LECTURE 1: PRACTICE EXERCISES

MAT-67 SPRING 2024

Abstract. These practice problems correspond to the first lecture of MAT-67 Spring 2024, delivered on April 1st 2024.

The following are practice problems. They are not to be submitted, they are for your own practice. Solutions will be posted soon.

Problem 1. For each of the following eight systems of equations, decide whether the system is *linear* or *non-linear*.

(1)
$$\begin{cases} 3x_1 + 2x_2 - 4.7x_3 = 5 \\ -x_1 + 9.1x_2 - 2x_3 = 10 \end{cases}$$

(2)
$$\begin{cases} 3x_1^7 + 2x_2 - x_1x_3 = 0 \\ -x_1 + 9.1x_2 - 2x_3 = 10 \end{cases}$$

(3)
$$\begin{cases} x_1x_2 = 1 \\ x_1 + x_2 = -1 \end{cases}$$

(4)
$$\begin{cases} x_2 + \sqrt{x_3} = 1 \\ x_1 - 2x_3 = 10 \\ \cos(x_2) - x_3 = 0 \end{cases}$$

(5)
$$\begin{cases} e^{x_1}x_2 + 4x_3 + 7x_4 = -16 \\ x_2 - x_3 + x_4 = 10 \\ x_1 + x_2x_4 - 8x_3 = 1 \end{cases}$$

(6)
$$\begin{cases} x_1 - x_2 + 5x_3 - 9x_4 = 1 \\ 3x_1 - 8x_2 + 5x_3 - 9x_4 = 0 \\ -4x_1 + 5x_2 + 5x_3 - \ln(2)x_4 = 1 \\ 5x_1 - x_2 + 10x_3 - 9x_4 = \cos(105) \end{cases}$$

(7)
$$\begin{cases} e^3x_2 + 4x_3 + \sin(54)x_4 = -\ln(\cos(1 + e^7)) \\ \tan(32)x_2 - x_3 + x_4 = 10 \\ x_1 + x_2 - x_4 - 8x_3 = 1 \end{cases}$$

(8)
$$\begin{cases} x_1 + x_2 = -2 \\ (x_1 + x_2)^2 - 2x_1x_2 = 10 \end{cases}$$

Problem 2. By direct calculation, discuss whether each of the following linear systems of equations have *no solution*, *a unique solution* or *infinitely many solutions*.

(1) The following linear system in two unknown variables $x_1, x_2 \in \mathbb{R}$:

$$\begin{cases} x_1 + x_2 = 0 \\ x_1 + x_2 = 1 \end{cases}$$

(2) The following linear system in two unknown variables $x_1, x_2 \in \mathbb{R}$:

$$\begin{cases} x_1 + x_2 = 0 \\ x_1 - x_2 = 1 \end{cases}$$

(3) The following linear system in three unknown variables $x_1, x_2, x_3 \in \mathbb{R}$:

$$\begin{cases} x_1 + x_2 = 0 \\ x_1 + x_3 = 1 \end{cases}$$

(4) The following linear system in three unknown variables $x_1, x_2, x_3 \in \mathbb{R}$:

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1 + 4x_2 + x_3 = 2 \\ x_1 + x_2 + 5x_3 = -12 \end{cases}$$

(5) The following linear system in three unknown variables $x_1, x_2, x_3 \in \mathbb{R}$:

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ 2x_1 + 2x_2 + 6x_3 = 0 \\ x_1 + x_2 + 5x_3 = 2 \end{cases}$$

(6) The following linear system in three unknown variables $x_1, x_2, x_3 \in \mathbb{R}$:

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ 2x_1 + 2x_2 + 6x_3 = 0 \\ x_1 + x_2 + 5x_3 = 0 \end{cases}$$

Problem 3. For each of the linear systems in Problem 2, write a linear map $f : \mathbb{R}^n \to \mathbb{R}^m$ and an m-tuple $(y_1, \ldots, y_m) \in \mathbb{R}^m$, for some $n, m \in \mathbb{R}$, (all depending on the given system) such that solving that given linear system is equivalent to finding $(x_1, \ldots, x_n) \in \mathbb{R}^n$ in the domain of f such that

$$f(x_1, \ldots, x_n) = (y_1, \ldots, y_m).$$