1. Let $G$ be a group of order 25. Prove that $G$ has at least one subgroup of order 5, and that if it contains only one subgroup of order 5 then it is a cyclic group.

2. Is it possible to construct an injective homomorphism (a) from $\mathbb{Z}_3$ to $\mathbb{Z}_4$? (b) From $S_3$ to $S_4$?

3. A finite group $G$ contains an element $x$ of order 10 and also an element $y$ of order 6. What can be said about the order of $G$?

4. Let $\varphi : G_1 \to G_2$ be a group homomorphism. Suppose that $|G_1| = 18, |G_2| = 15$ and that $\varphi$ is not the trivial homomorphism. What is the order of the kernel?