DRP Spring Seminar: Numerical Methods — What they didn't tell you
Spring — 2025

Instructor Information

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Class Information

Dates: TBD
Time: TBD
Classroom: TBD

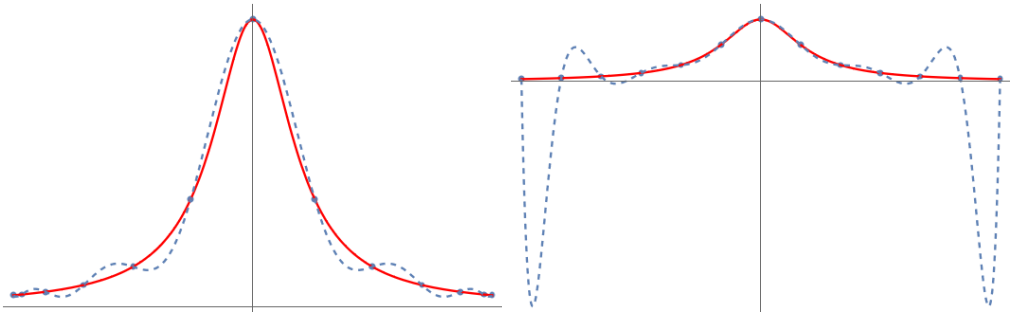

Figure 1: Approximation with this class (left) and without this class (right)

Course Description

Numerical methods are the algorithms that we use to solve math problems using computers. If you want to take a derivative, compute an integral, or solve a system of equations then there is a numerical method out there for you! Unfortunately, many numerics classes are prevented from getting to "the good stuff" due to a broad focus.

In this seminar, we will get straight to the heart of the subject by focusing on one and only one question: how do we approximate functions on the set $[-1, 1]$? Through this, a whole world of practical algorithms for solving real-world problems will become available to us. Meetings will include a brief discussion about a new topic followed by students working in small groups to solve related problems. Students are encouraged to read the textbook listed below, but will not be required to. It's truly an excellent book. A final presentation or project will be required.

Prerequisites:

1. Knowledge of at least one programming language including one vector library (NumPy, MatLab, Mathematica, Octave, or similar)
2. Strong knowledge of linear algebra including eigenvectors, eigenvalues, and abstract vector spaces

3. Strong knowledge of calculus particularly sequences and series. Firm understanding of Taylor remainder theorem. Differential equations will be mentioned, but only very basic knowledge is needed
4. Strong familiarity with complex numbers

Sources

Approximation Theory and Approximation Practice by Nick Trefethen — Trefethen distributes a PDF of all the chapters we will need!

Course Objectives

After this course, you should be able to...

- Approximate continuous functions on $[-1, 1]$ and, as a result of this...
- Compute numerical derivatives of functions,
- Compute numerical integrals of functions,
- Compute numerical roots of functions,
- Compute numerical extrema of functions.

Tentative Schedule

The following is a *tentative* schedule for the course.

1. Linear Algebra review — Inner Products, Orthonormal Bases, and Projection,
2. Approximation Via Projection — Fourier and Chebyshev Series,
3. The Fast Fourier Transform — The Most Important Algorithm of the 20th Century,
4. Applications of the FFT — Fast Interpolation and Multipoint Evaluation,
5. Approximation Via Interpolation — Chebyshev Interpolation,
6. Interpolation Versus Projection — How much do we lose?,
7. Numerical Calculus — Integrals and Derivatives,
8. Numerical Root Finding — Return of the Characteristic Polynomial,
9. Spectral Methods — Turning Differential Equations into Linear Systems,
10. Presentations.