

MAT 150C, Spring 2021  
Homework 6

**Due before 12:10 on Monday, May 24**

*Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.*

1. Suppose that  $a$  is not a square in the field  $F$ . Prove that there is an automorphism  $\sigma$  of the field  $F[\sqrt{a}]$  such that

$$\sigma(x + y\sqrt{a}) = x - y\sqrt{a} \text{ for all } x, y \in F$$

2. Suppose that  $\sigma(z) = z$  for some  $z \in F[\sqrt{a}]$  and  $\sigma$  as in Problem 1. Prove that  $z$  is an element of  $F$ .
3. Prove that  $(2 - \sqrt{3})^n + (2 + \sqrt{3})^n$  is an integer for all  $n$ . *Hint: use the automorphism  $\sigma$  of  $\mathbb{Q}[\sqrt{3}]$  from Problems 1 and 2.*
4. Prove that the distance between  $(2 + \sqrt{3})^n$  and the nearest integer becomes arbitrary small for  $n \rightarrow \infty$ .