\int \frac{1}{x (\log x)^2} dx = \int \frac{1}{x \left(\frac{\ln x}{\ln 10}\right)^2} dx$

$= (\ln 10)^2 \int \frac{1}{x (\ln x)^2} dx$ (Let $u = \ln x \rightarrow du = \frac{1}{x} dx$)

$= (\ln 10)^2 \int \frac{1}{u^2} du$

$= (\ln 10)^2 \int u^{-2} du$

$= (\ln 10)^2 \cdot \left(\frac{u^{-1}}{-1} + c \right)$

$= (\ln 10)^2 \left(\frac{(\ln x)^{-1}}{-1} + c \right)$

49.) $\frac{d^2y}{dx^2} = 2e^{-x} \rightarrow \frac{dy}{dx} = -2e^{-x} + c$

and $y'(0) = 0 \rightarrow 0 = -2 \cdot e^0 + c \rightarrow c = 2 \rightarrow$

$\frac{dy}{dx} = -2e^{-x} + 2 \longrightarrow y = 2e^{-x} + 2x + c$

and $y(0) = 1 \rightarrow 1 = 2 \cdot e^0 + 2(0) + c \rightarrow$

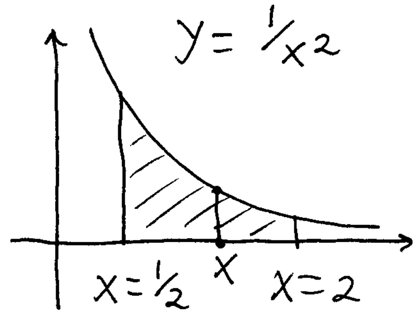
$c = -1 \longrightarrow y = 2e^{-x} + 2x - 1$

51.) $\frac{dy}{dx} = 1 + \frac{1}{x} \rightarrow y = x + \ln|x| + c$

and $y(1) = 3 \rightarrow 3 = 1 + \cancel{\ln 1}^{0} + c \rightarrow$

$c = 2 \rightarrow y = x + \ln|x| + 2$.

53.)



SHELL METHOD:

$Vol = 2\pi \int_{\frac{1}{2}}^{2} (x)\left(\frac{1}{x^2}\right) dx$

radius height

$= 2\pi \int_{1/2}^{2} \frac{1}{x}\, dx = 2\pi \ln|x| \Big|_{1/2}^{2}$

$= 2\pi (\ln 2 - \ln 1/2)$

$= 2\pi (\ln 2 - (\cancel{\ln 1}^{0} - \ln 2))$

$= 2\pi (2 \ln 2)$

$= 4\pi \ln 2$

55.) $y = \frac{x^2}{8} - \ln x$ for $4 \le x \le 8$ $\rightarrow$

$$\text{Arc} = \int_4^8 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_4^8 \sqrt{1 + \left(\frac{1}{4}x - \frac{1}{x}\right)^2}\, dx$$

$$= \int_4^8 \sqrt{1 + \frac{1}{16}x^2 - \frac{1}{2} + \frac{1}{x^2}}\, dx$$

$$= \int_4^8 \sqrt{\frac{1}{16}x^2 + \frac{1}{2} + \frac{1}{x^2}}\, dx$$

$$= \int_4^8 \sqrt{\left(\frac{1}{4}x + \frac{1}{x}\right)^2}\, dx$$

$$= \int_4^8 \left(\frac{1}{4}x + \frac{1}{x}\right) dx$$

$$= \left(\frac{1}{4}\frac{x^2}{2} + \ln|x|\right)\Big|_4^8$$

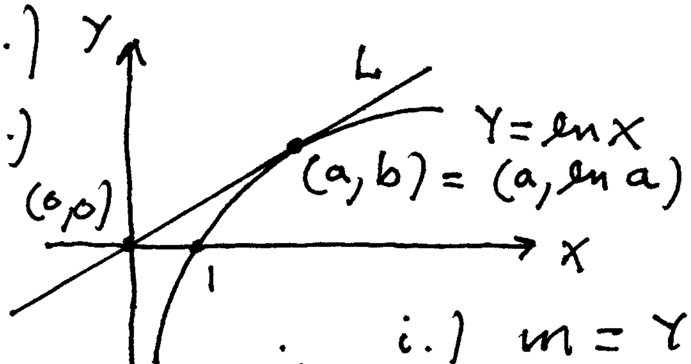
$$= \frac{1}{8}(64) + \ln 8 - \left(\frac{1}{8}(16) + \ln 4\right)$$

$$= 8 - 2 + \ln 8 - \ln 4$$

$$= 6 + \ln(8/4) = 6 + \ln 2$$

67.)

a.)



$\xrightarrow{D} y' = \frac{1}{x}$, so slope of line L is

i.) $m = y'(a) = \frac{1}{a}$

ii.) $m = \frac{\ln a - 0}{a - 0}$

$\Bigg\} \rightarrow$

$$\frac{\ln a}{a} = \frac{1}{a} \rightarrow \ln a = 1 \rightarrow a = e \, ;$$

so line L is $y = mx + b \rightarrow \boxed{Y = \frac{1}{e} x}$

b.) It's clear from the graph that $\ln x < \frac{x}{e}$ for $x \neq e$ (and $\ln x = \frac{x}{e}$ if $x = e$)

c.) assume $x > 0$, then

$$\ln x < \frac{x}{e} \rightarrow e \ln x < x \rightarrow$$

$$\ln x^e < x \quad (\text{for } x \neq e)$$

d.) assume $x > 0$, then

$$\ln x^e < x \rightarrow e^{\ln x^e} < e^x \rightarrow$$

$$x^e < e^x \quad (\text{for } x \neq e)$$

e.) Let $x = \pi$ in part d. :

$$\pi^e < e^\pi$$

68.) $\ln x = 1 \rightarrow \ln x - 1 = 0$ so let $f(x) = \ln x - 1 \xrightarrow{D} f'(x) = \frac{1}{x}$, and Newton's Method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\ln x_n - 1}{1/x^n}$$

$= x_n - (x_n \ln x_n - x_n) \rightarrow$

$\boxed{x_{n+1} = 2x_n - x_n \ln x_n}$; start with $x_0 = 2$ and estimate e to 9 decimal places (I used an EXCEL spreadsheet to fill out the following table

n	x(n)	x(n+1)
0	2	2.6137056389
1	2.6137056389	2.7162439264
2	2.7162439264	2.7182810644
3	2.7182810644	2.7182818285
4	2.7182818285	2.7182818285
5	2.7182818285	2.7182818285

$e \approx 2.7182818285$

70.) b.) Show $\log_b x = \frac{\ln a}{\ln b} \log_a x$:

Let $y = \log_b x \longrightarrow b^y = x \rightarrow \log_a b^y = \log_a x$

$\rightarrow y \cdot \log_a b = \log_a x \longrightarrow y = \frac{1}{\log_a b} \cdot \log_a x$

$\rightarrow y = \frac{1}{\frac{\ln b}{\ln a}} \cdot \log_a x$ (By Rules From Class)

$\rightarrow y = \frac{\ln a}{\ln b} \cdot \log_a x$, i.e., $\log_b x = \frac{\ln a}{\ln b} \cdot \log_a x$ ▨