

MAT 17A Fall 2023
Solutions to homework 6

1. (10 points) Find the derivative of $f(x) = (\arcsin(x))^5$.

Solution: By Chain Rule we have

$$f'(x) = 5(\arcsin(x))^4 \cdot (\arcsin(x))' = 5(\arcsin(x))^4 \cdot \frac{1}{\sqrt{1-x^2}}.$$

2. (10 points) When a cold drink is taken from a refrigerator, its temperature is $5^\circ C$. After 25 minutes in a $20^\circ C$ room its temperature has increased to $10^\circ C$. What is the temperature of the drink after 50 minutes?

Solution: Let $T(x)$ be the temperature after x minutes, then $T(x) = T_s + Ae^{-kx}$. Here $T_s = 20$ is the room temperature, and $T(0) = 5 = T_s + A = 20 + A$, so $A = -15$. We get $T(x) = 20 - 15e^{-kx}$. To find k , we plug in $x = 25$ and get

$$10 = T(25) = 20 - 15e^{-25k}, \quad 15e^{-25k} = 20 - 10 = 10,$$

$$e^{-25k} = \frac{10}{15} = \frac{2}{3}, \quad -25k = \ln(2/3),$$

and $k = -\frac{1}{25} \ln(2/3)$. Now $-50k = \frac{50}{25} \ln(2/3) = 2 \ln(2/3)$, so

$$e^{-50k} = e^{2 \ln(2/3)} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Finally,

$$T(50) = 20 - 15e^{-50k} = 20 - 15 \cdot \frac{4}{9} = 20 - \frac{60}{9} = \frac{120}{9} = \frac{40}{3}.$$

Answer: $\frac{40}{3} \approx 13.3$.

3. (10 points) Use linear approximation to estimate $\arctan(0.1)$.

Solution: We have $f(x) = \arctan(x)$ and $f'(x) = \frac{1}{1+x^2}$, so $f(0) = 0$ and $f'(0) = 1$. Therefore $f(x) \approx 0 + 1(x - 0) = x$ for x close to 0, and $\arctan(0.1) \approx 0.1$.