MAT 17A, practice problems for Midterm 2

1. Find the derivatives of the following functions:
   a) $f(x) = \frac{\sin x}{\ln x}$
   b) $f(x) = xe^{\cos x}$
   c) $f(x) = e^{\ln(2+x)-\ln(1+x)}$.
   d) $f(x) = (x+1)\sqrt[3]{x}$
   e) $f(x) = \sqrt{\frac{x-1}{x+1}}$.
   f) $f(x) = (x^2+1)\arctan(x)$
2. Find the derivative of $y(x)$ using implicit differentiation, if
   a) $3x^2 + 2y^2 = 10$
   b) $\cos(x) + \cos(y) = 0.5$
   c) $\frac{x}{y} - \frac{y}{x} = 1$
3. Find the equation of the tangent line to the graph of $f(x) = x^4 e^{-x}$ at a point $(1, e^{-1})$.
4. Show that the function $f(x) = x + \sin(x)$ is increasing everywhere by computing $f'(x)$.
5. A radioactive material has half-life time of 100 days. Find the formula for its mass $m(t)$ after t days, if $m(0) = 50$ gram.
6. The temperature $T(x)$ (Celsius) of the cup of coffee after x minutes is given by the formula
$$T(x) = 20 + 60e^{-3x}$$
a) What was the initial temperature $T(0)$?
b) What is the temperature of the room?
c) Find the derivative $T'(x)$. Is the temperature increasing or decreasing?
7. Use linear approximation to estimate $\sqrt[3]{8.1}$.