# Meeting 2/18

Thursday, February 18, 2021 3:27 PM

$\{x^5 = y^2\}$ $m = 2$

$y = xv$

$x^5 - x^2v^2 = 0$ $\quad x^2(x^3 - v^2) = 0$

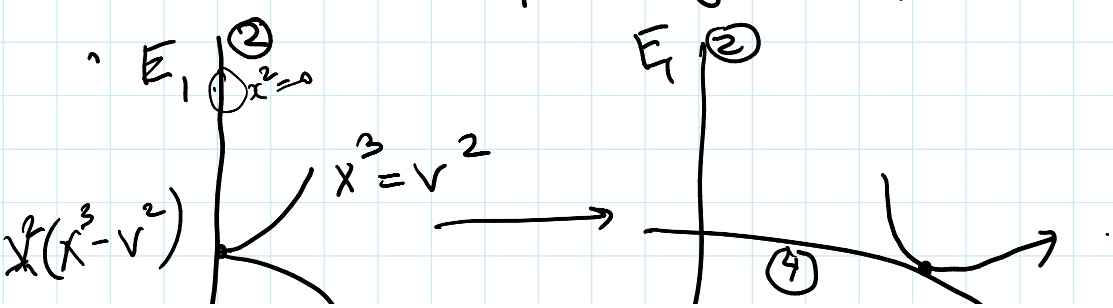
① Multiplicity of the exceptional divisor

$m(f)$

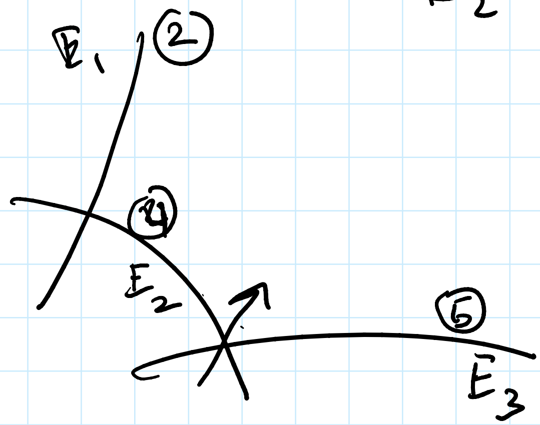
$\{f = 0\}$ multiplicity of $f$ = smallest possible intersection number with a line (= intersection # with generic line through 0)

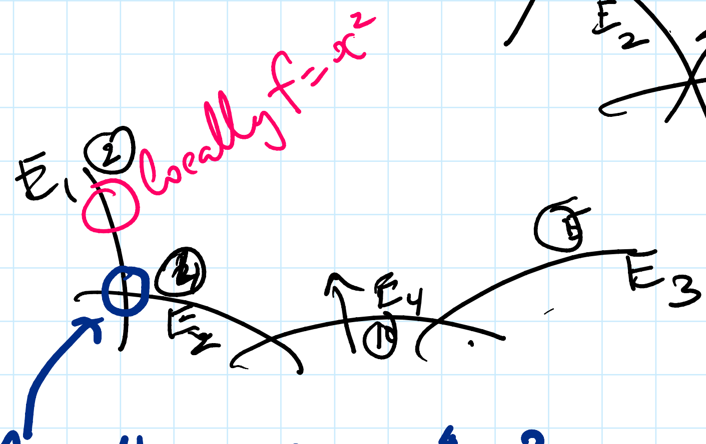
= smallest degree of a monomial in $f$.

<u>Fact</u> After one blow-up, exceptional divisor appears in the strict transform with multiplicity $m(f)$.




↗ arrow = strict transform.




locally $f = x^4 y^2$

In general, we get a tree of exceptional divisors with some multiplicities, and the function

$$X \xrightarrow[\text{resolution map}]{\pi} \mathbb{C}^2 \xrightarrow{f} \mathbb{C}$$

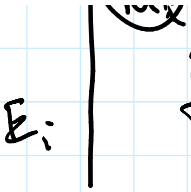
complicated surface

$$\tilde{f} = f \circ \pi$$

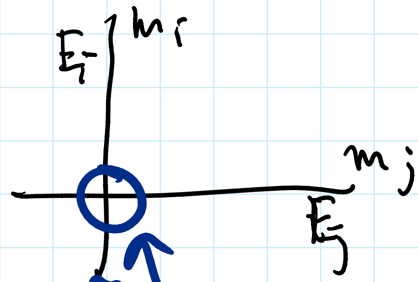
locally, $\tilde{f}$ has the following behaviour:

$(m_i)$

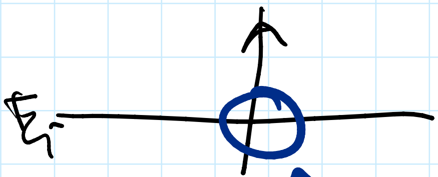
$\tilde{f} \sim x^{m_i}$ $\quad E_i \}^{m_i}$




$\tilde{f} \sim x^{m_i}$




$\tilde{f} = x^{m_i} y^{m_j}$




$\tilde{f} = x^{m_i} y$

If we blow up sufficiently many times to avoid triple intersection.

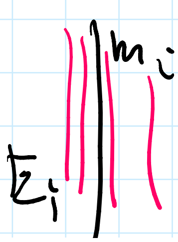
(= normal crossing divisor).

__Application__ We can use the resolution to describe the Milnor fiber:

$V_\varepsilon = \{ f = \varepsilon \} = \{ \tilde{f} = \varepsilon \}$

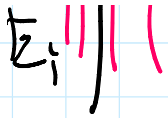
(Since $V_\varepsilon$ does not go through the origin, it is unchanged under the resolution).

$\tilde{f} = \varepsilon$ has the following local models in $X$:

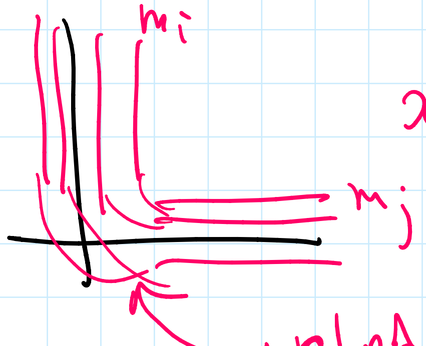



$\tilde{f} = x^{m_i} = \varepsilon$

$x = \sqrt[m_i]{\varepsilon} \Rightarrow m$-parallel



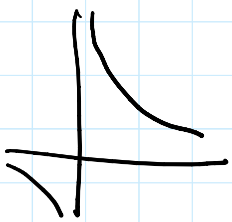

$x = \sqrt[m_i]{\varepsilon} \Rightarrow m_i$-parallel lines near $E_i$, or, equivalently $m_i$-fold cover of $E_i$




$x^{m_i} y^{m_j} = \varepsilon$

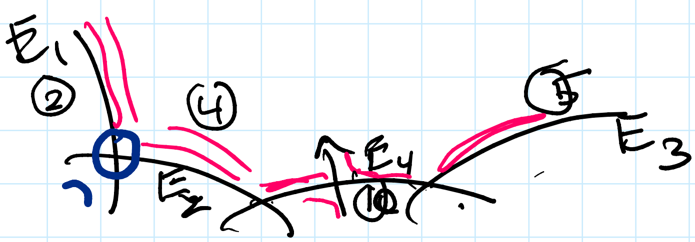
glued together near intersection points by a bunch of cilinders.

$xy = \varepsilon$



Milnor fiber gets decomposed into covers over $\mathring{E}_i$

$\mathring{E}_i$ — $E_i$ − intersection points.




**Thm** (A'Campo) $\boxed{\chi(V_\varepsilon) = \sum_i m_i \chi(\mathring{E}_i)}$

**Proof** We have $m_i$-fold

that we have $m_i$-fold

cover over $\mathring{E}_i$ and a bunch of annuli

over the intersection points which do

not contribute to $\chi$. ∎

<u>Ex:</u> $\mathring{E}_1 = \mathbb{P}^1 \setminus pt \qquad \chi(\mathring{E}_1) = 2-1 = 1$

$\mathring{E}_2 = \mathbb{P}^1 \setminus 2pt \qquad \chi(\mathring{E}_2) = 2-2 = 0$

$\mathring{E}_3 = \mathbb{P}^1 \setminus pt \qquad \chi(\mathring{E}_3) = 1$

$\mathring{E}_4 = \mathbb{P}^1 \setminus pt \qquad \chi(\mathring{E}_4) = 2-3 = -1$

By A'Campo's formula we get

$\chi(V_\varepsilon) = m_1 + m_3 - m_4 =$

$= 2 + 5 - 10 = -3$

On the other hand, $\mu = (2-1)(5-1)$

$= 4$

$\chi = 1 - \mu = -3$