

MAT 150C, Spring 2021  
Practice problems for Midterm 1

**This sheet contains more problems than the actual midterm.**

1. Suppose that  $\rho : G \rightarrow GL(V)$  is a representation of  $G$  and  $f : H \rightarrow G$  is a group homomorphism. Define

$$\rho' : H \rightarrow GL(V), \rho'(h) = \rho(f(h)).$$

- a) Prove that  $\rho'$  is a representation of  $H$ .
  - b) Prove that if  $\rho'$  is irreducible then  $\rho$  is irreducible.
  - c) Give an example where  $\rho$  is irreducible but  $\rho'$  is not.
2. Describe all irreducible representations and compute their characters for the groups (a)  $\mathbb{Z}_4$  (b)  $\mathbb{Z}_2 \times \mathbb{Z}_2$ .
3. Describe all 2-dimensional representations of the group  $S_3$  and compute their characters.
4. The action of  $D_n$  on diagonals of the regular  $n$ -gon defines a representation of  $D_n$ . Compute the character of this representation for (a)  $n = 4$  (b)  $n = 5$  (c)  $n = 6$ . In all these cases, find the multiplicity of the trivial representation.
5. For some representation of an 8-element group the character values are 7,6,5,0,0,0,0,0. Is it possible? *Hint: compute  $\langle \chi, \chi \rangle$ .*
- 6\*. Let  $\rho : G \rightarrow GL(n)$  be a representation of a finite group  $G$ . Prove that  $|\chi_\rho(g)| \leq n$  for all  $g$  in  $G$ .