

**THEORY OF NUMBERS, Math 115 A**  
**Homework 6 Due Wednesday November 15 2002**

1. (6.1 6) What is the remainder when  $7 \cdot 8 \cdot 9 \cdot 15 \cdot 16 \cdot 17 \cdot 23 \cdot 24 \cdot 25 \cdot 43$  is divided by 11?
2. (6.1 12) Using Fermat's little theorem, find the least positive residue of  $2^{1000000}$  modulo 17.
3. (6.1 34) Show that if  $p$  is a prime and  $0 < k < p$ , then  $(p - k)!(k - 1)! \equiv (-1)^k$  modulo  $p$ .
4. (6.1 40,41) Using the fact that  $p$  divides the binomial coefficient  $\binom{p}{k}$  when  $k$  is less than  $p$  show that if  $a, b$  are integers then  $(a + b)^p \equiv a^p + b^p$ . Use this to prove Fermat's little theorem.
5. (6.2 2) Show that 45 is a pseudoprime to the bases 17 and 19.
6. (6.2 20) Show that if  $n$  is a Carmichael number then  $n$  is square free.
7. (6.2 1 computer exploration) Determine for which positive integers  $n$ ,  $n \leq 100$ , the integer  $n2^n - 1$  is prime.
8. (6.3 6) Find the last digit of the decimal expansion of  $7^{999,999}$ .
9. (6.3 10) Show that  $a^{\phi(b)} + b^{\phi(a)} \equiv 1 \pmod{ab}$  if  $a, b$  are relatively prime positive integers.
10. (6.3 2 computational exploration) Find  $\phi(n)$  for all integers less than 1000. What conjecture can you make about the values of  $\phi(n)$ .