

MAT 17A, practice problems for Midterm 2

- Find the derivatives of the following functions:
 - $f(x) = \frac{\sin x}{\ln x}$
 - $f(x) = xe^{\cos x}$
 - $f(x) = e^{\ln(2+x) - \ln(1+x)}$
 - $f(x) = (x+1)\sqrt[3]{x}$
 - $f(x) = \sqrt{\frac{x-1}{x+1}}$
 - $f(x) = (x^2 + 1) \arctan(x)$
- Find the derivative of $y(x)$ using implicit differentiation, if
 - $3x^2 + 2y^2 = 10$
 - $\cos(x) + \cos(y) = 0.5$
 - $\frac{x}{y} - \frac{y}{x} = 1$
- Find the equation of the tangent line to the graph of $f(x) = x^4e^{-x}$ at a point $(1, e^{-1})$.
- After an oil spill, a round spot of oil is growing on the surface of the ocean. Today the radius of the oil spot is 1km and the rate of growth of the radius is 0.1 km/day. What is the rate of growth for the area of the spot?
- A radioactive material has half-life time of 100 days. Find the formula for its mass $m(t)$ after t days, if $m(0) = 50$ gram.
- The temperature $T(x)$ (Celsius) of the cup of coffee after x minutes is given by the formula
$$T(x) = 20 + 60e^{-3x}$$
 - What was the initial temperature $T(0)$?
 - What is the temperature of the room?
 - Find the derivative $T'(x)$. Is the temperature increasing or decreasing?
- Use linear approximation to estimate $\sqrt[3]{8.1}$.
- Show that the function $f(x) = x + \sin(x)$ is increasing everywhere by computing $f'(x)$.