Topology of Singularities
Problem set 2

1. Resolve the following plane curve singularities by sequences of blow-ups:
   (a) $x^5 = y^2$
   (b) $x^3 = y^5$
   (c) $x^4 = y^5$
   (d) $y(y - x^n) = 0$
   (e) $x^4 = y^6$
   (f) $(x^2 - y^3)(x^3 - y^2) = 0$
   (g) $y^p - x^{p+1} = 0$

   In each of these cases, first make a sequence of blow-ups until the strict transform is smooth. Next, make additional blow-ups until the total transform becomes a *normal crossings divisor*, that is, all components are smooth and transverse, and there are no triple intersections.

2. For all singularities from Problem 1, describe the corresponding links. For links with several components, compute the linking numbers between components.

3. Consider the singularity defined by parametrization

$$\{x(t) = t^4, y(t) = t^6 + t^7\}$$

(a) Find the equation $\{f(x, y) = 0\}$ for this singularity.
Hint: Compute $x^3 - y^2$.
(b) Find the semigroup of this singularity
(c) Compute the Alexander polynomial of its link.

4. For all singularities from Problem 1:
(a) Describe the intersection form on the resolution
(b) Compute the multiplicities of all components of the exceptional divisor
(c) Use A'Campo formula to compute the Milnor number
(d) Compare the results of (c) with the genera of links from Problem 2.