University of California Davis **Name (Print):** ____________________

Differential Equations MAT 108 **Student ID (Print):** ____________________

Midterm Examination I **October 25 2024**

Time Limit: 50 Minutes

This examination document contains 5 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, the Internet, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

Do not write in the table to the right.

1. (25 points) Show that the following formulas hold:
   (a) (15 points) Prove that

$$\sum_{k=1}^{n}(4k+3) = 2n^2 + 5n, \quad \forall n \in \mathbb{N}.$$

   (b) (10 points) Show that $n \leq 2^n, \quad \forall n \in \mathbb{N}$.

2. (25 points) Solve the following two parts:

   (a) (10 points) Consider the sequence $(x_n)_n \in \mathbb{N}$ given by the recursion

   $$x_{n+1} = 2x_n + 2^n, \quad x_1 = 1.$$

   Find the first 5 terms of the sequence.

   (b) (15 points) Show that a closed formula for x_n as in (a) is given by $x_n = n \cdot 2^{n-1}$.

3. (25 points) Solve the following two parts:

   (a) (10 points) Show that 7 divides $n^7 + 21n^6 + 20n + 49$ for all $n \in \mathbb{N}$.

   (b) (15 points) Show that there are no solutions $x, y, z \in \mathbb{Z}$ to the equation

$$x^4 + y^4 + z^4 = 1054279.$$

4. (25 points) For each of the sentences below, circle **the unique correct answer**. (You do *not* need to justify your answer.)

   (a) (5 points) The last two digits of $(10252024)^{2048}$ are

   (a) 24. (b) 54. (c) 74. (d) 76.

   (b) (5 points) If $A, B \subseteq X$ are subsets of a set X, then $A^c \cap B^c$ equals

   (a) $A^c \cup B^c$. (b) $A \cap B$. (c) $(A \cap B)^c$. (d) $(A \cup B)^c$.

   (c) (5 points) Let $x = 11023$, then $x^8 \pmod{11}$ is

   (a) 0 (b) 1 (c) 2 (d) 3

   (d) (5 points) The number $\binom{9}{4}$ equals

   (a) $\binom{13}{4}$ (b) $\binom{9}{4}$ (c) $\binom{13}{5}$ (d) $\binom{9}{5}$ (e) None of the above.

   (e) (5 points) The coefficient in front of x^3y^{18} in $(x+y)^{21}$ is

   (a) 1. (b) 18. (c) 210. (d) 816. (e) 1330.