MAT 150C, Spring 2021
Homework 2
Due before 12:10 on Monday, April 12

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. a) Prove that for odd n all reflections in the dihedral group D_n are conjugate to each other.
b) Prove that for even n there are exactly two conjugacy classes of reflections in D_n.

2. Use problem 1 to describe all 1-dimensional representations of D_n.

3. Recall that the **averaging** operator for a representation $\rho : G \to GL(n)$ is defined as

$$\mathrm{Av}_G = \frac{1}{|G|} \sum_{g \in G} \rho(g).$$

Compute $\mathrm{Av}_{S_3}(v)$ where $v = (x_1, x_2, x_3)$ is a vector in the 3-dimensional permutation representation of S_3.

4. Prove that for any $n > 1$ the sum of all complex roots of unity of degree n equals 0. *Hint: Use a one-dimensional representation of the cyclic group of order n.*