MAT 150C, Spring 2021 Homework 1

Due before 12:10 on Monday, April 5

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Consider the cyclic group $G_n = \langle x | x^n = 1 \rangle$.

a) Describe all one-dimensional complex representations of G_n .

b) Prove that every complex representation of G_n has a one-dimensional invariant subspace.

2. a) Prove that there is a two-dimensional representation of G_4 such that

$$x \mapsto \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

b) Find all invariant subspaces for the corresponding **real** representation.

c) Find all invariant subspaces for the corresponding **complex** representation.

3. Consider the standard two-dimensional representation of the dihedral group D_n . For which n is this an irreducible **complex** representation.