Section 8.2

1.) $\int x \cdot \sin\left(\frac{x}{2}\right) dx$ (Let $u = x$, $dv = \sin\left(\frac{x}{2}\right) dx$
$du = dx$, $v = -2\cos\left(\frac{x}{2}\right)$)

$= -2x\cos\left(\frac{x}{2}\right) - {}^{-}2\int \cos\left(\frac{x}{2}\right) dx$

$= -2x\cos\left(\frac{x}{2}\right) + 2 \cdot 2\sin\left(\frac{x}{2}\right) + c$

$= -2x\cos\left(\frac{x}{2}\right) + 4\sin\left(\frac{x}{2}\right) + c$

4.) $\int x^2 \sin x\, dx$ (Let $u = x^2$, $dv = \sin x\, dx$
$du = 2x\, dx$, $v = -\cos x$)

$= -x^2\cos x - {}^{-}2\int x\cos x\, dx$
(Let $u = x$, $dv = \cos x\, dx$
$du = dx$, $v = \sin x$)

$= -x^2\cos x + 2[x\sin x - \int \sin x\, dx]$

$= -x^2\cos x + 2x\sin x - 2\cdot(-\cos x) + c$

$= -x^2\cos x + 2x\sin x + 2\cos x + c$

5.) $\int_1^2 x\ln x\, dx$ (Let $u = \ln x$, $dv = x\, dx$
$du = \frac{1}{x}dx$, $v = \frac{x^2}{2}$)

$= \frac{x^2}{2}\ln x\Big|_1^2 - \frac{1}{2}\int_1^2 x\, dx$

$= 2\ln 2 - \frac{1}{2}\cancel{\ln 1}^{\,0} - \frac{1}{2}\cdot\frac{x^2}{2}\Big|_1^2$

$= 2\ln 2 - \frac{1}{4}(4-1) = 2\ln 2 - \frac{3}{4}$

8.) $\int xe^{3x} dx$ (Let $u = x$, $dv = e^{3x} dx$
$du = dx$, $v = \frac{1}{3}e^{3x}$)

$= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x} dx = \frac{1}{3}xe^{3x} - \frac{1}{3}\cdot\frac{1}{3}e^{3x} + c$

9.) $\int x^2 e^{-x}\,dx$ (Let $u = x^2$, $dv = e^{-x}\,dx \rightarrow$
$du = 2x\,dx$, $v = -e^{-x}$)

$= -x^2e^{-x} - -2\int xe^{-x}\,dx$ (Let $u = x$, $dv = e^{-x}\,dx \rightarrow$
$du = dx$, $v = -e^{-x}$)

$= -x^2e^{-x} + 2\left[-xe^{-x} - -\int e^{-x}\,dx\right]$

$= -x^2e^{-x} - 2xe^{-x} + 2(-e^{-x}) + C$

11.) $\int \arctan y\,dy$ (Let $u = \arctan y$, $dv = dy \rightarrow$
$du = \frac{1}{1+y^2}\,dy$, $v = y$)

$= y\arctan y - \int \frac{y}{1+y^2}\,dy$ (Let $u = 1+y^2 \xrightarrow{D} \ldots$)

$= y\arctan y - \frac{1}{2}\ln|1+y^2| + C$

12.) $\int \arcsin y\,dy$ (Let $u = \arcsin y$, $dv = dy \longrightarrow$
$du = \frac{1}{\sqrt{1-y^2}}\,dy$, $v = y$)

$= y\arcsin y - \int \frac{y}{\sqrt{1-y^2}}\,dy$ (Let $u = 1-y^2 \xrightarrow{D} \ldots$)

$= y\arcsin y - \frac{-1}{2}\cdot 2(1-y^2)^{1/2} + C$

$= y\arcsin y + (1-y^2)^{1/2} + C$

13.) $\int x\sec^2 x\,dx$ (Let $u = x$, $dv = \sec^2 x\,dx$
$du = dx$, $v = \tan x$)

$= x\tan x - \int \tan x\,dx$

$= x\tan x - \ln|\sec x| + C$

15.) $\int x^3 e^x \, dx$ (Let $u = x^3$, $dv = e^x \, dx$
$du = 3x^2 \, dx$, $v = e^x$)

$= x^3 e^x - 3\int x^2 e^x \, dx$ (Let $u = x^2$, $dv = e^x \, dx$
$du = 2x \, dx$, $v = e^x$)

$= x^3 e^x - 3\left[x^2 e^x - 2\int x e^x \, dx\right]$

$= x^3 e^x - 3x^2 e^x + 6\int x e^x \, dx$

(Let $u = x$, $dv = e^x \, dx$
$du = dx$, $v = e^x$)

$= x^3 e^x - 3x^2 e^x + 6\left[x e^x - \int e^x \, dx\right]$

$= x^3 e^x - 3x^2 e^x + 6x e^x - 6 \cdot e^x + C$

23.) $\int e^{2x} \cos 3x \, dx$

(Let $u = e^{2x}$, $dv = \cos 3x \, dx$
$du = 2e^{2x} \, dx$, $v = \frac{1}{3}\sin 3x$)

$= \frac{1}{3} e^{2x} \cdot \sin 3x - \frac{2}{3}\int e^{2x} \sin 3x \, dx$

(Let $u = e^{2x}$, $dv = \sin 3x \, dx$
$du = 2e^{2x} \, dx$, $v = \frac{-1}{3}\cos 3x$)

$$= \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left[\frac{-1}{3}e^{2x}\cos 3x + \frac{2}{3}\int e^{2x}\cos 3x\,dx\right]$$

$$= \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}\int e^{2x}\cos 3x\,dx \rightarrow$$

$$\underline{\int e^{2x}\cos 3x\,dx} = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x$$

$$- \underline{\frac{4}{9}\int e^{2x}\cos 3x\,dx} \rightarrow$$

(TWIST) $$\frac{13}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x + C \rightarrow$$

$$\int e^{2x}\cos 3x\,dx = \frac{3}{13}e^{2x}\cdot\sin 3x + \frac{2}{13}e^{2x}\cdot\cos 3x + C$$

26.) $\int_0^1 x\sqrt{1-x}\,dx$ (Let $u = x$, $dv = (1-x)^{1/2}dx$

$du = dx$, $v = \frac{-2}{3}(1-x)^{3/2}$)

$$= \frac{-2}{3}x(1-x)^{3/2}\Big|_0^1 - \frac{-2}{3}\int_0^1 (1-x)^{3/2}\,dx$$

$$= 0 - 0 + \frac{2}{3}\cdot\left(\frac{-2}{5}\right)(1-x)^{5/2}\Big|_0^1$$

$$= \frac{-4}{15}(1-x)^{5/2}\Big|_0^1 = 0 - \frac{-4}{15}(1)^{5/2} = \frac{4}{15}$$

27.) $\int_0^{\frac{\pi}{3}} x\tan^2 x\,dx = \int_0^{\frac{\pi}{3}} x\cdot(\sec^2 x - 1)\,dx$

$$= \int_0^{\frac{\pi}{3}} x\cdot\sec^2 x\,dx - \int_0^{\frac{\pi}{3}} x\,dx$$

(Let $u = x$, $dv = \sec^2 x\,dx$

$du = dx$, $v = \tan x$)

$$= \left[x\tan x\Big|_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}}\tan x\,dx\right] - \frac{x^2}{2}\Big|_0^{\frac{\pi}{3}}$$

$$= \frac{\pi}{3}\tan\frac{\pi}{3} - 0 - \ln|\sec x|\Big|_0^{\frac{\pi}{3}} - \frac{1}{2}\cdot\frac{\pi^2}{9}$$

$$= \frac{\pi}{3} \cdot \sqrt{3} - \left(\ln|\sec \tfrac{\pi}{3}| - \ln|\sec 0|\right) - \frac{\pi^2}{18}$$

$$= \frac{\pi}{\sqrt{3}} - \left(\ln 2 - \cancel{\ln 1}^{\,0}\right) - \frac{\pi^2}{18}$$

$$= \frac{\pi}{\sqrt{3}} - \ln 2 - \frac{\pi^2}{18}$$

29.) $\int \sin(\ln x)\,dx$ (Let $u = \ln x \rightarrow x = e^u$ and $du = \frac{1}{x}\,dx \rightarrow du = \frac{1}{e^u}\,dx$ $\rightarrow e^u\,du = dx$)

$$= \int e^u \sin u \, du$$

(Let $w = e^u$, $dv = \sin u\,du$
$dw = e^u\,du$, $v = -\cos u$)

$$= -e^u \cos u - {}^{-}\int e^u \cos u \, du$$

(Let $w = e^u$, $dv = \cos u\,du$
$dw = e^u\,du$, $v = \sin u$)

$$= -e^u \cos u + \left[e^u \sin u - \int e^u \sin u \, du\right] \rightarrow$$

$$\underline{\int e^u \sin u \, du} = -e^u \cos u + e^u \sin u - \underline{\int e^u \sin u \, du}$$

(TWIST) $\rightarrow$ $2\int e^u \sin u \, du = -e^u \cos u + e^u \sin u + C$

$$\rightarrow \int e^u \sin u \, du = \frac{-1}{2} e^u \cos u + \frac{1}{2} e^u \sin u + C$$

$$= \frac{-1}{2} e^{\ln x} \cos(\ln x) + \frac{1}{2} e^{\ln x} \sin(\ln x) + C$$

$$= \frac{-1}{2} x \cos(\ln x) + \frac{1}{2} x \sin(\ln x) + C \rightarrow$$

$$\int \sin(\ln x)\,dx = \frac{-1}{2} x \cos(\ln x) + \frac{1}{2} x \sin(\ln x) + C$$

30.) $\int z(\ln z)^2\,dz$

$\left(\text{Let } u=(\ln z)^2,\ dv = z\,dz \right.$

$\left. du = 2(\ln z)\cdot\frac{1}{z}\,dz,\ v=\frac{z^2}{2}\right)$

$= \frac{z^2}{2}(\ln z)^2 - \int z\ln z\,dz$

$\left(\text{Let } u=\ln z,\ dv = z\,dz \right.$

$\left. du = \frac{1}{z}\,dz,\ v=\frac{z^2}{2}\right)$

$= \frac{z^2}{2}(\ln z)^2 - \left[\frac{z^2}{2}\cdot\ln z - \frac{1}{2}\int z\,dz\right]$

$= \frac{z^2}{2}(\ln z)^2 - \frac{z^2}{2}\cdot\ln z + \frac{1}{2}\cdot\frac{z^2}{2} + C$

31.) $\int x\sec^2(x^2)\,dx$ $\left(\text{Let } u = x^2 \xrightarrow{D} \right.$

$\left. du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx\right)$

$= \frac{1}{2}\int \sec^2 u\,du = \frac{1}{2}\tan u + C = \frac{1}{2}\tan(x^2)+C$

33.) $\int x(\ln x)^2\,dx$ $\left(\text{Let } u=(\ln x)^2,\ dv = x\,dx \right.$

$\left. \rightarrow du = 2(\ln x)\cdot\frac{1}{x}\,dx,\ v=\frac{1}{2}x^2\right)$

$= \frac{1}{2}x^2(\ln x)^2 - 2\cdot\frac{1}{2}\int \frac{x^2}{x}\ln x\,dx$

$= \frac{1}{2}x^2(\ln x)^2 - \int x\ln x\,dx$ $\left(\text{Let } u=\ln x,\ dv = x\,dx \right.$

$\left. \rightarrow du = \frac{1}{x}\,dx,\ v=\frac{1}{2}x^2\right)$

$= \frac{1}{2}x^2(\ln x)^2 - \left[\frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx\right]$

$= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{1}{2}\cdot\frac{1}{2}x^2 + C$

34.) $\int \frac{1}{x(\ln x)^2}\,dx$ (Let $u = \ln x \xrightarrow{D} du = \frac{1}{x}\,dx$)

$= \int \frac{1}{u^2}\,du = \frac{-1}{u} + c = \frac{-1}{\ln x} + c$

35.) $\int \frac{\ln x}{x^2}\,dx$ (Let $u = \ln x$, $dv = \frac{1}{x^2}\,dx \rightarrow$
$du = \frac{1}{x}\,dx$, $v = {}^{-1}\!/x$)

$= \frac{-1}{x}\ln x - {}^{-}\!\int \frac{1}{x^2}\,dx = \frac{-1}{x}\ln x + \frac{-1}{x} + c$

37.) $\int x^3 e^{x^4}\,dx$ (Let $u = x^4 \xrightarrow{D} du = 4x^3\,dx$
$\longrightarrow \frac{1}{4}\,du = x^3\,dx$)

$= \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + c = \frac{1}{4}e^{x^4} + c$

38.) $\int x^5 e^{x^3}\,dx$ (Let $u = x^3 \xrightarrow{D} du = 3x^2\,dx$
$\longrightarrow \frac{1}{3}\,du = x^2\,dx$)

$= \int x^2 \cdot x^3 \cdot e^{x^3}\,dx$

$= \frac{1}{3}\int u e^u\,du$ (Let $w = u$, $dv = e^u\,du \rightarrow$
$dw = du$, $v = e^u$)

$= \frac{1}{3}\left[u e^u - \int e^u\,du\right] = \frac{1}{3}u e^u - \frac{1}{3}e^u + c$

$= \frac{1}{3}x^3 e^{x^3} - \frac{1}{3}e^{x^3} + c$

39.) $\int x^3\sqrt{x^2+1}\,dx$ (Let $u = x^2+1 \xrightarrow{D} du = 2x\,dx$
$\longrightarrow \frac{1}{2}\,du = x\,dx$)

$x^2 = u - 1$

$= \int x \cdot x^2\sqrt{x^2+1}\,dx$

$= \frac{1}{2}\int (u-1)u^{1/2}\,du = \frac{1}{2}\int \left(u^{3/2} - u^{1/2}\right)du$

$$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + c$$

$$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + c$$

41.) $A = \int \sin 3x \cos 2x \, dx$ (Let $u = \sin 3x$, $dv = \cos 2x \, dx$
$\rightarrow du = 3\cos 3x \, dx$, $v = \frac{1}{2}\sin 2x$)

$$= \frac{1}{2}\sin 3x \sin 2x - \frac{3}{2}\int \cos 3x \sin 2x \, dx$$

(Let $u = \cos 3x$, $dv = \sin 2x \, dx$
$\rightarrow du = -3\sin 3x$, $v = -\frac{1}{2}\cos 2x$)

$$= \frac{1}{2}\sin 3x \sin 2x$$
$$- \frac{3}{2}\left[-\frac{1}{2}\cos 3x \cos 2x - \frac{3}{2}\int \sin 3x \cos 2x \, dx\right]$$

$$= \frac{1}{2}\sin 3x \sin 2x + \frac{3}{4}\cos 3x \cos 2x$$
$$+ \frac{9}{4}\underbrace{\int \sin 3x \cos 2x \, dx}_{A}\text{; then}$$

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$$-\frac{5}{4}A = \frac{1}{2}\sin 3x \sin 2x + \frac{3}{4}\cos 3x \cos 2x + C \longrightarrow$$

$$A = \int \sin 3x \cos 2x \, dx$$
$$= -\frac{2}{5}\sin 3x \sin 2x - \frac{3}{5}\cos 3x \cos 2x + C$$

44.) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$ (Let $u = \sqrt{x} \xrightarrow{D} du = \frac{1}{2\sqrt{x}} \, dx$
$\rightarrow 2\,du = \frac{1}{\sqrt{x}} \, dx$)

$$= 2\int e^u \, du = 2e^u + c = 2e^{\sqrt{x}} + c$$

$$u=\sqrt{x}$$

45.) $\int \cos\sqrt{x}\,dx \quad (\text{Let } x=u^2 \xrightarrow{D} dx = 2u\,du)$

$= 2\int u\cos u\,du \quad (\text{Let } w=u,\ dv=\cos u\,du \rightarrow$
$dw = du,\ v=\sin u)$

$= 2\left[u\sin u - \int \sin u\,du\right]$

$= 2\sqrt{x}\sin\sqrt{x} - 2(-\cos u) + c$

$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + c$

$$u=\sqrt{x}$$

46.) $\int \sqrt{x}\,e^{\sqrt{x}}\,dx \quad (\text{Let } x=u^2 \xrightarrow{D} dx = 2u\,du)$

$= 2\int u\cdot u e^u\,du = 2\int u^2 e^u\,du$

$(\text{Let } w=u^2,\ dv=e^u\,du \longrightarrow$
$dw=2u\,du,\ v=e^u)$

$= 2\left[u^2e^u - 2\int ue^u\,du\right] = 2u^2e^u - 4\int ue^u\,du$

$(\text{Let } w=u,\ dv=e^u\,du \longrightarrow$
$dw=du,\ v=e^u)$

$= 2u^2e^u - 4\left[ue^u - \int e^u\,du\right]$

$= 2u^2e^u - 4ue^u + 4e^u + c$

$= 2xe^{\sqrt{x}} - 4\sqrt{x}\,e^{\sqrt{x}} + 4e^{\sqrt{x}} + c$

47.) $\int_0^{\frac{\pi}{2}} \theta^2\sin 2\theta\,d\theta \quad (\text{Let } u=\theta^2,\ dv=\sin 2\theta\,d\theta \rightarrow$
$du = 2\theta\,d\theta,\ v=-\frac{1}{2}\cos 2\theta)$

$= -\frac{1}{2}\theta^2\cos 2\theta\Big|_0^{\frac{\pi}{2}} - -\int_0^{\frac{\pi}{2}} \theta\cos 2\theta\,d\theta$

$(\text{Let } u=\theta,\ dv=\cos 2\theta \longrightarrow$
$du=d\theta,\ v=\frac{1}{2}\sin 2\theta)$

$$= \frac{-1}{2}\left(\frac{\pi}{2}\right)^2 \cancel{\cos}^{-1}\pi - \frac{-1}{2}(0)^2 \cancel{\cos 0}^{0}$$

$$+ \left[\frac{1}{2}\theta \sin 2\theta \Big|_0^{\frac{\pi}{2}} - \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin 2\theta \, d\theta\right]$$

$$= \frac{1}{8}\pi^2 + \frac{1}{2}\left(\frac{\pi}{2}\right) \cancel{\sin}^{0}\pi - \frac{1}{2}(0) \cancel{\sin}^{0} 0$$

$$- \frac{1}{2} \cdot \frac{-1}{2} \cos 2\theta \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{1}{8}\pi^2 + \frac{1}{4}(\cancel{\cos}^{-1}\pi - \cancel{\cos}^{1} 0)$$

$$= \frac{1}{8}\pi^2 - \frac{1}{2}$$

49.) $\int_{\frac{2}{\sqrt{3}}}^{2} t \operatorname{arcsec} t \, dt$ (Let $u = \operatorname{arcsec} t$, $dv = t\,dt$

$\rightarrow du = \frac{1}{|t|\sqrt{t^2-1}} dx$, $v = \frac{1}{2}t^2$)

$$= \frac{1}{2}t^2 \operatorname{arcsec} t \Big|_{2/\sqrt{3}}^{2} - \frac{1}{2}\int_{\frac{2}{\sqrt{3}}}^{2} \frac{t}{\sqrt{t^2-1}} dt$$

$$= 2 \operatorname{arcsec} 2 - \frac{2}{3} \operatorname{arcsec} \frac{2}{\sqrt{3}}$$

$$- \frac{1}{2} \cdot \frac{1}{2} \cdot 2(t^2-1)^{\frac{1}{2}} \Big|_{2/\sqrt{3}}^{2}$$

$$= 2 \cdot \frac{\pi}{3} - \frac{2}{3} \cdot \frac{\pi}{6} - \frac{1}{2}\left(\sqrt{3} - \sqrt{\frac{1}{3}}\right)$$

$$= \frac{2}{3}\pi - \frac{1}{9}\pi - \frac{1}{2}\sqrt{3} + \frac{1}{2\sqrt{3}}$$

$$= \frac{5}{9}\pi - \frac{1}{2}\sqrt{3} + \frac{1}{2\sqrt{3}}$$

50.) $\int_0^{1/\sqrt{2}} 2x \arcsin(x^2)\,dx$ (Let $u = \arcsin(x^2)$

$\xrightarrow{D} du = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x\,dx = \frac{2x}{\sqrt{1-x^4}}\,dx$,

$dv = 2x\,dx$, $v = x^2$)

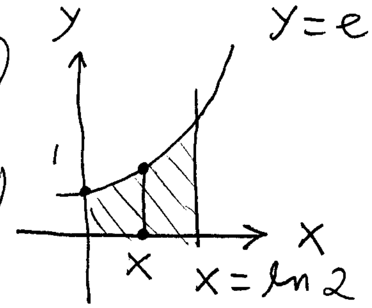
$= x^2 \arcsin(x^2) \Big|_0^{1/\sqrt{2}} - 2\int_0^{1/\sqrt{2}} \frac{x^3}{\sqrt{1-x^4}}\,dx$

$= \frac{1}{2}\arcsin\left(\frac{1}{2}\right) - (0)\arcsin 0$

$- 2 \cdot \frac{-1}{4} \cdot 2(1-x^4)^{1/2} \Big|_0^{1/\sqrt{2}}$

$= \frac{1}{2}\cdot\frac{\pi}{6} + \left(\sqrt{\frac{3}{4}} - \sqrt{1}\right) = \frac{1}{12}\pi + \frac{\sqrt{3}}{2} - 1$

55.) (SHELL METHOD)



$Vol = 2\pi \int_0^{\ln 2} (\text{radius})(\text{height})\,dx$

$= 2\pi \int_0^{\ln 2} (\ln 2 - x)e^x\,dx$

(Let $u = \ln 2 - x$, $dv = e^x\,dx$
$du = -dx$, $v = e^x$)

$= 2\pi \left\{ (\ln 2 - x)e^x \Big|_0^{\ln 2} - -\int_0^{\ln 2} e^x\,dx \right\}$

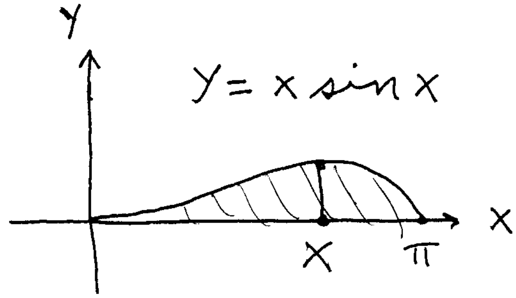
$= 2\pi \left\{ (0 - \ln 2) + e^x \Big|_0^{\ln 2} \right\}$

$= 2\pi \left\{ -\ln 2 + e^{\ln 2} - e^0 \right\}$

$= 2\pi \left\{ -\ln 2 + 2 - 1 \right\}$

$= 2\pi(1 - \ln 2)$

58.) a.)



(SHELL METHOD)

$$Vol = 2\pi\int_0^{\pi} (\text{radius})(\text{height})\, dx$$
$$= 2\pi\int_0^{\pi} x\cdot x\sin x\, dx$$
$$= 2\pi\int_0^{\pi} x^2\sin x\, dx$$

(Let $u = x^2$, $dv = \sin x\, dx$
$du = 2x\, dx$, $v = -\cos x$)

$$= 2\pi\left\{ -x^2\cos x\Big|_0^{\pi} - -2\int_0^{\pi} x\cos x\, dx\right\}$$

(Let $u = x$, $dv = \cos x\, dx$
$du = dx$, $v = \sin x$)

$$= 2\pi\left\{ (-\pi^2(-1) - 0) + 2\left(x\sin x\Big|_0^{\pi} - \int_0^{\pi} \sin x\, dx\right)\right\}$$
$$= 2\pi\left\{ \pi^2 + 2\left((\pi(0) - 0) + \cos x\Big|_0^{\pi}\right)\right\}$$
$$= 2\pi\left\{ \pi^2 + ((-1) - (1))\right\} = 2\pi(\pi^2 - 2)$$

64.) $\int x^n \sin x\, dx$ (Let $u = x^n$, $dv = \sin x\, dx$
$du = n x^{n-1}\, dx$, $v = -\cos x$)

$$= -x^n\cos x - -\int n\cdot x^{n-1}\cos x\, dx$$
$$= -x^n\cos x + n\int x^{n-1}\cos x\, dx$$

65.) $\int x^n e^{ax}\, dx$ (Let $u = x^n$, $dv = e^{ax}\, dx$
$du = n x^{n-1}\, dx$, $v = \frac{1}{a}e^{ax}$)

$$= \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax}\, dx$$

66.) $\int (\ln x)^n \, dx$ (Let $u = (\ln x)^n$, $dv = dx$
$du = n(\ln x)^{n-1} \cdot \frac{1}{x}\, dx$, $v = x$)

$$= x(\ln x)^n - n \int (\ln x)^{n-1}\, dx$$

69.) $\int_a^b \left(\int_x^b f(t)\, dt \right) dx$ (Let $u = \int_x^b f(t)\, dt$,
$dv = dx$, $du = D\left(-\int_b^x f(t)\, dt\right) = -f(x)\, dx$,
$v = x$)

$$= x \int_x^b f(t)\, dt \Big|_a^b - -\int_a^b x f(x)\, dx$$

$$= \underbrace{b \int_b^b f(t)\, dt}_{0} - a \int_a^b f(t)\, dt + \int_a^b x f(x)\, dx$$

$$= \int_a^b x f(x)\, dx - \int_a^b a f(t)\, dt$$

$$= \int_a^b x f(x)\, dx - \int_a^b a f(x)\, dx$$

$$= \int_a^b (x f(x) - a f(x))\, dx$$

$$= \int_a^b (x - a) f(x)\, dx$$

70.) $\int \sqrt{1-x^2}\, dx$ (Let $u = \sqrt{1-x^2}$, $dv = dx$,
$du = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x)\, dx = \frac{-x}{\sqrt{1-x^2}}\, dx$, $v = x$)

$= x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}}\,dx$

$= x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}\,dx$

$= x\sqrt{1-x^2} - \int \left[\frac{(\sqrt{1-x^2})^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\right]dx$

$= x\sqrt{1-x^2} - \int \sqrt{1-x^2}\,dx + \int \frac{1}{\sqrt{1-x^2}}\,dx \rightarrow$

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$2\int \sqrt{1-x^2}\,dx = x\sqrt{1-x^2} + \int \frac{1}{\sqrt{1-x^2}}\,dx \rightarrow$

$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$

71.) $y = \arcsin x \rightarrow x = \sin y \rightarrow$

$\int f^{-1}(x)\,dx = x f^{-1}(x) - \int f(y)\,dy \rightarrow$

$\int \arcsin x\,dx = x \arcsin x - \int \sin y\,dy$

$= x \arcsin x - (-\cos y) + C$

$= x \arcsin x + \cos(\arcsin x) + C$

72.) $y = \arctan x \rightarrow x = \tan y \rightarrow$

$\int f^{-1}(x)\,dx = x f^{-1}(x) - \int f(y)\,dy \rightarrow$

$\int \arctan x\,dx = x \arctan x - \int \tan y\,dy$

$= x \arctan x - \ln|\sec y| + c$

$= x \arctan x - \ln|\sec(\arctan x)| + c$

74.) $y = \log_2 x \rightarrow x = 2^y \rightarrow$

$\int f^{-1}(x)\,dx = x f^{-1}(x) - \int f(y)\,dy$

$= x \log_2 x - \int 2^y\,dy$

$= x \log_2 x - \frac{1}{\ln 2} 2^y + c$

$= x \log_2 x - \frac{1}{\ln 2} \cdot 2^{\log_2 x} + c$

$= x \log_2 x - \frac{1}{\ln 2} \cdot x + c$