The Surprising Math of Voting

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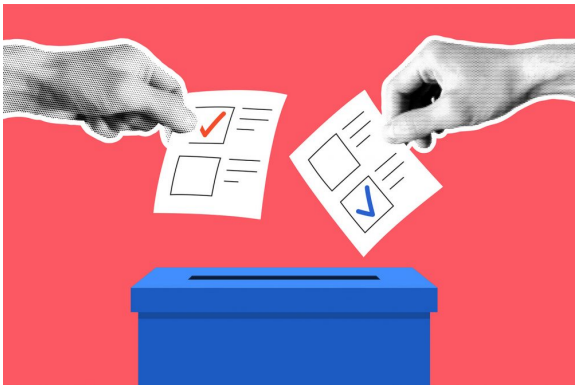
1 Seminar Description

Imagine you and your friends are planning a pizza night, and you are trying to find the best night that works for everyone. Each person has their own complicated preferences: there are days you love, days you *can* make it but would prefer other days, and days you simply cannot go. So how do you choose a date that fairly takes into account everyone's preferences?

The answer, surprisingly, is that you cannot. *Social choice theory* is the study of these types of situations and the best (and worst) ways to go about them. In this seminar, we will take a beginner-friendly deep dive into *voting theory*, which analyzes what voting systems most accurately represent the electorate. We will look at many different systems, including approval voting (similar to a when2meet), ranked choice voting, and Condorcet voting (where you see which choice is preferred most in head-to-head contests). We will see why each system has its flaws, and look at



various situations where there is no clear way to decide a winner. Most importantly, *you* will be able to present on a specific voting theory concept, and become the expert in that topic.

2 Logistics and Schedule

We have designed this seminar to be accessible to all students interested in math and voting. In particular, we assume no previous knowledge and will approach things in an accessible way to all.

Each week we will meet for approximately 1 hour. At the beginning, we will spend a few weeks giving the background of *social choice theory*, with *voting theory* being our specific focus. In the following weeks, students will pick a specific voting system or property for a 30 minute presentation in a later week.

During each meeting from weeks 3-9, a student group will present the week's topic. They will be provided with some resources to start from along with problems that will be the focus of the end of the weekly seminar meeting. Their responsibility will be to prepare a presentation about 30 minutes long that prepares the seminar to work on problems (chosen by instructors and shared with presenters ahead of time) in the last 20 minutes of seminar.

Over the course of the meetings, we will build up our knowledge of voting systems so that, on the final day, we can have a conversation/debate about what voting system everyone prefers, and why. Everyone will have all the language and tools at this point to articulate the strengths and weaknesses of their preferred system.

Weekly Schedule Summary

Week 2:	DRP leaders will give background for half of the class, and then students will work on problems related to the information taught.
Weeks 3-9:	Students take turns presenting a voting system or property, and in the second half of class the presenter will help students with practice problems related to the topic.
Week 10:	Whole class discussion on what is the best voting system. We will then look at some interesting results regarding voting as a whole.

3 Textbook and Planned Topics

Although we do not plan to assign reading (except what is necessary for presentations), we will use *The Mathematics of Politics* by Robinson and Ullman as a reference throughout the course. Interested students are encouraged to use the textbook to gain a deeper perspective of the topics of the class.

Likely topics (we will not get to all of these, nor is this list exhaustive):

- Background
  - Profile spaces
  - Social choice functions
  - Properties of social choice functions

- Voting Systems
  - First-past-the-post (plurality)
  - Ranked choice voting
  - Approval voting
  - Condorcet Method
  - Score voting
  - Antiplurality method
- Important results
  - Arrow's Impossibility Theorem
  - Median voter theorem
  - Gibbard's theorem