MAT 215A, Winter 2023
Homework 2
Due before 10:00 on Wednesday, January 25

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. Prove that the Möbius band is homotopy equivalent to a circle.

2. Let G be a connected graph with V vertices and E edges. Prove that G is homotopy equivalent to a graph with 1 vertex and $E - V + 1$ edges.

3. Prove that homotopy equivalence is indeed an equivalence relation on topological spaces. *You can use without proof any properties of homotopies discussed in class, provided that you state them explicitly and clearly.*

4. Prove that the following spaces are pariwise homotopy equivalent:
a) S^2 with North and South poles identified.
b) The union of S^2 with a vertical segment connecting North and South poles.
c) The union of the torus T^2 with a meridional disk.
d) The union of the torus T^2 with a longitudinal disk.