Math 21D (De Loera)
Mid-term exam 1
February 4th 2008

Name:
Student ID#
Section number

DO NOT TURN THIS PAGE UNTIL INSTRUCTED TO DO SO
Fill in the information above (your name, etc), DO IT NOW!

Show your work on every problem. Correct answers with no support work will not receive full credit. Be organized and use the notation appropiately. No calculators are allowed, nor is any assistance from classmates, notes, or books. You should only have a writing and an erasing implement on your desk. No cell phones please

Please write legibly!!

#	Student's Score	Maximum possible Score
1		7
2		7
3		5
4		9
5		7
Total points		35

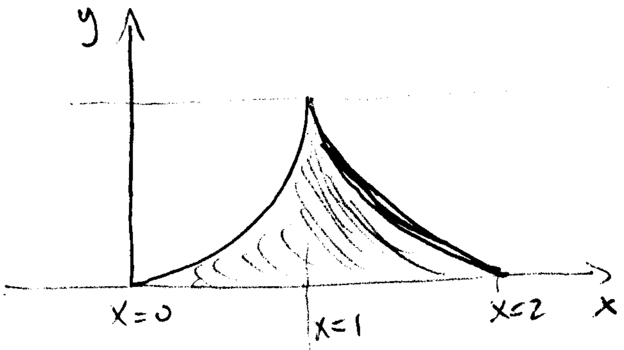
1. Sketch the region of integration of the following integral and write an equivalent integral in the reversed order of integration. Then evaluate the new integral

$$\int_0^1 \int_{\sqrt{y}}^{2-\sqrt{y}} xy\,dx\,dy.$$

y ranges from 0 to 1

x ranges from $x = \sqrt{y}$ to $x = 2 - \sqrt{y}$

thus bounding curves $x^2 = y$, $(x-2)^2 = y$



thus we need a sum of two integrals

$$\int_0^1 \int_0^{x^2} xy\,dy\,dx +$$

$$\int_1^2 \int_0^{(x-2)^2} xy\,dy\,dx$$

2. A perfect sphere of radius 2 is centered at (0,0,0). We remove from the sphere a vertical cylinder of radius 1 with equation $x^2 + z^2 = 1$. What is the volume of the remaining sphere?

Note that removing the cylinder $x^2+z^2=1$ gives same answer as removing cylinder $x^2+y^2=1$, but then we can directly apply polar coordinates!

We use double integral to find the volume of the top half of the cored sphere, then double the result

Surface $\Rightarrow \sqrt{4-r^2} = z$

$$\text{volume} = 2 \int_0^{2\pi} \int_1^2 \left(\sqrt{4-r^2}\right) r\,dr\,d\theta$$

$$= 2 \int_0^{2\pi} \left[-\frac{(4-r^2)^{3/2}}{3} \right]_1^2 d\theta = 2 \int_0^{2\pi} \sqrt{3}\,d\theta$$

$$= 4\sqrt{3}\,\pi$$

3. Evaluate the integral $\int_0^\pi \int_0^\pi \int_0^\pi cos(x+y+z)dxdydz$

$$= \int_0^{\pi} \int_0^{\pi} [\sin(z+y+\pi) - \sin(z+y)]\, dy\, dz$$

$$= \int_0^{\pi} [-\cos(z+y+\pi)]\Big|_0^{\pi} dz - \int_0^{\pi} [-\cos(z+y)]\Big|_0^{\pi} dz$$

$$= \int_0^{\pi} \left(-\cos(z+2\pi) + \cos(z+\pi) - \cos(z) + \cos(z+\pi)\right) dz$$

$$= 0$$

4. (a) Write a triple integral in Cartesian coordinates for the function $f(x, y, z) = 6 + 4y$ over the region of the first octant bounded by the cone $z = \sqrt{x^2 + y^2}$ and the cylinder $x^2 + y^2 = 1$ and the coordinate planes. DO NOT EVALUATE

(b) Write triple integrals in cylindrical AND spherical coordinates for the same function and region as in part (a). DO NOT EVALUATE.

(c) Evaluate just the cylindrical version of the integrals.

a) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} (6+4y)\, dz\, dy\, dx$

b) cylindrical $\int_0^{\frac{\pi}{2}} \int_0^1 \int_0^r (6 + 4r\sin\theta)\, dz\, r\, dr\, d\theta$

spherical $\int_0^{\frac{\pi}{2}} \int_{\pi/4}^{\pi/2} \int_0^{\csc\phi} (6 + 4\rho\sin\phi\sin\theta)(\rho^2 \sin\phi)\, d\rho\, d\phi\, d\theta$

c) $\int_0^{\frac{\pi}{2}} \int_0^1 \int_0^r (6 + 4r\sin\theta)\, dz\, r\, dr\, d\theta = \int_0^{\frac{\pi}{2}} \int_0^1 (6r^2 + 4r^3\sin\theta)\, dr\, d\theta$

$= \int_0^{\pi/2} [2r^3 + r^4 \sin\theta]\Big|_0^1 d\theta = \int_0^{\frac{\pi}{2}} (2+\sin\theta)\, d\theta = [2\theta - \cos\theta]\Big|_0^{\frac{\pi}{2}}$

$= \pi + 1$

5. Of the 3-dimensional body of problem four, assume its density is 1. Write triple integrals in Cartesian coordinates that describe 1) The mass of the body, 2) the center of mass, and 3) the moment of inertia about the y-axis. DO NOT EVALUATE ANY OF THE INTEGRALS.

(1) $$M = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} dz\,dy\,dx$$

(2) The coordinates of the center of mass are

$$\bar{x} = \frac{1}{M} \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} x\,dz\,dy\,dx$$

$$\bar{y} = \frac{1}{M} \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} y\,dz\,dy\,dx$$

$$\bar{z} = \frac{1}{M} \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} z\,dz\,dy\,dx$$

where M is the mass given in (1)

(3) $$I_y = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} x^2 + z^2\,dz\,dy\,dx$$