MATH 22AL
Lab # 6

1 Objectives

In this LAB you will explore the following topics using MATLAB.

- Investigate properties of the null space of a matrix.
- Learn how to find a basis for null-space of a matrix.
- Investigate properties of the column space of a matrix.
- Learn how to find a basis for column space of a matrix.
- Investigate properties of the row space of a matrix.
- Learn how to find a basis for row space of a matrix.

2 What to turn in for this lab

Please save and submit your MATLAB session.
Important: Do not Change the name of the Variables that you supposed to type in MATLAB, type as you are asked.

3 Recording and submitting your work

The following steps will help you to record your work and save and submit it successfully.

- Open a terminal window.
  - In Computer LAB (2118 MSB) click on terminal Icon at the bottom of the screen
  - In Windows OS, Use Putty
  - In MAC OS, Use terminal window of MAC.
- Start a MATLAB Session that is :
  - Type "textmatlab" Press Enter
- Enter your information that is :
  - Type "diary LAB6.text"
  - Type "% First Name:" then enter your first name
  - Type "% Last Name:" then enter your Last name
  - Type "% Date:" then enter the date
  - Type "% Username:" then enter your Username for 22AL account
- Do the LAB that is :
  - Follow the instruction of the LAB.
  - Type needed command in MATLAB.
  - All commands must be typed in front of MATLAB Command " >> "..
- Close MATLAB session Properly that is :
  - When you are done or if you want to stop and continue later do the following:
  - Type "save" Press Enter
  - Type "diary off" Press Enter
  - Type "exit" Press Enter
- Edit Your Work before submitting it that is :
  - Use pico or editor of your choice to clean up the file you want to submit:
  - in command line of pine type "pico LAB6.text"
  - Delete the errors or insert missed items.
  - Save using "ˆ o= control key then o"
  - Exit using "ˆ x= control key then x"
- Send your LAB that is :
  - Type "ssh point" : Press enter
  - Type submitm22al LAB6.text

4 Background, Reading Part : Row vectors and column vectors

Let $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix}$.
Matrix B has 2 rows and 3 columns. Each of the two rows

$$r_1 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

and

$$r_2 = \begin{bmatrix} 4 & 0 & 1 \end{bmatrix}$$

of B are referred as a row vector of B . They are 3-tuple of real numbers, so they belong to R^3.
In general if A is a $m \times n$ matrix, then the rows of A are n-tuples of real numbers and therefore they are vectors in R^n.
Similarly the columns of B are vectors in R^2. You may write the column vectors as

$$C_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \qquad C_3 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

or as their transpose as

$$C_1^t = \begin{bmatrix} 1 & 4 \end{bmatrix}, \qquad C_2^t = \begin{bmatrix} 2 & 0 \end{bmatrix} \text{ and } \qquad C_3^t = \begin{bmatrix} 3 & 1 \end{bmatrix}.$$

or as

$$C_1 = (1, 4), \qquad C_2 = (2, 0) \qquad \text{and} \qquad C_3 = (3, 1).$$

The main idea is to understand that they are vectors in R^2.

In general if A is a $m \times n$ matrix, then the columns are m-tuples of real numbers and therefore they are in R^m.

We are interested in vector space spanned by the row vectors and vector space spanned by the column vectors of A.

5 Row space, Column space, Null space

Definition : The vector space spanned by the row of $A_{m\times n}$ is a subspace of R^n and is called **Row space of** A and is denoted by $row(A)$.

Note: For some matrices the row space of A is R^n and for some it is not.

Definition : The vector space spanned by the columns of A is a subspace of R^m and is called **th column space of** A **and is denoted by** $col(A)$**.**

Note: For some matrices the column space of A is R^m and for some it is not.

We are interested in studying $row(A)$ and $col(A)$. In particular we want to find bases for $row(A)$ and $col(A)$.

Note: Since column vectors of A are row vectors of $A^t = A'$ we will study the row space in more details. To study the column space of A we need to consider the row space of A^t.

Definition : There is also another subspace of R^n which we are interested to study. This subspace is the set of all solutions of the linear system $AX = 0$ and is called **null-space of A**. The Null space of A is denoted by $null(A)$ and is a subspace of R^n.

6 A closer look at Row Space

Definition : The set of all linear combination of the row vectors of a $m \times n$ matrix A, is a subspace of R^n, and is called row space of A.

Note: Since the set of scalars (a quantity that can multiply vectors in the vector spaces) is infinite, we can not list all vectors in row(A) or col(A), we usually determine bases for them.

Definition : The dimension of the row space of A is called $rank(A)$.

Definition : The **dimension** of a vector space defined as the minimum number of coordinates needed to specify any vector within the vector space, which is equal to the number of the vectors in a basis for that vector space.

Note:

- The dimension of row space of A = The dimension of column space of A.
- If $rank(A) = n$ the row space of A is an n-dimensional subspace of R^n, so $row(A) = R^n$.
- If $rank(A) < n$ then the row vectors of A do not span R^n.
- For a matrix $A_{m \times n}$ rank of A is less than or equal to minimum of m and n.

MAT LAB could be used to find a basis for $row(A)$

End of reading Materials

Start Typing in MATLAB

Example 1: **Let** $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix}$.

Type :

$B = [1\ 2\ 3; 4\ 0\ 1]$.

Before continuing using MATLAB consider the set of all linear combinations of the row vectors of B. This is a subspace of R^3 **spanned by the vectors** $r_1 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ **and** $r_2 = \begin{bmatrix} 4 & 0 & 1 \end{bmatrix}$.

First note that the two vectors r_1 **and** r_2 **are linearly independent (Why?).**

Enter your answer as:

% ANS1= type your answer

So these two vectors r_1 **and** r_2 **form a basis for the row space of** B**. Since** B **is a simple matrix of small size, you should also be able to justify that the row space of** B **is a subspace of** R^3 **but it is not** R^3**, which means that row vectors of** B **do not span** R^3**. In other words there are vectors in** R^3 **which are not in the row space of** B**.**

For large matrices finding out this information is not that simple. You could get some information about the dimension of the row space or column space using the size of the matrix. Another useful MATLAB command is $rref(B)$.

7 Background, Reading Part : How to use MATLAB to study the row space of a matrix A?

- **Enter your matrix A in MATLAB.**
- **Find** $rref(A)$**.**
- **Non-zero rows of** $rref(A)$ **form a basis for** $row(A)$.
- **Find** $rank(A)$ **by typing :** $rank(A)$ **it should be equal to the number of the non-zero rows of** $rref(A)$.

End of reading Materials

Start Typing in MATLAB

Example 2: **Let** $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix}$.

Type :

$B = [1\ 2\ 3; 4\ 0\ 1]$.

Type :

$REFB = rref(B)$

Type :

$RANKB = rank(B)$

Type two vectors that form a basis for the row space of B. Type your answer as R1B for the first vector and R2B for the second vector:

$R1B =$

$R2B =$

Example 3:

Enter $A = B'$ **or type** $A = \begin{bmatrix} 1 & 4 \\ 2 & 0 \\ 3 & 1 \end{bmatrix}$ **and find** $RA = rref(A)$ **you should get**

$RA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

So the row space of A is a 2-dimensional subspace of R^2. That is $row(A) = R^2$ (why?). Enter your answer as:

% ANS2= type your answer

Type :

$REFA = rref(A)$

Type :

$RANKA = rank(A)$

Type two vectors that form a basis for the row space of A. Type your answer as R1A= for the first vector and R2A= for the second vector:

$R1A =$

$R2A =$

Notice that

- **The matrix in example 3 is the transpose of the matrix in example 2. Both sub spaces (row space and column spaces) have the same dimension (= 2) but they are subspaces of different vector spaces.**
- **The $row(A)$ in example 2 is a two-dimensional subspace of R^3. But $row(A)$ in example 3 is a two-dimensional subspace of R^2.**
- **he subspace in example 3, $row(B) = row(A')$ is the column space of the matrix in example 2. In general one can prove that dimension of the row space of a matrix is equal to the dimension of the column space of the matrix, which is called $rank(A)$.**

NOTE

- **Row operations do not change the solution set of a matrix.**
- **Row operations do not change the row space of a matrix.**
- **Row operations can change the column space of a matrix.**

End of reading Materials

Start Typing in MATLAB

7.1 *Exercise 1*

Enter the matrix C $= \begin{bmatrix} 1 & 0 & 2 & 3 \\ 4 & -1 & 0 & 2 \\ 0 & -1 & -8 & -10 \end{bmatrix}$ **in MATLAB. (you know how to enter a matrix in MATLAB)**

a) **Find a basis for the** $row(C)$**.**

Type vectors that form a basis for the row space of C**. Type your answer as R1C for the first vector and R2C for the second vector and R3C, ... as many as needed:**

$R1C =$

$R2C =$

$R3C =$

b) **Find a basis for the** $row(C^t)$**.**

Type vectors that form a basis for the row space of C^t**. Type your answer as R1CT for the first vector and R2CT for the second vector and R3C, ... as many as needed:**

$R1CT =$

$R2CT =$

$R3CT =$

c) **Find a basis for the** $col(C)$**.**

Type vectors that form a basis for the column space of C**. Type your answer as C1C for the first vector and C2C for the second vector and R3C, ... as many as needed:**

$C1C =$

$C2C =$

$C3C =$

d) **Find a basis for the** $col(C^t)$**.**

Type vectors that form a basis for the column space of C^t**. Type your answer as C1CT for the first vector and C2CT for the second vector and C3CT, ... as many as needed:**

$C1CT =$

$C2CT =$

$C3CT =$

d) **Find** $rank(C)$ **and** $rank(C^t)$ **by typing**

RanC=$rank(C)$

and

RanCt=$rank(C')$

Write down dimension of $row(C)$ **and** $col(C)$ **as:**

Drowspace= type your answer here
Dcolspace= type your answer here

8 Null space

Definition The set of all vectors v that satisfies $A\mathbf{v} = \mathbf{0}$ is called the null-space of $A_{n\times m}$.

You should be able to prove that this set is a subspace of R^n.(By showing that it is closed under addition and multiplication.)

8.1 Background, Reading Part : How to use MATLAB to find a basis Null space of A

Enter your matrix A in MATLAB.

1. Enter the following matrix:

$$A1 = \begin{bmatrix} 1 & 3 & 0 & 2 & 6 & 3 & 1 \\ -2 & -6 & 0 & -2 & -8 & 3 & 1 \\ 3 & 9 & 0 & 0 & 6 & 6 & 2 \\ -1 & -3 & 0 & 1 & 0 & 9 & 3 \end{bmatrix}$$

2. Find $rref(A1)$ by typing $RRA1 = rref(A1)$ you will get

$$RRA1 = \begin{bmatrix} 1 & 3 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3. Solve for leading variables:

$$\begin{aligned} x_1 &= -3x_2 \quad -2x_5 \\ x_4 &= -2x_5 \\ x_6 &= -\tfrac{1}{3}x_7 \end{aligned}$$

Set $x_2 = r, x_3 = s, x_5 = t$ and $x_7 = w$ we obtain

$$\begin{bmatrix} x_1 = & -3r & & -2t & \\ x_4 = & & -2t & & \\ x_6 = & & & & -\frac{1}{3}w \end{bmatrix}$$

In matrix form we can write this as

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -3r-2t \\ r \\ s \\ -2t \\ t \\ \frac{1}{3}w \\ w \end{bmatrix} = r\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 0 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + w\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{3} \\ 1 \end{bmatrix}$$

So the vectors

$$\mathbf{v_1} = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \mathbf{v_3} = \begin{bmatrix} -2 \\ 0 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \mathbf{v_4} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{3} \\ 1 \end{bmatrix}$$

form a basis for the solution space of $AX = 0$.

Definition **The solution space of $AX = 0$ is called the null-space of A and denoted by $null(A)$.**

Definition **Dimension of the null-space of A is called nullity of A and denoted by $nullity(A)$.**

NOTE: in the above example $rank(A) = 3$ and $nullity(A) = 4$. Recall the dimension theorem : Is A is an $m \times n$ matrix then $rank(A) + nullity(A) = n$.

Example 6 of section 1.2, Example 10 of section 5.4, Example 4 of section 5.5 of your text book show how to solve $AX = 0$ and how to find a basis for the null-space of A.
End of reading Materials

Start Typing in MATLAB

8.2 Exercise 2

Enter the matrix A2= $\begin{bmatrix} 1 & 0 & 2 & 3 \\ 4 & -1 & 0 & 2 \\ 0 & -1 & -8 & -10 \end{bmatrix}$

a.) Use the method shown above to find a basis for the null-space of A2 (this may involve MATLAB and some pencil & paper. Then enter (type) your answer as column vectors:

$A2N1 =$ **your first vector in basis**

$A2N2 =$ **your second vector in basis**

b.) Check your work by calculating A*A2N1 and A*A2N2. You should get zero vector. to do this type:

$AN1 = A2 * A2N1$

$AN2 = A2 * A2N2$

c.) Check to see if any linear combination of $A2N1$ **and** $A2N2$ **is in** $null(A2)$**. You could do this by typing (for example)**

A2LC= A2* (3*A2N1 -2 *A2N2)

Try part c.) for two other numbers different from 3 and -2.

Start Typing in MATLAB

8.3 An other way of finding a basis for null-space of a matrix .

Let $B2 = [A2^t \quad I]$**, you need to type this as :**

B2=[A2' eye(4)]

then find $F2 = rref(B2)$ **by typing**

F2=rref(B2)

You will get a matrix of the form

$$\begin{bmatrix} R & * \\ 0 & N \end{bmatrix}$$

Note that in this matrix $\begin{bmatrix} R \\ 0 \end{bmatrix}$ **is the** $rref(A')$ **and** $\begin{bmatrix} * \\ N \end{bmatrix}$ **is corresponding** $rref$ **of I.**

Now check to see if the rows of N form a basis for the null-space of $A2$**, by typing**

A2X1= F2(3,4:7)

A2NS1= A2*A2X1'

Repeat this for the second vector:

A2X2= F2(4,4:7)

A2NS2= A2*A2X2'

Start Typing in MATLAB

8.4 Exercise 3

3.1) Enter the matrix A2= $\begin{bmatrix} 1 & 0 & 2 & 3 \\ 4 & -1 & 0 & 2 \\ 0 & -1 & -8 & -10 \end{bmatrix}$

Find a basis for $null(A2)$, you may use the ones from the previous section or compute it again. Call these vectors $N1$ and $N2$.

Recall that any linear combination of the basis vectors will be in the subspace.

That means $k1 * N1 + k2 * N2$ should be in the null-space of A2 no matter what k1 and k2 are. so $A2(k1 * N1 + k2 * N2) = 0$.

We can demonstrate this with the following commands:

*for i = 1:10, k1 = round(10*rand(1)); k2 = -round(10*rand(1));*

*V(:,i) = k1*N1 + k2*N2;*

*AV(:,i) = A2*V(:,i);*

end

Explanation: *The rand function gives numbers between zero and 1 so multiplying by 10 and rounding off gives integers between zero and 10. Thus k1 and k2 are randomly chosen numbers. Each column of the matrix V is a vector which is a linear combination of our basis vectors $N1$ and $N2$. The corresponding column in AV is the vector A2*(that column of V).*

Claim *AV should be a matrix of zeros.*

Type:

A2V= AV

to check what the loop did.

Wow! It works! All of the columns of V are in the null-space of A2!

3.2) *Which of the following vectors are in the null-space of A2? (you need to enter these vectors in MATLAB as matrices, column vectors).*

- *v1= (5, -35, 81, 2)*
- *v2= (38, 104, 17, -24)*
- *v3= (-9, -26, -3, 5)*
- *v4= (1, 0, 1, 1)*

Use MATLAB, then type the following and enter your answer in the following format:

If you decided v1 is in the null-space of A2 type :

% A2v1= is in null(A2)

otherwise type:

% A2v1= is NOT in null(A2) .

(note the answers are case sensitive.)

9 Background, Reading Part : COLUMN SPACE

Recall that the column space of a matrix $A_{m\times n}$ is the span of the columns of A which is the set of all all possible linear combinations of the columns of the matrix.
if $A_1, A_2, \cdots, A_n$ are columns of A, then for any given vector

$$x = (x_1, x_2, \cdots, x_n)$$

the vector

$$A\mathbf{x} = x_1A_1 + x_2A_2 + \cdots + x_nA_n$$

is a linear combination of the columns of A and is in column space of A.

Using this equation you can write any linear combination of the columns of A as $A\mathbf{x}$ for some $\mathbf{x}$.

So the column space is the set of vectors $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ is consistent.

Finding basis for column space of given matrix

- **One way to find a basis for column space of a matrix A is to find a basis for the row space of A^t.**
- **The following method not only gives you a basis for column space of A, it will give you a basis consist of column vectors of A.**

Start Typing in MATLAB

9.1 How to use MATLAB to find a basis for $col(A)$ consisting of column vectors.

a) Enter the following matrix A in MATLAB

$$AA = \begin{bmatrix} 3 & 9 & -7 & -2 & 6 & -3 & -1 \\ 2 & 6 & 0 & 8 & 4 & 12 & 4 \\ 2 & 6 & 5 & 18 & 4 & 33 & 11 \\ 3 & 9 & -2 & 8 & 6 & 18 & 6 \end{bmatrix}$$

2. Find $rref(AA)$ you will get

$$A = \begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Leading ones are in the columns 1, 3 and 6. The corresponding columns in the original matrix AA form a basis for column space of AA.

That is $w1 = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 3 \end{bmatrix}, \quad w2 = \begin{bmatrix} -7 \\ 0 \\ 5 \\ -2 \end{bmatrix}, \quad w3 = \begin{bmatrix} -3 \\ 12 \\ 33 \\ 18 \end{bmatrix}$

So to find a basis for the column space of A type rref(AA) , then find those columns of AA that correspond to the leading ones of rref(AA) .

End of reading Materials

Start Typing in MATLAB

9.2 Exercise 4

Let $AB = \begin{bmatrix} 3 & 9 & -7 & -2 & 6 & -3 & -1 \\ 2 & 6 & 0 & 8 & 4 & 12 & 4 \\ 2 & 6 & 5 & 18 & 4 & 33 & 11 \\ 3 & 9 & -2 & 8 & 6 & 18 & 6 \end{bmatrix}$

a.)

(Enter AB in MATLAB.)

Type:

ABRF= rref(AB)

Decide what is the rank of AB **and what is nullity of** AB**, then enter them in the following format:**

RANKAB= your answer for rank of AB

NULLAB= your answer for nullity of AB

Recall that to show that a vector b **in column space of a matrix** AB **you need to show that the linear system** $(AB)X = b$ **is consistent. you may do this in one of the following way:**

- **Show that** $rank(AB) = rank([AB \;\; b])$**.**
- **Finding** $rref(AB)$

b.) **Which of the following vectors is in the column space of AB? (you need to enter these vectors as column matrices in MATLAB.)**

1. **W1=(6, 4, 4, 6)**
2. **W2=(6, 16, 4, 0)**
3. **W3=(-4, 16, 40, 20)**

Type:

% W1 = true if W1 is in column space of AB.
% W1 = false if W1 is not in column space of AB.
% W2 = true if W2 is in column space of AB.
% W2= false if W2 is not in column space of AB.
% W3 = true if W3 is in column space of AB.
% W3 = false if W3 is not in column space of AB.

10 How to Find Basis for ROW SPACE of AB Using Column Space of $(AB)^t = (AB)'$

Let AB be defined as before. $AB = \begin{bmatrix} 3 & 9 & -7 & -2 & 6 & -3 & -1 \\ 2 & 6 & 0 & 8 & 4 & 12 & 4 \\ 2 & 6 & 5 & 18 & 4 & 33 & 11 \\ 3 & 9 & -2 & 8 & 6 & 18 & 6 \end{bmatrix}$

Note that row space of AB = column space of (AB)'.

Use MATLAB to find a basis for the row space of AB consist of row vectors of AB**.**
Then enter your basis vectors as:
ABV1 = Type your first row vector of basis of row space of AB
ABV2 = Type your second row vector of basis of row space of AB
ABV3 = Type your third row vector of basis of row space of AB

11 How to Find independent Columns of Matrix AB

Let AB be defined as before. $AB = \begin{bmatrix} 3 & 9 & -7 & -2 & 6 & -3 & -1 \\ 2 & 6 & 0 & 8 & 4 & 12 & 4 \\ 2 & 6 & 5 & 18 & 4 & 33 & 11 \\ 3 & 9 & -2 & 8 & 6 & 18 & 6 \end{bmatrix}$

Type :

ABR = rref(AB)
To see the Reduced Row-Echelon Form of AB**.**
Use ABR to find a basis for Column space of AB

Then enter your basis vectors as:

% ABW1 = Type your first column vector of basis of column space AB
% ABW2 = Type your second column vector of basis of column space AB
% ABW3 = Type your third column vector of basis of column space AB

Now type

[R1, pivcol] = rref(AB)
This command will provide you rref(AB) and pivot columns of AB**. The columns of** AB **that are independent and form a basis for the column space.**
You can use MATLAB to give you a matrix composed of the independent columns of AB
Type:
$UAB = AB(:, pivcol)$

12 How to Find independent Columns of Matrix AB

You may use this to find a basis for the row space:
Type:
[R2, pivcol] = rref(AB')
This command will provide you rref(AB') and pivot columns of AB' which are row vectors of AB. The columns of AB that are independent and form a basis for the column space.

Then enter your basis vectors as:

% ABU1 = Type your first Row vector of basis of row space of AB
% ABU2 = Type your second Row vector of basis of row space AB
% ABU3 = Type your third Row vector of basis of row space AB

Note that these are Rows of AB that form a basis for row space of AB.

You Can use MATLAB to give you a matrix composed of the independent rows of AB
Type:
$C = AB'$
$UAB1 = C(:, pivcol)$

13 Using m-file to find a basis for null-space of AB

Use Pico to create a new file called nulbasis.m in your home directory:
to do this , Type:
pico nulbasis.m
When file is opened enter the following code in the file(copy and paste) .

========================= Beginning of the code ==================

```
function N = nulbasis(A)


% nulbasis Basis for nullspace.
% % N = nulbasis(A) returns a basis for the nullspace of A
% in the columns of N. The basis contains the n-r special
% solutions to Ax=0. free col is the list of free columns.
%
% Example:
%
% >> A = [1 2 0 3;
%         [0 0 1 4];
%
% >> N = nulbasis(A)
%
% N = [-2 -3]
%     [ 1 0]
%     [ 0 -4]
%     [ 0 1]
%
% See also fourbase.

[R, pivcol] = rref(A, sqrt(eps));
[m, n] = size(A);
r = length(pivcol);
freecol = 1:n;
freecol(pivcol) = [];
N = zeros(n, n-r);
N(freecol, : ) = eye(n-r);
N(pivcol, : ) = -R(1:r, freecol);
```

========================= End of the code ==================

Then Save the file and exit pico. In your MATLAB enter matrix AB, then type

nulbasis(AB)

The result will be a 7 by 4 matrix, columns of this matrix form an basis for the nullspace of AB.

Then enter your basis vectors as:

% ABN1 = Type your first column vector of basis of nullspace of AB
% ABN2 = Type your second column vector of basis of nullspace of AB
% ABN3 = Type your third column vector of basis of nullspace of AB
% ABN4 = Type your forth column vector of basis of nullspace of AB

Now type these vectors without % as

ABN1 = Type your first column vector of basis of nullspace of AB
ABN2 = Type your second column vector of basis of nullspace of AB
ABN3 = Type your third column vector of basis of nullspace of AB
ABN4 = Type your forth column vector of basis of nullspace of AB

type

NN= [ABN1 ABN2 ABN3 ABN4]

to get NN whose columns forming a Basis for nullspace of AB. So columns of NN are in nullspace of AB. Type

OO= AB*NN

to confirm that. Explain what you see and how did you confirm it.

Use, save, diary off, exit to end your lab, then use pico to edit it before submitting.