From last time:

- Defined Φ_W, the map induced by a cobordism W
- Defined knot concordance and slice genus g_*
- Used Φ_W to show $|s(K)| \leq 2g_*(K) \leq 2g(K)$

Compute q-deg Φ_W

$$q\text{-deg} = \deg_A - |V| - \#\text{circles} - n_+ - 2n_-$$

1-handle maps have q-deg $+1$

0-handle maps have #circles increases,

so q-deg -1

2-handle maps $\Rightarrow$ q-deg $= -1$

$$\begin{aligned} q\text{-deg of } \Phi_W &= -\#\text{ 0-handles} \\ &\quad + \#\text{ 1-handles} \\ &\quad - \#\text{ 2-handles} \\ &= -\chi(W) \end{aligned}$$

Cor: For a positive knot K,

$$s(K) = 2g_*(K) = 2g(K)$$

Cor: (Milnor conj) K a (p,q)-torus knot

$$\Rightarrow g_*(K) = \frac{(p-1)(q-1)}{2}$$

0-handle map:

$$i(s_0) = s_0 \otimes \tfrac{1}{2}(a-b) = s_0 \otimes 1$$

$\Rightarrow$ # circles increases by 1,

$\Rightarrow$ q-deg decreases by 1

2-handle map

$$\varepsilon : A \longrightarrow \mathbb{C}$$

$\Rightarrow$ $\deg_A$ decreases by 2

circles decreases by 1

$\Rightarrow$ q-deg decreases by $2-1=1$