4-manifolds

Thursday, May 7, 2020 2:00 PM

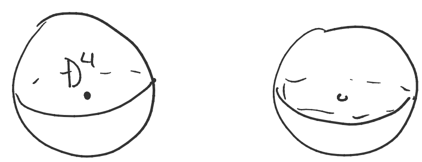
Handles: 4-dimensional K-handle $D^K \times D^{4-K}$

attached along $\underbrace{\partial D^k}_{} \times D^{4-K}$

$S^{K-1} \times D^{4-k}$

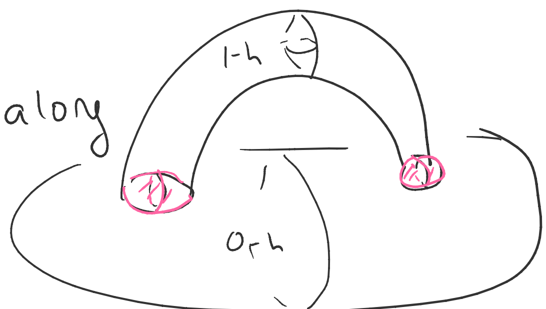
$K = 0, 1, 2, 3, 4$

0-handle : $D^0 \times D^4$



attached along $\partial D^0 = \emptyset$
$ \; S^{-1}$ ''

1-handle : $D^1 \times D^3$ attached along $S^0 \times D^3$

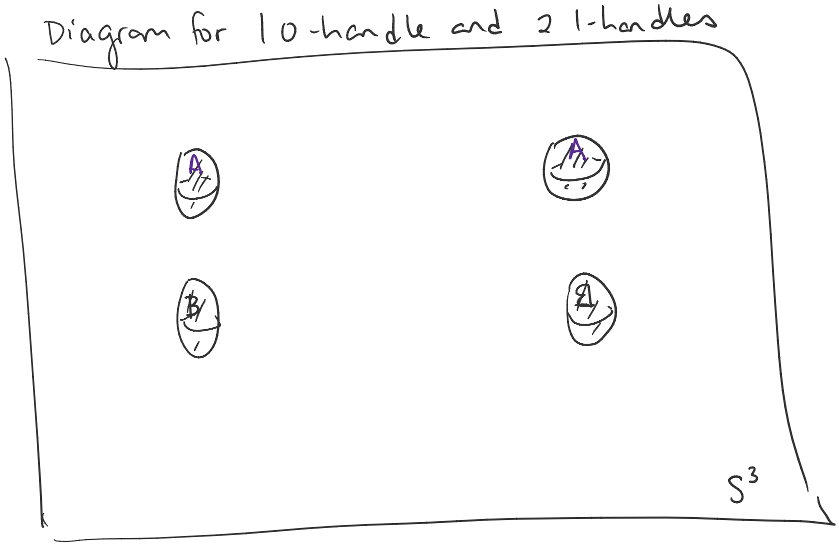


Note boundary of 0-handle is S^3

Usually assume have a single 0-handle

Glue 1-handles to $\partial(\text{0-handle}) = S^3 \leftarrow$ page we draw in

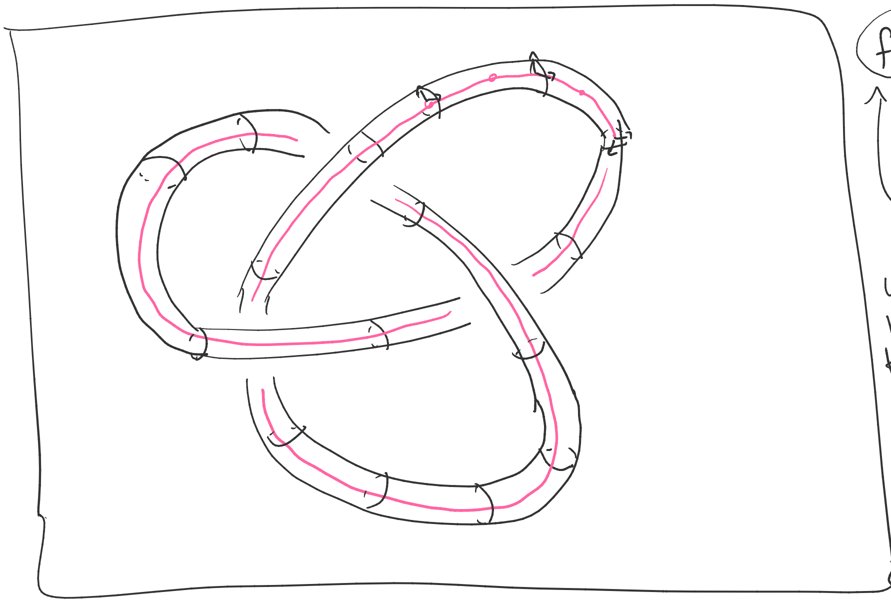
Diagram for 1 0-handle and 2 1-handles



2-handles $D^2 \times D^2$ glued along

$S^1 \times D^2 \leftarrow$ solid torus

Beginner Case no 1-handles attaching 2-handles to a 0-handle



$f: S^1 \times D^2 \to S^3$ smooth embedding

$\uparrow^{x_2}_{\to x_1}$

to encode 4-manifold what do we need to remember to recover f up to isotopy?

$f(S^1 \times \{0\})$ is a knot keep track of this knot up to isotopy

to keep track of normal disk neighborhood embedding, need a framing

For each point in S^1 can compare $\partial_{x_1}, \partial_{x_2}$ on $S^1 \times D^2$

$df(\partial_{x_1}), df(\partial_{x_2})$ on $f(S^1 \times D^2)$

compare with chosen coordinate system for each pt in S^1

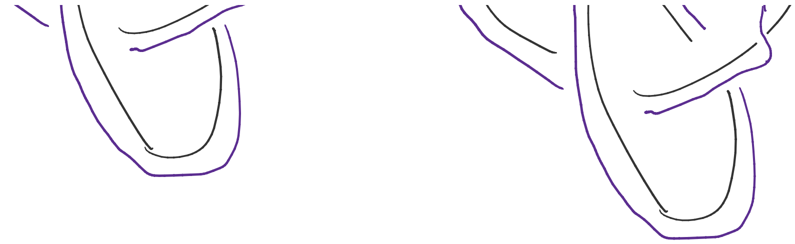
get a linear transformation taking one coord system to other

$$S^1 \longrightarrow GL^+(2,\mathbb{R}) \longrightarrow SO(2,\mathbb{R}) \simeq S^1$$

End conclusion: Normal bundle information of f is encoded by an element of $\pi_1(SO(2,\mathbb{R})) = \pi_1(S^1) \cong \mathbb{Z}$

A framing is an integer.

parallel copy of the knot gives a framing



<u>Convention:</u> 0-framing is Seifert framing

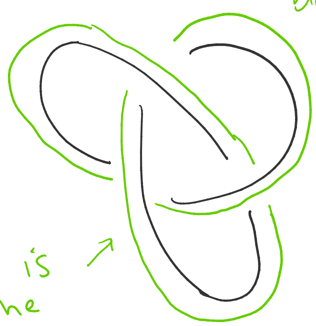
Pick a Seifert surface for knot

Want parallel copy of K lying on Seifert surface

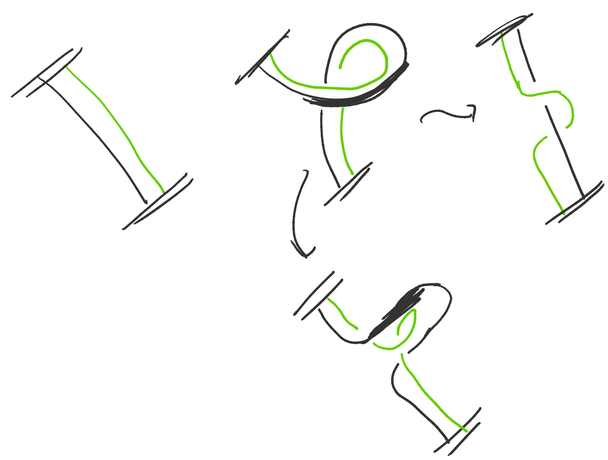
Each crossing contributes what looks like twisting in 0-framing

From a diagram -- blackboard framing

blackboard framing changes based on diagram

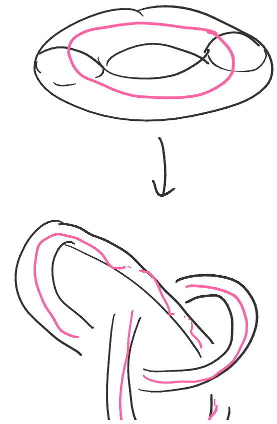


integer is writhe

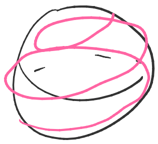


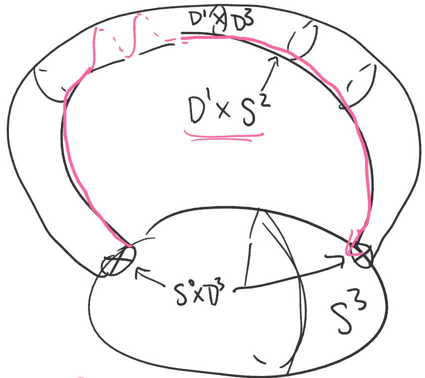
2-handle $D^2 \times D^2$

$S^1 \times D^2$

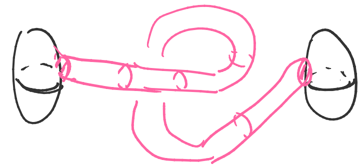


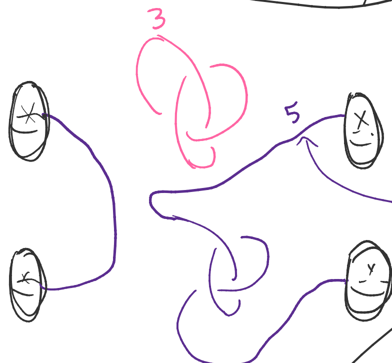
If I have both 0-handles and 1-handles



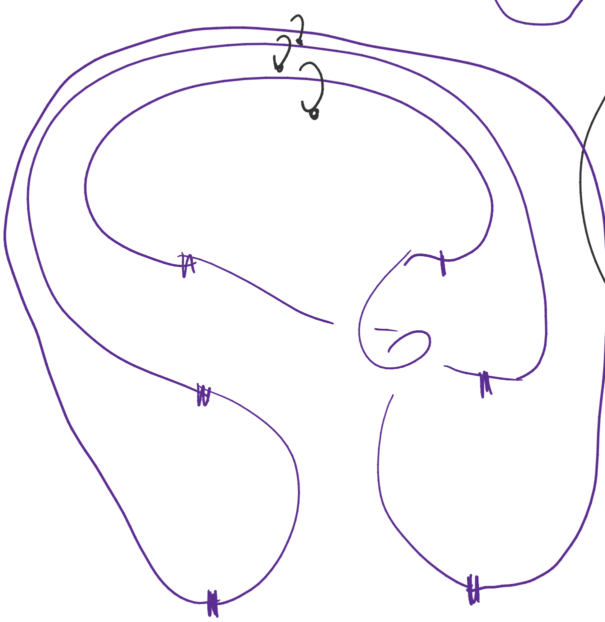


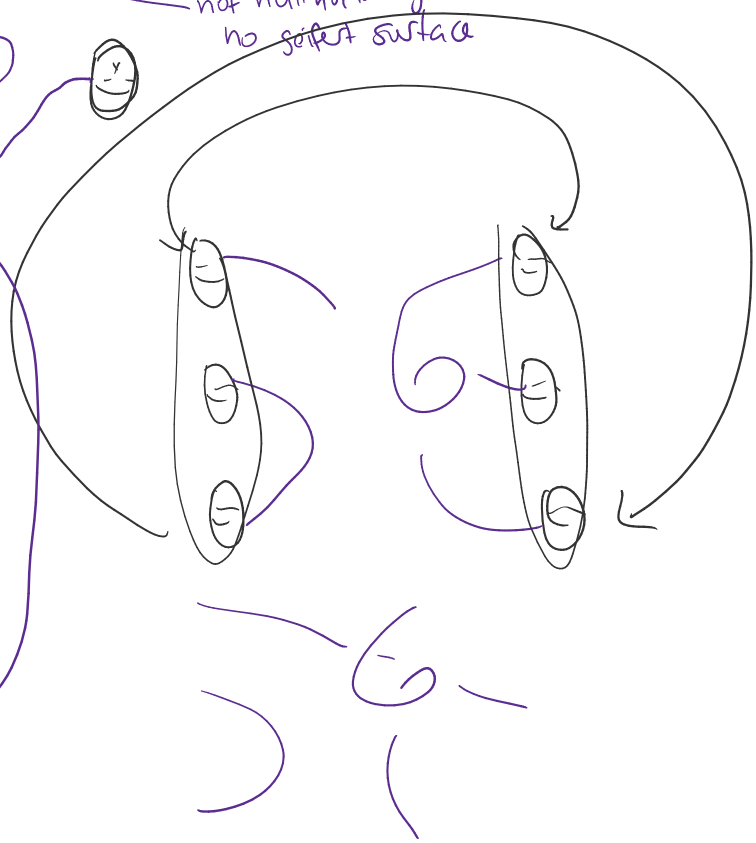
2-handle attached by gluing $S^1\times D^2$ into ∂(0-h $\cup$ 1-h's)



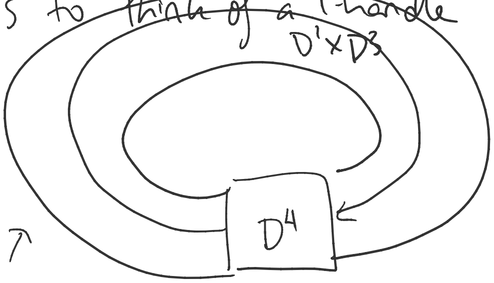


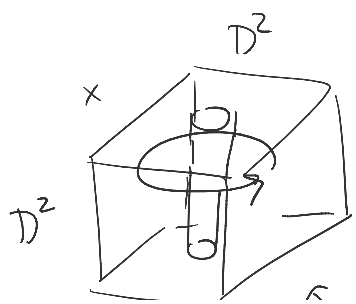
not nullhomologous
no seifert surface

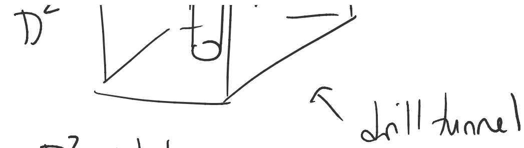




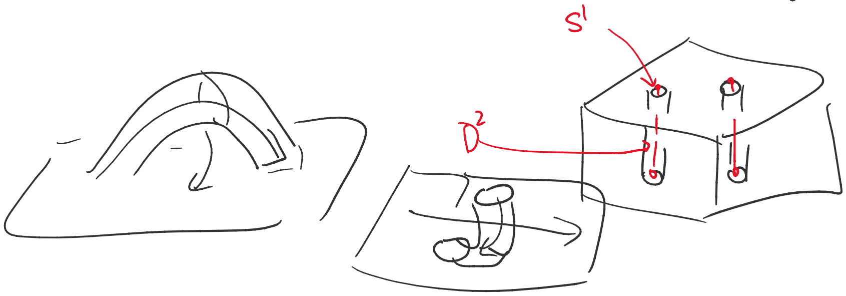
2 ways to ~~think of a 1-handle~~



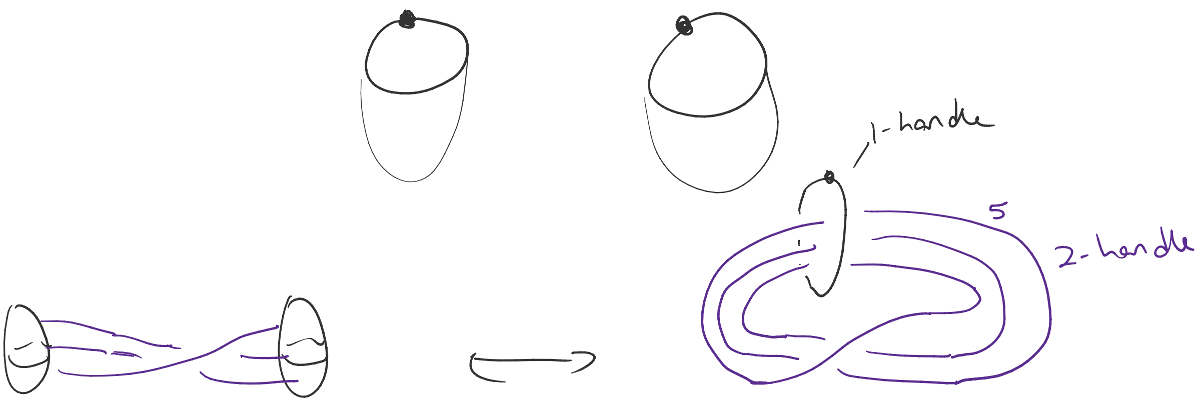




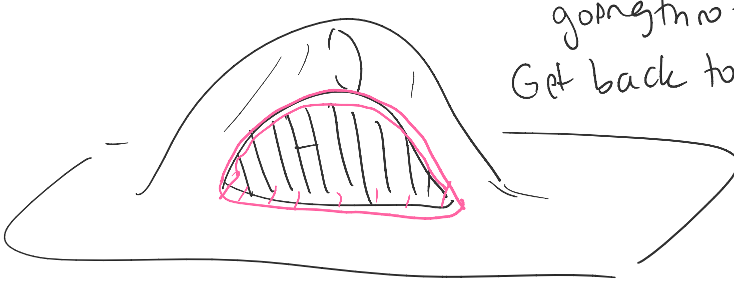
Drill out $D^2 \times$ hole
core $D^2 \times pt$



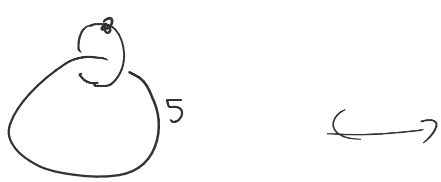
Can keep track of 1-handles by specifying a disk in 0-handle to drill out with unknotted boundary in S^3



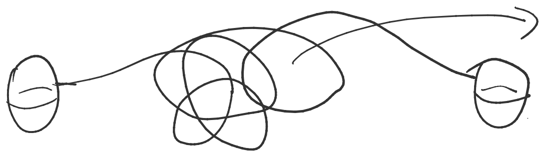
Attach 1-handle then 2-handle going through 1-handle exactly once
Get back to equivalent to attaching nothing



nothing

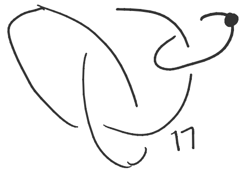


nothing



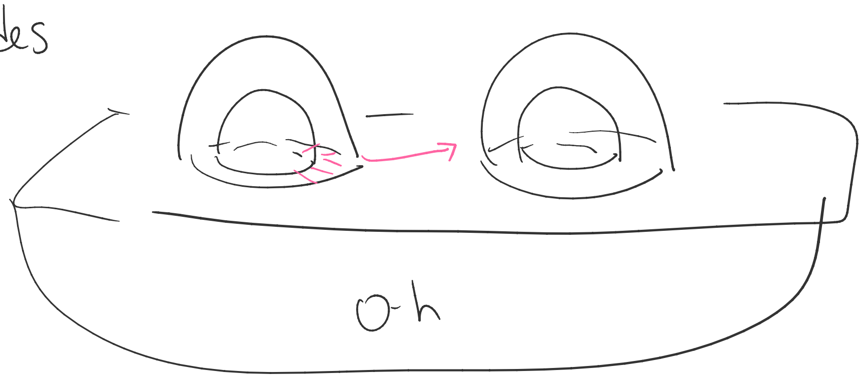
⟷ nothing

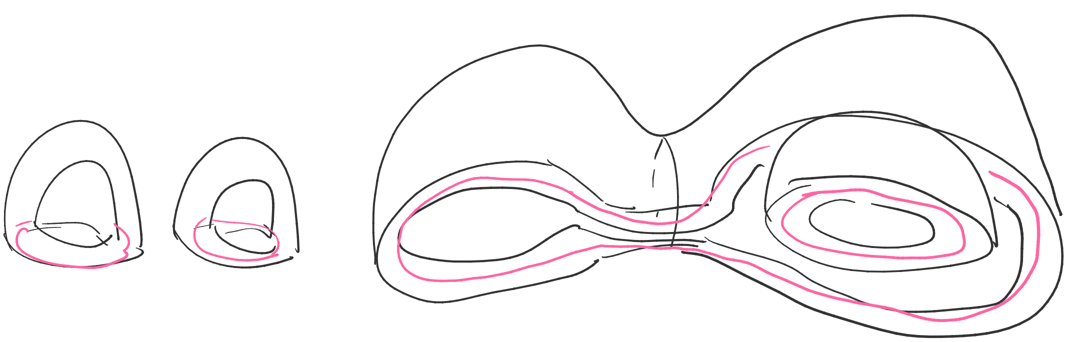
undo knotting
by isotopy



⟷ nothing

Handle slides

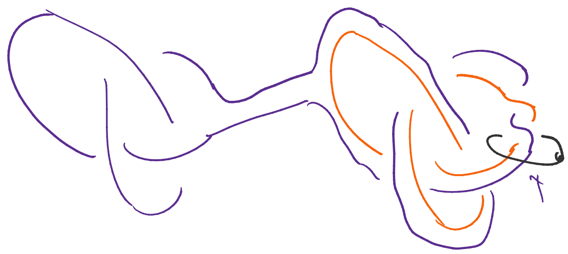


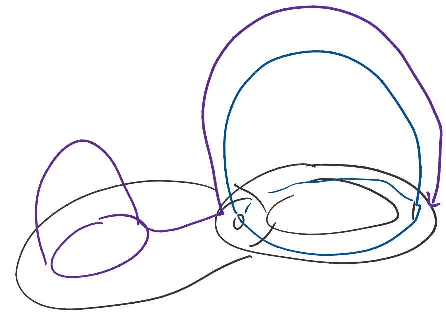


4-mflds:

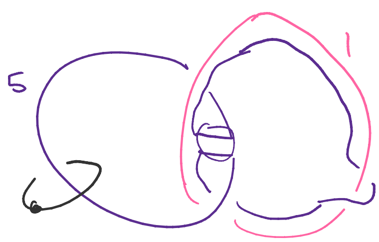
handleslide

preserves diffeo type of 4-mfld





$$\begin{matrix} A \\ B \end{matrix} \longrightarrow \begin{matrix} A \pm B \\ B \end{matrix}$$



framing

$A \cdot B = lk$

$2 \quad A^2 = \text{framing}_A$

$1 \quad B^2 = \text{framing}_B$

$(A \pm B) \cdot B = A \cdot B \pm B \cdot B$

$(A \pm B)^2 = A^2 \pm 2A \cdot B + B^2$

$\qquad 2 \pm 2 \cdot 1 \quad +1$

Attaching a 2-handle this changes the boundary 3-manifold by integral Dehn surgery