# LECTURE 3: PRACTICE EXERCISES

MAT-67 SPRING 2024

ABSTRACT. These practice problems correspond to the 3rd lecture of MAT-67 Spring 2024, delivered on April 5th 2024.

The following are practice problems. They are not to be submitted, they are for your own practice. Solutions will be posted soon.

**Problem 1**. Draw in the real line $\mathbb{R}$ and the real plane $\mathbb{R}^2$ the following maps $f$ by drawing vectors $v_i$ and their images $f(v_i)$.

(1) $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 5x$ and the vectors
$$v_1 = (1), \quad v_2 = (-3), \quad v_3 = (4)$$
(2) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $f(x_1, x_2) = (3x_1 - x_2, x_2)$ and the vectors
$$v_1 = (1, 0), \quad v_2 = (0, 1), \quad v_3 = (2, 5)$$
(3) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $f(x_1, x_2) = (3x_1, 5x_2)$ and the vectors
$$v_1 = (1, 0), \quad v_2 = (0, 1), \quad v_3 = (1, 1)$$
(4) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $f(x_1, x_2) = (x_1, x_1)$ and the vectors
$$v_1 = (1, 0), \quad v_2 = (0, 1), \quad v_3 = (1, 1)$$
(5) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $f(x_1, x_2) = (2x_1, 0)$ and the vectors
$$v_1 = (1, 0), \quad v_2 = (0, 1), \quad v_3 = (2, -3)$$
(6) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $f(x_1, x_2) = (-x_1, -x_2)$ and the vectors
$$v_1 = (1, 0), \quad v_2 = (0, 1), \quad v_3 = (5, 6)$$

**Problem 2**. Solve the following parts:

(1) Suppose that $f : \mathbb{R} \longrightarrow \mathbb{R}$ is a linear map such that $f(1) = 3$. Compute $f(4)$.

(2) Suppose that $f : \mathbb{R} \longrightarrow \mathbb{R}$ is a linear map such that $f(7) = -2$. Compute $f(5)$.

(3) Suppose that $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear map such that $f(1, 0) = (3, 4)$ and $f(0, 1) = (0, 2)$. Compute $f(-1, 5)$.

(4) Suppose that $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear map such that $f(2, 0) = (3, 1)$ and $f(0, 4) = (0, -1)$. Compute $f(7, 1)$.

(5) Suppose that $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear map such that $f(1, 1) = (1, 2)$ and $f(2, 3) = (-4, 9)$. Compute $f(1, 1)$.

(6) Suppose that $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a linear map. Compute $f(0, 0)$.

(7) Suppose that $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ is a linear map such that $f(1,0) = 3$ and $f(0,1) = 2$. Compute $f(-1,5)$.

**Problem 3**. **Prove**, with an argument, or **disprove**, with a counter-example, each of the statements sentences below.

(1) If a linear map $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is such that $f(1,0) = (1,0)$ and $f(0,1) = (2,5)$. Then $f(1,1) = (3,5)$.

(2) If a linear map $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is such that $f(1,0) = (1,0)$ and $f(2,0) = (2,5)$. Then $f(1,1) = (3,5)$.

(3) If a linear map $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is such that $f(1,3) = (1,0)$ and $f(-2,-6) = (0,1)$. Then $f(1,2) = (3,5)$.

(4) If a linear map $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is such that $f(1,0) = (1,0)$ and $f(0,1) = (2,5)$. Then $f(1,1) = (3,5)$.

(5) Any map $f : \mathbb{R} \longrightarrow \mathbb{R}$ of the form $f(x) = \alpha \cdot x$, for some $\alpha \in \mathbb{R}$ is linear.

(6) Any linear map $f : \mathbb{R} \longrightarrow \mathbb{R}$ is of the form $f(x) = \alpha \cdot x$, for some $\alpha \in \mathbb{R}$.

(7) Any linear map $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is of the form $f(x_1, x_2) = (\alpha_1 \cdot x_1, \alpha_2 \cdot x_2)$, for some $\alpha_1, \alpha_2 \in \mathbb{R}$.