LECTURE 5: PRACTICE EXERCISES

MAT-67 SPRING 2024

ABSTRACT. These practice problems correspond to the 5th lecture of MAT-67 Spring 2024, delivered on April 10th 2024.

The following are practice problems. They are not to be submitted, they are for your own practice. Solutions will be posted soon.

Problem 1. Show that these subsets $U \subseteq V$ are vector subspaces of V:

(1) The subset $U \subseteq V = \mathbb{R}^2$ of all vectors of the form
$$U = \{(x_1, x_2) \in V : x_1 + 4x_2 = 0\} \subseteq V.$$
(2) The subset $U \subseteq V = \mathbb{R}^4$ of all vectors of the form
$$V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 - x_2 + 7x_3 = 0, \quad 3x_2 - 8x_3 + 4x_4 = 0\} \subseteq V.$$
(3) The subset $U \subseteq V = \mathbb{R}^2$ of all vectors of the form
$$U = \{(x_1, x_2) \in V : x_1 + x_2 = -9\} \subseteq V.$$
(4) The solutions $(x_1, x_2, x_3) \in V = \mathbb{R}^3$ of the system of equations:
$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ 2x_1 + 4x_2 + 5x_3 = 0 \\ x_1 + x_2 + 5x_3 = 0 \end{cases}$$
(5) The subspace $U \subseteq V = \mathbb{R}[x]$ of polynomials of the form
$$U = \{p(x) \in \mathbb{R}[x] : p(0) + p(4) = 0\}$$
(6) The subspace $U \subseteq V = \mathbb{R}[x]$ of polynomials of the form
$$U = \{p(x) \in \mathbb{R}[x] : p(0) = 0, \quad p(2) - p(3.5) = 0\}$$
(7) Let $V = \{f : \mathbb{R} \to \mathbb{R} : \ f \text{ continuous}\}$ be the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Consider
$$U = \{f : \mathbb{R} \to \mathbb{R} : \ f \text{ differentiable}\}$$
(8) Let $V = \{f : \mathbb{R} \to \mathbb{R} : \ f \text{ continuous}\}$ be the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Consider
$$U = \{f : \mathbb{R} \to \mathbb{R} : f(0) = 0\}$$

Problem 2. Show that neither of these subsets U is a subspace of V:

(1) The subset $U \subseteq V = \mathbb{R}^2$ of all vectors of the form
$$U = \{(x_1, x_2) \in V : x_1 + 4x_2 = 0\} \subseteq V.$$
(2) The subset $U \subseteq V = \mathbb{R}^4$ of all vectors of the form
$$V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 - x_2 + 7x_3 = 0\} \subseteq V.$$
(3) The subset $U \subseteq V = \mathbb{R}^2$ of all vectors of the form
$$U = \{(x_1, x_2) \in V : x_1 + x_2 = -9\} \subseteq V.$$
(4) The subset $U \subseteq V = \mathbb{R}^4$ of all vectors of the form
$$V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 x_3 = 0\} \subseteq V.$$
(5) The solutions $(x_1, x_2, x_3) \in V = \mathbb{R}^3$ of the system of equations:
$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ 2x_1 + 2x_2 + 6x_3^2 = 0 \\ x_1^7 + x_2 + 5x_3 = 1 \end{cases}$$
(6) The subspace $U \subseteq V = \mathbb{R}[x]$ of polynomials of the form
$$U = \{p(x) \in \mathbb{R}[x] : p(0)p(4) = 0\}$$
(7) The subspace $U \subseteq V = \mathbb{R}[x]$ of polynomials of the form
$$U = \{p(x) \in \mathbb{R}[x] : p(x) \cdot (x^2 - x + 1) = 1\}$$
(8) Let $V = \{f : \mathbb{R} \to \mathbb{R} : f \text{ continuous}\}$ be the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Consider
$$U = \{f : \mathbb{R} \to \mathbb{R} : f'(0) = 0\}$$

Problem 3. Let $U_1, U_2 \subseteq V$ be subspaces of $V = \mathbb{R}^n$. Suppose that neither U_1 or U_2 are the zero subspace $\{0\} \subseteq V$. Show that the set-theoretic union $U_1 \cup U_2 \subseteq V$ is *not* a vector subspace.

Problem 4. Consider the subsets $U_1, U_2 \subseteq V$ of $V = \mathbb{R}^3$ given by
$$U_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}$$
$$U_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_3 = 0\}.$$
Show that the intersection $U_1 \cap U_2$ is *not* a vector subspace.