MAT 17A Fall 2023
Solutions to homework 6

1. (10 points) Find the derivative of \( f(x) = (\arcsin(x))^5 \).
   
   **Solution:** By Chain Rule we have
   
   \[
   f'(x) = 5(\arcsin(x))^4 \cdot (\arcsin(x))' = 5(\arcsin(x))^4 \cdot \frac{1}{\sqrt{1-x^2}}.
   \]

2. (10 points) When a cold drink is taken from a refrigerator, its temperature is 5° C. After 25 minutes in a 20° C room its temperature has increased to 10° C. What is the temperature of the drink after 50 minutes?
   
   **Solution:** Let \( T(x) \) be the temperature after \( x \) minutes, then \( T(x) = T_s + Ae^{-kx} \). Here \( T_s = 20 \) is the room temperature, and \( T(0) = 5 = T_s + A = 20 + A \), so \( A = -15 \). We get \( T(x) = 20 - 15e^{-kx} \). To find \( k \), we plug in \( x = 25 \) and get
   
   \[
   10 = T(25) = 20 - 15e^{-25k}, \quad 15e^{-25k} = 20 - 10 = 10,
   \]
   
   \[
   e^{-25k} = \frac{10}{15} = \frac{2}{3}, \quad -25k = \ln(2/3),
   \]
   
   and \( k = -\frac{1}{25} \ln(2/3) \). Now \( -50k = \frac{50}{25} \ln(2/3) = 2 \ln(2/3) \), so
   
   \[
   e^{-50k} = e^{2\ln(2/3)} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}
   \]
   
   Finally,
   
   \[
   T(50) = 20 - 15e^{-50k} = 20 - 15 \cdot \frac{4}{9} = 20 - \frac{60}{9} = \frac{120}{9} = \frac{40}{3}.
   \]

   **Answer:** \( \frac{40}{3} \approx 13.3 \).

3. (10 points) Use linear approximation to estimate \( \arctan(0.1) \).
   
   **Solution:** We have \( f(x) = \arctan(x) \) and \( f'(x) = \frac{1}{1+x^2} \), so \( f(0) = 0 \) and \( f'(0) = 1 \). Therefore \( f(x) \approx 0 + 1(x - 0) = x \) for \( x \) close to 0, and \( \arctan(0.1) \approx 0.1 \).