

Math 21D  
practice midterm exam  
February 2008

Name: \_\_\_\_\_  
Student ID# \_\_\_\_\_  
Section number \_\_\_\_\_

**DO NOT TURN THIS PAGE UNTIL INSTRUCTED TO DO SO**

**FILL IN ABOVE INFORMATION (your name, etc) NOW!!**

Show your work on every problem. Correct answers with no support work will not receive full credit. Be organized and use the notation appropriately. No calculators are allowed, nor is any assistance from classmates, notes, or books. You should only have a writing and an erasing implement on your desk. No cell phones please.

*Please write legibly!!*

#	Student's Score	Maximum possible Score
1		7
2		7
3		7
4		7
5		5
Total points		35

1. Sketch the region of integration of the following integral and write an equivalent integral in the reversed order of integration. Then evaluate the new integral

$$\int_0^1 \int_{x^2}^x \sqrt{x} dy dx.$$

2. Integrate the function  $f(x, y) = 1/(1+x^2+y^2)^2$  over the region enclosed by the triangle with vertices  $(0, 0), (1, 0), (1, \sqrt{3})$  Hint: Use a trigonometric substitution to solve the final integral  $x = \tan(u)$ .
3. Find the volume of the region between the saddle  $z = xy$  and the cone  $z^2 = x^2 + y^2$  and above the portion of the disk whose boundary is  $x^2 + y^2 = 1$  that lies in the first quadrant.
4. Convert to cylindrical coordinates and evaluate:

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{(x^2+y^2)} 21xy^2 dz dy dx$$

5. The surface of an apple can be described in spherical coordinates by the equation  $\rho = 1 - \cos(\phi)$  (a revolution figure of half a cardioid). Apples have constant density 1, find the mass of the apple.