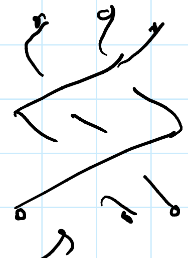
Intro meeting 10/1

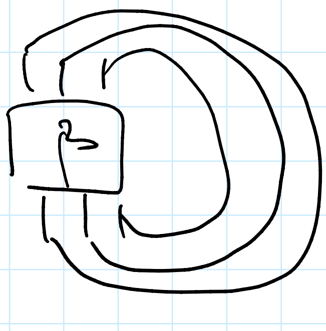
Thursday, October 1, 2020 11:34 AM

Braids



braid on n strands

= braid

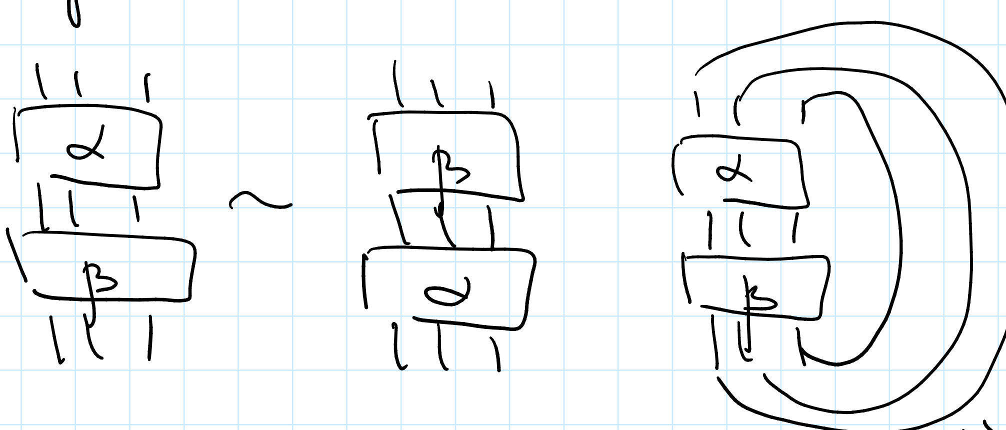


braid closure
=
link diagram

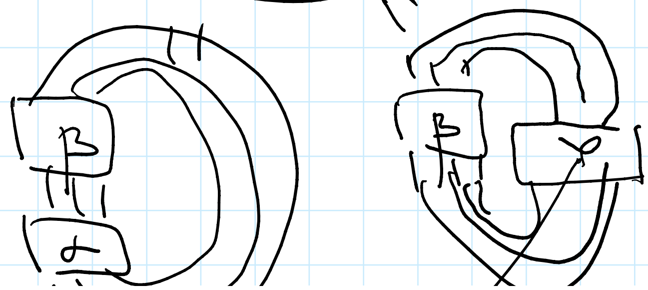
Thm (Alexander)
Any link is a closure of some braid

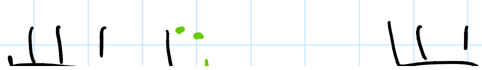
Thm (Markov) Two braids represent the same link, iff they are related by a sequence of moves:

(M1)
(conjugation in braid group)

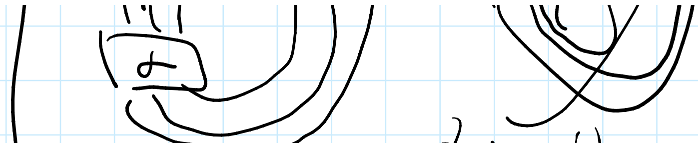


(M2) (stabilization)





M2−

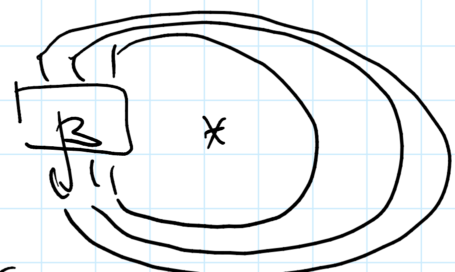


α upside down.

M2+

β braid → usual braid closure (M1) (M2)

→ annular closure (M1)



(annulus = plane minus ∗)

annular braids = conj. classes in braid group

→ transverse link with respect to standard contact structure in S^3

Thm (Orevkov Shevchishin)

(M1, M2+) conj. positive stabilization.

Link invariants ⟷ braid invariants which do not change under these moves.

these moves.

- Usual link invt $\Longleftrightarrow$ braid invt which does not change under M1, M2
- Annular link invt $\Longleftrightarrow$ M1 (just the conj)
- Transverse link invt $\Longleftrightarrow$ M2+, M1

One direction: define contact structure, transverse links .....

Another direction: use Khovanov homology to get interesting invariants of braids, annular links, transverse links...

Annular Khovanov homology [Grigsby, Wehrli, Licata, ...]

Plamenevskaya invariant class in Kh(L) which does not change under M1, M2+ [Plamenevskaya, Ng...]

Kauffman bracket

Link diagram $\rightsquigarrow \mathbb{Q}(q)$

Link diagram $\rightsquigarrow \mathbb{C}(q)$

link diagram

containing pieces

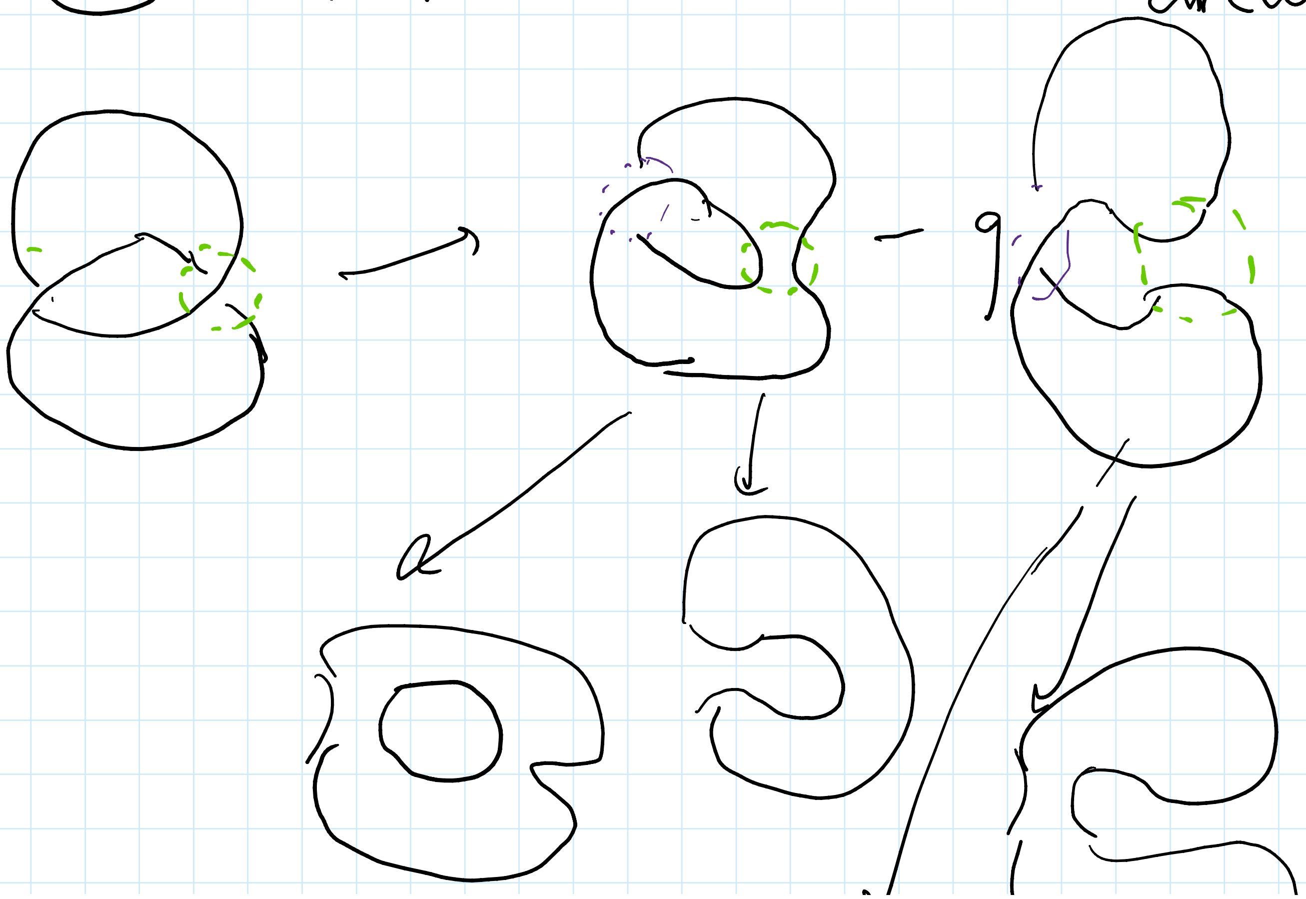
If we do this at every crossing, we have a "lot of resolutions"

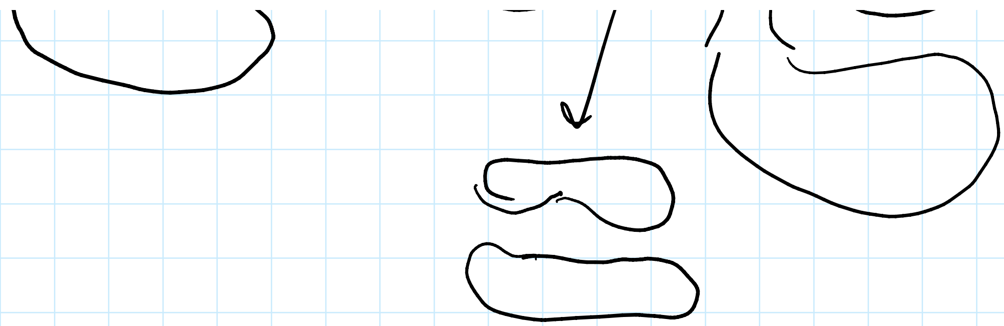
usual link diagram

$\longrightarrow$ bunch of circles

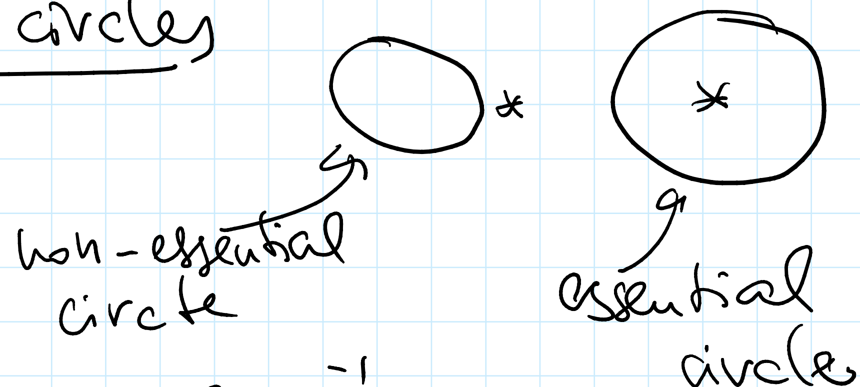
$\bigcirc = -q - q^{-1}$

$\langle L \rangle$ = linear combination of evaluation of these circles.



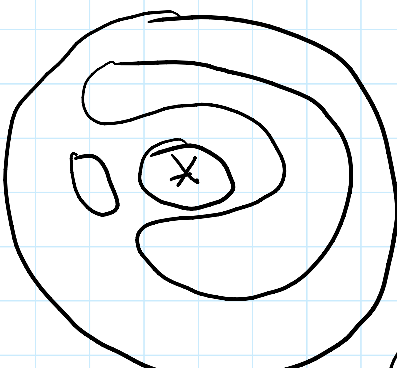


In the annular link diagram, we get <u>two types of circles</u>



non-essential $\longrightarrow -q-q^{-1}$

essential $\longrightarrow V$



$\longrightarrow$ 2 essential
2 non-essential

$\rightsquigarrow V^2 \cdot (-q-q^{-1})^2$

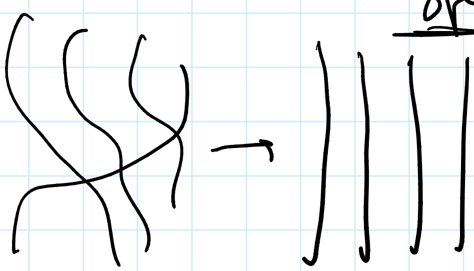
(~ Jones poly).

Kauffman bracket in a circle $\longrightarrow$
$\longrightarrow$ polynomial in V and q

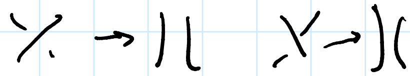
Can get to normal Kauffman bracket by evaluating at $V = -q-q^{-1}$

by evaluating at $V = -q - q^{-1}$.

Plamenevskaya: braid closure has a special resolution



n vertical strands.



$\rightsquigarrow$ special class in Kh.