

Surreal Numbers
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Seminar Description: Happiness, eternal friendship, and wealth beyond measure can all be yours even if you take this seminar. Surreal numbers are an extension of the real numbers that, among other weird properties, include infinity in a principled way. We will start from quite literally nothing, then use a couple simple rules to build up a number system that includes infinities, infinitesimals, and everything in between. Hopefully we'll also figure out how to do math with these numbers. This is the kind of seminar you should attend if it annoys you that infinity plus one is not a bigger number than infinity.

Prerequisites: MAT 108 or similar familiarity with proof writing

Required Readings: None, though additional resources will be provided at the end of the seminar.

Homework: Problems will be generated based on questions that arise during our weekly meetings, and will inform the topics we discuss in the next meeting. Solutions do not need to be formally written up, but students should at least attempt the weekly problems and be prepared to share what ideas they came up with.

Before starting, students should review properties of the empty set, in particular recognizing when statements are vacuously true.

Outline:

- Meeting 1:
 - Introductions
 - Definition of a number and less than or equal to
- Meeting 2:
 - Properties of less than or equal to, inductive proofs
- Meeting 3:
 - Students present last week's homework
 - Surreal addition and subtraction
- Meeting 4:
 - Students present last week's homework
 - Infinite, infinitesimal, and other non-dyadic surreal numbers
- Meeting 5:
 - Students present last week's homework
 - ?????
- Meeting 6:
 - Students present last week's homework
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- Meeting 7:
 - Students present last week's homework
 - ?????
- Meeting 8:
 - Students present last week's homework
 - ?????