

MAT 150C, Spring 2021
Homework 7

Due before 12:10 on Friday, June 4

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Suppose that $\cos(\alpha)$ is a constructible number. Prove that $\sin(\alpha)$ is constructible.
2. Suppose that $\cos(\alpha)$ is a constructible number. Prove that $\cos(2\alpha)$ and $\cos(\frac{\alpha}{2})$ are constructible.
3. Let $z = e^{\frac{2\pi i}{5}}$ be the fifth root of unity, and $x = \cos(\frac{2\pi}{5})$.
 - a) Prove that $z + z^4 = 2x$, and $z^2 + z^3 = 2(2x^2 - 1)$.
 - b) Use the equation $1 + z + z^2 + z^3 + z^4 = 0$ and part (a) to find an algebraic equation for x . Solve it and find an explicit formula for x .
4. Use problem 3 to construct a regular pentagon by ruler and compass.