MATH 22AL
Lab # 4

1 Objectives

In this LAB you will explore the following topics using MATLAB.

- Properties of invertible matrices.
- Inverse of a Matrix
- Explore LU Factorization

2 Recording and submitting your work

The following steps will help you to record your work and save and submit it successfully.

- Open a terminal window.
  - In Computer LAB (2118 MSB) click on terminal Icon at the bottom of the screen
  - In Windows OS, Use Putty
  - In MAC OS, Use terminal window of MAC.
- Start a MATLAB Session that is :
  - Type "textmatlab" Press Enter
- Enter your information that is :
  - Type " diary LAB4.text"
  - Type "% First Name:" then enter your first name
  - Type "% Last Name:" then enter your Last name
  - Type "% Date:" then enter the date
  - Type "% Username:" then enter your Username for 22AL account
- Do the LAB that is :
  - Follow the instruction of the LAB.
  - Type needed command in MATLAB.
  - All commands must be typed in front of MATLAB Command " >> "..
- Close MATLAB session Properly that is :
  - When you are done or if you want to stop and continue later do the following:
  - Type "save" Press Enter
  - Type "diary off" Press Enter

- Type "exit" Press Enter

- Edit Your Work before submitting it that is :
  - Use pico or editor of your choice to clean up the file you want to submit:
  - in command line of pine type "pico LAB4.text"
  - Delete the error
  - Properties of invertible matrices.
  - Inverse of a Matrix
  - Explore LU Factorization s or insert missed items.
  - Save using "ˆ o= control key then o"
  - Exit using "ˆ x= control key then x"

- Send your LAB that is :
  - Type "ssh point" : Press enter
  - Type submitm22al LAB4.text

3 Inverse of A

3.1 Reading:

Suppose$A$ is a square matrix. The inverse is written A^{-1} and is defined to be a matrix when A is multiplied by A^{-1} the result is the identity matrix I.
Note: The following facts about the inverse of a matrix are information from your 22A class. Section 2.5. Please review them before continuing the LAB

- Not all square matrices have inverses.
- A square matrix which has an inverse is called invertible or nonsingular
- A square matrix without an inverse is called non invertible or singular.
- The inverse exists if and only if elimination produces n pivots
- The matrix A cannot have two different inverses.
- If A is invertible, the one and only solution to $Ax = 0$ is $x = A^{-1}x$
- Suppose there is a nonzero vector x such that $Ax = 0$ then A cannot have an inverse.
- A 2 by 2 matrix is invertible if and only if $ad - bc$ is not zero
- A diagonal matrix has an inverse provided no diagonal entries are zero.
- If A and B are invertible then so is AB. The inverse of a product $(AB)^{-1} = B^{-1}A^{-1}$
- For square matrices, an inverse on one side is automatically an inverse on the other side.
- The right-inverse equals the left-inverse. $BA = I$ and $AC = I$ then $B = C$
- If A doesn't have n pivots, elimination will lead to a zero row.
- An invertible matrix A can't have a zero row!
- A triangular matrix is invertible if and only if no diagonal entries are zero.
- The Gauss-Jordan method solves $AA^{-1} = I$ to find the n columns of A^{-1}.

3.2 Working with MATLAB

3.2.1 Basic Properties of inverse

type	A= [1 2 3 ; 3 4 1; 9 4 2]	To enter a 3 by 3 matrix.
type	AI = inv(A)	to find the inverse of A
type	AII = inv(AI)	Explain what is happening by typing % before your answer.
type	AR =rref(A)	to see the Reduced Row Echelon Form of A. What type of the matrix you are getting? Is this true for all invertible matrices A? Explain by typing % before your answer.
type	B(:,1) = A(:,1)	to define the first column of B as first column of A.
type	B(:,2) = A(:,2)	to define the second column of B as second column of A.
type	B(:,3) = A(:,1) + A(:,2)	to define the third column of B as the sum of the first two columns of A.
type	BR =rref(B)	to see the Reduced Row Echelon Form of B. Do you see a row of zeros?
type	BI = inv(B)	Does the inverse of B exist? Explain by typing % before your answer.

3.2.2 Inverse of Diagonal Matrices

type	C=	[0 0 0 3; 0 0 2 0; 0 5 0 0 ; 4 0 0 0]
type	inv(C)	To see the inverse of C.
type	format rat	to see the numbers in fraction format.
type	inv(C)	To see the inverse of C.
type	D = [0 0 3; 0 8 0; 5 0 0]	
type	inv(D)	to see inverse of D

Without typing it in the MATLAB guess what the inverse of E= [0 0 1; 0 3 0; 7 0 0] should be? Explain by typing % before your answer. Enter your answer as EI=

3.2.3 Inverse of Block Matrices

type	J= E*EI	If you did not get an Identity matrix, try it again, until you get it right.
type	F=	[3 2 0 0 ; 4 3 0 0 ; 0 0 6 5 ; 0 0 7 6].
type	FI= inv(F)	to find inverse of F

Any thing interesting? Explain by typing % before your answer.

type	F1I= inv([3 2; 4 3])
type	F1I= inv([6 5; 7 6])

Explain the relation of the last two inverses with the inverse of matrix F, by typing % before your answer.

3.2.4 An Interesting Observation

type E1 = 5*eye(4) - ones(4,4)

type E1I = inv(E1) to see the inverse of E1

type E2 = 6*eye(5) - ones(5,5)

type E2I = inv(E2) to see the inverse of E1

type E3 = 3*eye(2) - ones(2,2)

Generalize your observation. Representing a $n \times n$ identity matrix by I_n and a $n \times n$ matrix of ones by O_n write an inverse for $(n+1)I_n - O_n$, it will be in the form of $k(I_n + O_n)$.

Enter your response by typing % before your answer. Your answer has to be in the form of INV$((n+1)I_n - O_n) = k(I_n + O_n)$. where the k is replaced by the value you choose.

4 Using Gauss- Jordan Elimination to calculate H^{-1}

You may find inverse of a $n \times n$ matrix H by forming a new matrix as $K = [A\ I]$ then using $rref(K)$, if H is invertible, H will transform to I and I will transform to H^{-1}.

4.1 Calculating H^{-1}

type H= [1 3 ; 2 7]

type H1= [H eye(2)] to form $[H\ I]$

type H2=rref(H1) To find Reduced Row Echelon Form of $[H\ I]$

type HI= H2(:,[3 4]) to extract inverse of H

type inv(H) To find the inverse of H using MATLAB command

type $L = [2\ 1\ 0; 1\ 2\ 1; 0\ 1\ 2]$ To enter 3×3 matrix L.

type $J = [1\ 3\ -5; 3\ 2\ 1; 0\ 1\ 2]$ To enter 3×3 matrix L.

type $LI1 = inv(L)$ to find inverse of L

type $LI2 = rref([L\ \ eye(3)])$ to calculate inverse of L

type $LI3 = LI2(:, 4:6)$ to extract the inverse of L

type $JI1 = inv(J)$ to find inverse of J

type $JI2 = rref([J\ \ eye(3)])$ to calculate inverse of J

type $JI3 = JI2(:, 4:6)$ to extract the inverse of J

4.2 Inverse of LJ

type $LJ = L * J$ to find LJ

type $LJI = inv(L * J)$ to calculate inverse of LJ

type LJ2= inv(L) *inv(J) $L^{-1}J^{-1}$

type LJ3= inv(J) *inv(L) $J^{-1}L^{-1}$

4.2.1 True , False questions

With these observation, determine which one of the following statements is true which one is false:

For each statement enter your response after % as (for example a.) is True) or (for example a.) is False) .

- **a.)** **If** A **and** B **are given invertible matrices, then** $(AB)^{-1} = (A)^{-1}(B)^{-1}$
- **b.)** **For any given invertible matrices** A **and** B **we have** $(AB)^{-1} = (B)^{-1}(A)^{-1}$
- **c.)** **For any given matrices** A **and** B **we have** $(AB)^{-1} = (B)^{-1}(A)^{-1}$
- **d.)** **There is a group of matrices in which for any given invertible matrices** A **and** B **in that group, we have** $(AB)^{-1} = (A)^{-1}(B)^{-1}$**.**

5 LU Factorization

5.1 Reading

Using Gaussian elimination we can express any square matrix as the product of a permutation of a lower triangular matrix and an upper triangular matrix. $M = PLU$. If A is invertible, Usually we choose the lower triangular matrix L with diagonal entries 1 and if we choose both L and U to have diagonal entries 1, then we need a diagonal matrix in the middle and decomposition becomes as $M = PLDU$, which is unique. One of the applications of LU decomposition or factorization is in solving a linear system $AX = b$. This can be seen as $LUX = b$. Assume $UX = Y$ the linear system becomes as $LY = b$ which can be solves easily for Y. Then the system $UX = Y$ can be solved for X. You can read more on LU-Factorization in section 2.6 of your text book

5.2 Using MATLAB

You can find LU factorization of a matrix in MATLAB using lu(A), the matrix L returned by MATLAB is a permutation of a lower triangular matrix.

5.2.1 Note

[L,U] = lu(A) stores an upper triangular matrix in U and a "potentially lower triangular matrix" (i.e. a product of lower triangular and permutation matrices) in L, so that A = L*U. A can be rectangular.

[L,U,P] = lu(A) returns unit lower triangular matrix L, upper triangular matrix U, and permutation matrix P so that P*A = L*U.

5.2.2 Examples

type	A= [8 2 9; 4 9 4; 6 7 9]	To enter A.
type	[L U] = lu(A)	To see LU factorization of A.
type	[L U P] = lu(A)	To see LU factorization of A with the permutation matrix P. Note that matrix P in this case will be an identity matrix.
type	A1= L*U	To check your answer.
type	B= [6 7 9; 4 9 4; 8 2 9]	To enter B. Note that first and third row of A are interchanged.
type	[L U P] = lu(B)	To see LU factorization of B with the permutation matrix P.
type	[L U] = lu(B)	To see LU factorization of B. Notice that L needs row exchange (permutation) to become a lower triangular matrix.
type	M= [2 1 0; 1 2 1; 0 1 2]	To enter M.
type	[L U] = lu(M)	To see LU factorization of M.
type	[L U P] = lu(M)	To see LU factorization of M with the permutation matrix.
type	N= L*U	To check your answer.
type	O= [1 3 0; 3 11 4; 0 4 9]	To enter O.
type	[L U P] = lu(O)	To see LU factorization of O with the permutation matrix.
type	[L U] = lu(O)	To see LU factorization of O to notice that L needs row exchange (permutation) to become a lower triangular matrix.

5.2.3 Solving AX=b Using LU factorization

When we need to solve several equations with the same coefficient matrix A, (as $AX = b_1, AX = b_2, Ax = b_3$), an efficient method for solving all of them is to find LU factorization of A, then use forward/back substitution to find X. Computational cost of this method is roughly $\frac{2}{3}n^3$ operation (flop).

We can also use LU factorization to find inverse of a nonsingular matrix A and use it to solve $AX = b$.

Here is how: Assume A is a nonsingular matrix, and A = P LU. First find the inverse of A using LU factorization as $A^{-1} = (PLU)^{-1} = U^{-1}L^{-1}P^T$. Then, $X = A^{-1}b = U^{-1}L^{-1}P^Tb$. Note that this method is not very efficient, its computational cost is $\frac{8}{3}n^3$ operation (flop).

5.2.4 Examples

type	L= [1 0 0 ; 2 1 0 ; 3 4 1]	To enter L.
type	U =[1 2 3 ; 0 4 5 ; 0 0 6]	To enter U.
type	S = L*U	to create S
type	b =[14 51 152] ′	to enter b

To solve $SX = b$ use LU- Factorization. consider $SX = LUX = b$. Set $Y = UX$, so you have $SX = LUX = L(UX) = L(Y) = b$. Solve $LY = b$ first, this can be done by backward substitution. Do it by hand and check it with MATLAB.

type $Y = L\backslash b$ **To solve $LY = b$ for Y.**

Now you have $UX = Y$ which you can solve using forward substitution. Do it on paper then, check you answer using MATLAB

type $X = U\backslash Y$ **To solve $UX = Y$ for X.**

This the end of the LAB 4, follow the directions to close your diary file save your variables, edit your work, then send it to the TA.