MAT 17A Fall 2023
Solutions to homework 5

Find the derivatives of the following functions:

1. (10 points)

   \[ f(x) = \sqrt{\frac{x}{x^2 + 4}} \]

   **Solution:** First, we compute the derivative of the inside function using the Quotient Rule:
   \[
   \left( \frac{x}{x^2 + 4} \right)' = \frac{1(x^2 + 4) - x(2x)}{(x^2 + 4)^2} = \frac{x^2 + 4 - 2x^2}{(x^2 + 4)^2} = \frac{4 - x^2}{(x^2 + 4)^2}.
   \]
   Now we apply Chain Rule
   \[
   \left( \sqrt{\frac{x}{x^2 + 4}} \right)' = \left( \left( \frac{x}{x^2 + 4} \right)^{1/2} \right)' = \frac{1}{2} \left( \frac{x}{x^2 + 4} \right)^{-1/2} \cdot \frac{4 - x^2}{(x^2 + 4)^2}.
   \]

2. (10 points)

   \[ f(x) = \frac{\ln x}{x^2} \]

   **Solution:** We apply Quotient Rule:
   \[
   \left( \frac{\ln x}{x^2} \right)' = \frac{\frac{1}{x} \cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x \ln x}{x^4} = \frac{1 - 2 \ln x}{x^3}.
   \]

3. (10 points) A curve is given by the equation \(2x^3 + x^2 y - xy^3 = 2\). Find the equation of the tangent line to this curve at the point (1, 1).

   **Solution:** We take the derivatives of the both sides of the equation:
   \[
   6x^2 + 2xy + x^2 y' - 1 \cdot y^3 - x \cdot 3y^2 y' = 0.
   \]
   Next we solve for \( y' \):
   \[
   6x^2 + 2xy - y^3 = y'(-x^2 + 3xy^2), \quad y' = \frac{6x^2 + 2xy - y^3}{-x^2 + 3xy^2}.
   \]
   At \( x = y = 1 \) we get \( y' = \frac{6+2-1}{1+3} = \frac{7}{2} \). Therefore the slope of the tangent line equals \( 7/2 \) and the equation of the tangent line is
   \[ y = \frac{7}{2}(x - 1) + 1. \]