MAT 150C, Spring 2021
Practice problems for Midterm 2

This practice sheet contains more problems than the actual exam.

1. Find the minimal polynomial for $\sqrt{2}+\sqrt{3}$ over $\mathbb{Q}$.

2. Find all irreducible polynomials over $\mathbb{Z}_2$ of degrees (a) 1 (b) 2 (c) 3 (d) 4.

3. Prove that the polynomial x^3-3x+1 is irreducible over $\mathbb{Q}$.

4. Decompose the polynomial x^4+1 into irreducible factors over (a) $\mathbb{Q}$ (b) $\mathbb{Q}[\sqrt{2}]$ (c) $\mathbb{Q}[i]$.

5. Compute $\frac{1}{1+\sqrt[3]{2}}$ in the field $\mathbb{Q}[\sqrt[3]{2}]$.

6. Let $\chi_n : SU(2) \to \mathbb{C}$ be the character of the $(n+1)$ dimensional irreducible representation V_n.
(a) Prove that $f(A) = \chi_1(A) \cdot \chi_3(A)$ can be written as a sum of χ_i with nonnegative integer coefficients.

(b) Construct a representation of $SU(2)$ with character $f(A)$.

7*. Suppose that $p > 2$ is a prime number, consider the field $F = \mathbb{Z}_p$.

(a) Consider the function $f : F \to F$, $f(x) = x^2$. Prove that every nonzero element has either 0 or 2 preimages.

(b) Use (a) to compute the number of complete squares in F

(c) Use (b) to prove that for some $a \in F$ the polynomial $x^2 - a$ is irreducible.

(d) Prove that there exists a field with p^2 elements.