University of California Davis **Name (Print):** ____________________

Differential Equations MAT 108 **Student ID (Print):** ____________________

Practice Midterm Examination II **October 25 2024**

Time Limit: 50 Minutes

This examination document contains 5 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, the Internet, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

1. (25 points) Show that the following formulas hold:

   (a) (15 points) Prove that

$$\sum_{k=1}^{n} 2k = n(n+1), \quad \forall n \in \mathbb{N}.$$

   (b) (10 points) Show that

$$\sum_{k=1}^{n} (2k)^2 = \frac{2n(n+1)(2n+1)}{3}, \quad \forall n \in \mathbb{N}.$$

2. (25 points) Solve the following two parts:

   (a) (10 points) Consider the sequence $(x_n)_n \in \mathbb{N}$ given by the recursion

   $$x_{n+1} = 2x_n - 1, \quad x_1 = 3.$$

   Find the first 5 terms of the sequence.

   (b) (15 points) For the sequence in (a), find a closed formula for the nth term x_n.

3. (25 points) Solve the following two parts:

   (a) (10 points) Let $x \in \mathbb{Z}$, prove that we must have one of the following three options: either $x^3 \equiv 0 \pmod 9$, $x^3 \equiv -1 \pmod 9$ or $x^3 \equiv 1 \pmod 9$.

   (b) (15 points) Show that there are no solutions $x, y, z \in \mathbb{Z}$ to the equation

$$x^3 + y^3 + z^3 = 2029.$$

4. (25 points) For each of the sentences below, circle **the unique correct answer**. (You do *not* need to justify your answer.)

   (a) (5 points) The residue of 70071^6 divided by 7 is

   (a) 0. (b) 1. (c) 2. (d) 6.

   (b) (5 points) The equation $x^2 + y^2 = 1$, $x, y \in \mathbb{Z}$ has

   (a) No solutions. (b) Exactly two solutions.
   (c) Infinitely many solutions. (d) None of the above.

   (c) (5 points) Let $x = 1298$, then $x \equiv \pmod 5$ is

   (a) 0 (b) 1

   (c) 2 (d) 3

   (d) (5 points) Let $x_n = 3x_{n-1}$ with $x_1 = 1$, then x_3 is

   (a) 1 (b) 3 (c) 9
   (d) 27 (e) None of the above.

   (e) (5 points) If $x \in \mathbb{Z}$, then x^{12} divided by 13 has resiude

   (a) Always 0. (b) Always 1. (c) Either 0 or 1. (d) 0,1 or 2. (e) 2.