MAT 150A, Fall 2023
Practice problems for Midterm 1

1. Decompose the product of permutations into non-intersecting cycles:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 1 & 3 & 2 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 3 & 4 & 6 & 1 \end{pmatrix}$$

2. Consider the permutation:

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 5 & 2 & 8 & 3 & 4 & 1 & 10 & 6 & 9 \end{pmatrix}$$

(a) Decompose f into non-intersecting cycles;
(b) Compute the sign of f;
(c) Compute the order of f;
(d) Compute the inverse permutation f^{-1}.
3. Find permutations in S_7 of orders 10, 11, 12 or prove that there are none.
4. Is the set of even integers a subgroup of $(\mathbb{Z}, +)$? The set of odd integers?
5. Find the orders of all elements in $\mathbb{Z}_6$.
6. Solve the equation $8x = 1 \mod 11$.
7. Find the orders of all elements in $\mathbb{Z}_{11}^*$ and prove that this group is cyclic.
8. Find all finite subgroups of $\mathbb{R}^*$. *Hint: find all elements of finite order.*
9. Give an example of a group G and two elements x, y in G such that

$$\mathrm{Ord}(xy) \neq \mathrm{LCM}(\mathrm{Ord}(x), \mathrm{Ord}(y)).$$

10**. Give an example of a group G and two elements x, y in G such that x and y have finite orders, but xy has infinite order.