MAT 17A Fall 2023
Solutions to homework 5

Find the derivatives of the following functions:

1. (10 points)

$$f(x) = \sqrt{\frac{x}{x^2+4}}$$

Solution: First, we conpute the derivative of the inside function using the Quotient Rule:

$$\left(\frac{x}{x^2+4}\right)' = \frac{1(x^2+4) - x(2x)}{(x^2+4)^2} = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}.$$

Now we apply Chain Rule

$$\left(\sqrt{\frac{x}{x^2+4}}\right)' = \left(\left(\frac{x}{x^2+4}\right)^{1/2}\right)' = \frac{1}{2}\left(\frac{x}{x^2+4}\right)^{-1/2} \cdot \frac{4-x^2}{(x^2+4)^2}$$

2. (10 points)

$$f(x) = \frac{\ln x}{x^2}$$

Solution: We apply Quotient Rule:

$$\left(\frac{\ln x}{x^2}\right)' = \frac{\frac{1}{x}\cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x\ln x}{x^4} = \frac{1-2\ln x}{x^3}.$$

3. (10 points) A curve is given by the equation $2x^3 + x^2y - xy^3 = 2$. Find the equation of the tangent line to this curve at the point $(1, 1)$.

Solution: We take the derivatives of the both sides of the equation:

$$6x^2 + 2xy + x^2y' - 1\cdot y^3 - x\cdot 3y^2y' = 0.$$

Next we solve for y':

$$6x^2 + 2xy - y^3 = y'(-x^2+3xy^2), \quad y' = \frac{6x^2+2xy-y^3}{-x^2+3xy^2}.$$

At $x = y = 1$ we get $y' = \frac{6+2-1}{-1+3} = \frac{7}{2}$. Therefore the slope of the tangent line equals $7/2$ and the equation of the tangent line is

$$y = \frac{7}{2}(x-1) + 1.$$